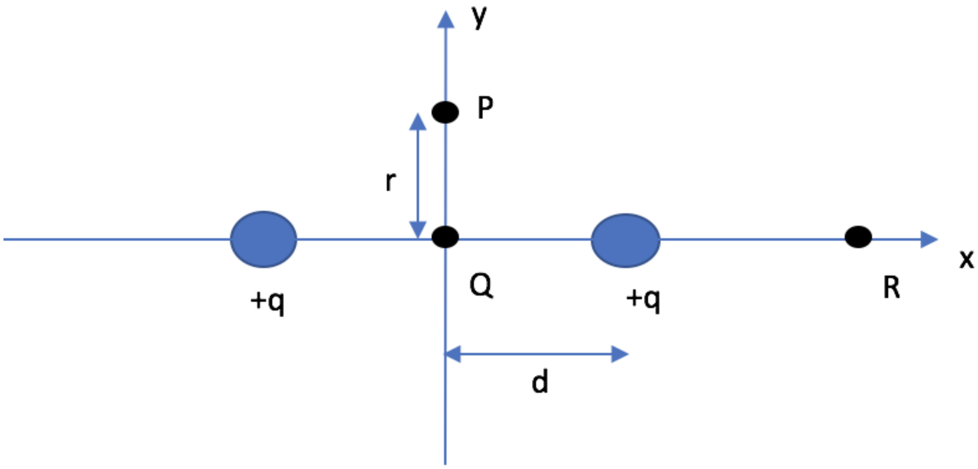
The next three questions pertain to the situation described below.

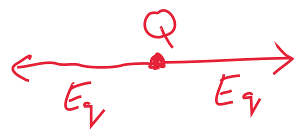
Consider the collection of two charges separated in the x-axis by a distance 2d:



1) Where should we place a negatively charged test particle so that it is not subject to a force?

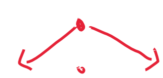
At point Q, the net electric field is zero

a. Point R
b. Point Q
c. Point P



2) Now place the negative test charge at point Q. Which statement below is correct?

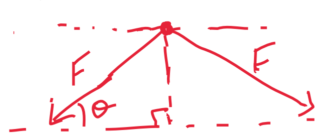
Displace charge slightly in +y direction; net force is in the -y direction



a. The test charge is stable to a slight perturbation in the x direction
b. The test charge is stable to a slight perturbation in the y direction
c. The test charge is stable to a slight perturbation in both x and y directions

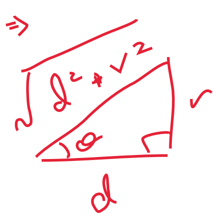
3) Write an expression for the force on a negatively charged test particle of charge -q at location P. ($\widehat{x}$ and $\hat{y}$ are unit vectors in the x and y directions.)

At P ⇒



by Coulomb's law ⇒

$$|F| = \frac{Kq^2}{d^2+r^2}$$



a. $-\frac{2kq^2}{(d^2+r^2)}$

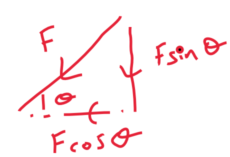
b. $-\frac{2kq^2 r}{(d^2+r^2)^{3/2}}\hat{y}$

c. $-\frac{2kq^2 d}{(d^2+r^2)^{3/2}}\hat{y}$

d. $-\frac{2kq^2 dr}{(d^2+r^2)^2}\widehat{x}$

e. $\frac{2kq^2 r}{(d^2+r^2)^{3/2}}\widehat{x}$

Note that the horizontal components cancel
Vertical components add and point in $-\hat{y}$ direction

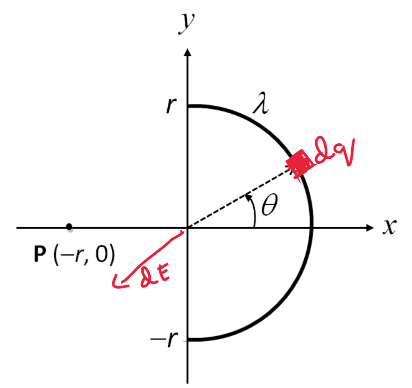


Net force $= -2F\sin\theta\,\hat{y}$

$$= -\frac{2Kq^2}{d^2+r^2}\,\frac{r\,\hat{y}}{(d^2+r^2)^{1/2}} = -\frac{2Krq^2}{(d^2+r^2)^{3/2}}\hat{y}$$

The next three questions pertain to the situation described below.

A uniformly charged semi-circular line with linear charge density λ is placed in the x-y plane as shown in the Figure on the right. Its radius is r and its total charge Q = 15 μC.



length of the line $= \frac{\cancel{2}\pi r}{\cancel{2}}$

so $\lambda = \dfrac{Q}{\pi r} = \dfrac{15\mu C}{\pi (5/100)m}$ ← pay attention to the units

$\lambda \simeq 95 \mu C/m$

4) If r = 5 cm, what is λ?

a. λ = 48 μC/m
b. λ = 95 μC/m
c. λ = 150 μC/m

5) What is the correct expression for the x-component of the total electric field at the origin, E_x, due to this charge?

Consider a charge element dq at position θ shown above

$dq = \lambda r d\theta \rightarrow dE_x = -\dfrac{K\,dq}{r^2}\cos\theta$

$dE_x = -\dfrac{K\lambda}{r}\cos\theta\, d\theta$

$E_x = -\displaystyle\int_{-\pi/2}^{\pi/2} \frac{K\lambda}{r}\cos\theta\, d\theta$

a. $E_x = -\int_{-\pi/2}^{\pi/2} \frac{k\lambda}{r^2} cos\theta\, d\theta$
b. $E_x = -\int_{-\pi/2}^{\pi/2} \frac{k\lambda}{r} cos\theta sin\theta\, d\theta$
c. $E_x = 0$
d. $E_x = -\int_{-\pi/2}^{\pi/2} \frac{k\lambda}{r} sin\theta\, d\theta$
e. $E_x = -\int_{-\pi/2}^{\pi/2} \frac{k\lambda}{r} cos\theta\, d\theta$

6) What kind of point charge should be placed at point **P**(-r, 0) in order to make the net electric field at the origin vanish?

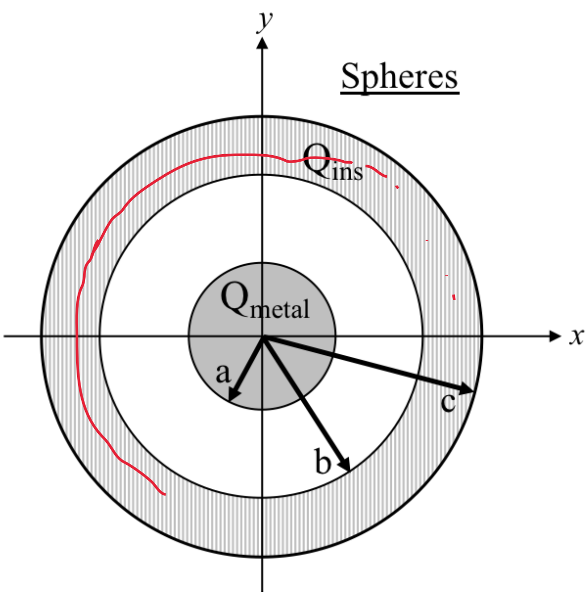
Note that $E_y = -\displaystyle\int_{-\pi/2}^{\pi/2} \frac{K\lambda}{r}\sin\theta\, d\theta = 0$

a. There is no such charge
b. Positive charge
c. Negative Charge

The electric field above is only along the x-direction and since $\lambda > 0$, E_x points in the negative x-direction. So we need to place a positive charge at point P to make net electric field to be zero.

The next four questions pertain to the situation described below.

A metal conducting sphere of radius a is centered on the origin. Concentric with it is a spherical shell made of insulating material of inner radius b and outer radius c. A total positive charge Q_{metal} is placed on the inner metal sphere, while a total negative charge Q_{insul} is uniformly distributed over the volume of the outer insulating sphere. The values of all parameters are given in the figure below. The figure is not drawn to scale.



a = 5 cm
b = 9 cm
c = 17 cm
Q_{metal} = 6 μC
Q_{insul} = -4 μC

7) Calculate the surface charge density on the inner metal sphere at a = 5 cm

a. σ = 190 μC/m^2
b. $\sigma = 2 \times 10^4$ μC/m^2
c. σ = 19 μC/m^2

$$\sigma = \frac{Q_{metal}}{4\pi a^2} = \frac{6\,\mu C}{4\pi (5/100)^2} = 190\,\mu C/m^2$$

8) Calculate the magnitude of the electric field E at a radius of 7 cm from the origin.

a. |E| = 1.1×10^7 N/C
b. |E| = 1.8×10^7 N/C
c. |E| = 7.7×10^5 N/C

at $r < b$, Use Gauss' law, $\oint \vec{E} \cdot d\vec{A} = Q_{metal}/\varepsilon_0$

$$|E| = \frac{Q_{metal}}{4\pi\varepsilon_0 r^2} = \frac{6 \times 10^{-6}}{4\pi (8.854 \times 10^{-12})(7 \times 10^{-2})^2}$$

9) Calculate the magnitude of the electric field E at a radius of 13 cm from the origin.

Note that 13 cm is for $b < r < c$

a. |E| = 8.2×10^6 N/C
b. |E| = 7.4×10^7 N/C
c. |E| = 2.4×10^6 N/C
d. |E| = 0 N/C
e. |E| = 5.3×10^6 N/C

$$|E| = \frac{Q_{enclosed}}{4\pi\varepsilon_0 r^2}$$

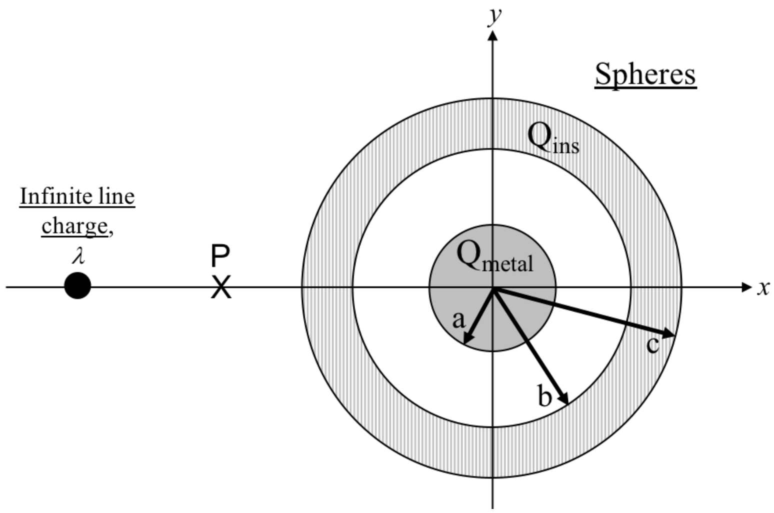
where $Q_{enclosed} = Q_{metal} + \rho V$

Note $V = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi b^3 = 6.15 \times 10^{-3} m^3$

and $\rho = \dfrac{Q_{ins}}{\frac{4}{3}\pi c^3 - \frac{4}{3}\pi b^3} = -228\,\mu C/m^3$

$$|E| = \frac{(6 + (-228)(6.15 \times 10^{-3}))\,10^{-6}}{4\pi\varepsilon_0 (13/100)^2}$$

10) An infinite line charge having $\lambda = 4\ \mu C/m$ is now added parallel to the z-axis and centered at (x,y) = (-30 cm, 0), as shown. Find the magnitude of electric field at a point P located at (x,y) = (-21.5 cm,0). (The figure is not drawn to scale. Here we assume the line charge does not affect the surface charge distribution on the metal sphere.)

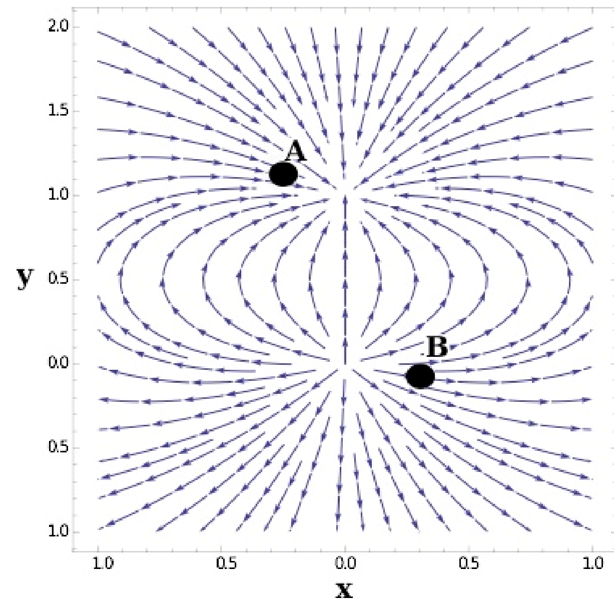


a = 5 cm
b = 9 cm
c = 17 cm
$Q_{metal} = 6\ \mu C$
$Q_{insul} = -4\ \mu C$

a. $|E| = 7.2 \times 10^6$ N/C
b. $|E| = 8.5 \times 10^5$ N/C
c. $|E| = 1.9 \times 10^6$ N/C
d. $|E| = 1.9 \times 10^5$ N/C
e. $|E| = 4.6 \times 10^5$ N/C

The next three questions pertain to the situation described below.

The electric field due to two point charges is given as a cross sectional view in the x-y plane.There are two points labeled A and B on the graph.



11) What is the position of the point charges?

a. A positive charge at (1,0) and a negative charge at (0,0)
b. A positive charge at (0,0) and a negative charge at (0,1)
c. A positive charge at (0,1) and a negative charge at (0,0)

12) Which of these lines is an equipotential?

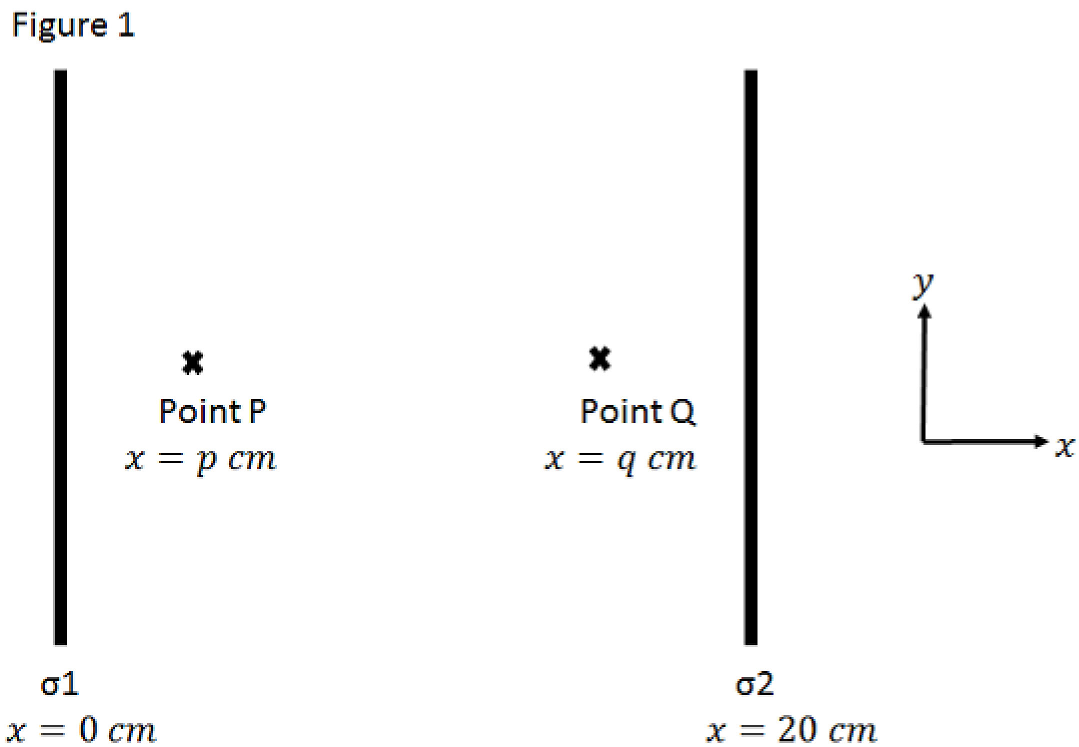
a. y=0.5
b. y=0
c. x=0.5
d. x=1
e. x=0

13) What is the relationship between the potential at A and B?

a. $V_A > V_B$
b. $V_A < V_B$
c. $V_A = V_B$

The next three questions pertain to the situation described below.

As seen in Figure 1, two charged, infinite plates have charge densities of $\sigma_1 = 5\ \mu C/m^2$ and $\sigma_2 = 9\ \mu C/m^2$ and are placed at x = 0 cm and x = 20 cm respectively.

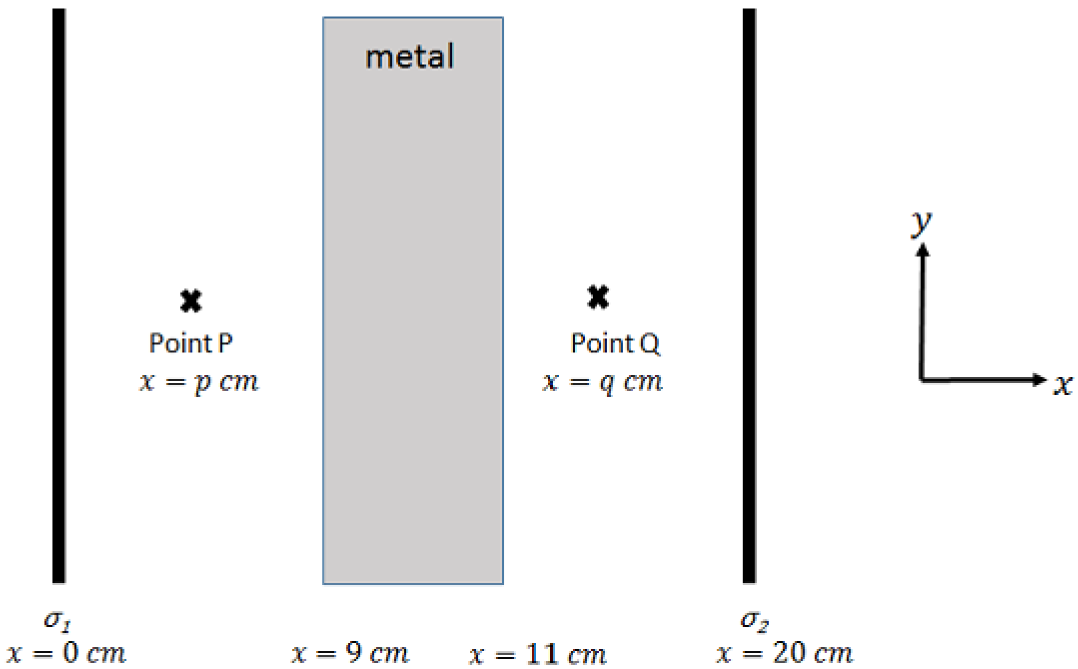


14) What is the potential difference between point Q ,(q cm , 0 cm) and point P, (p cm , 0 cm)? Let q = 17 cm and p = 2 cm.

a. $\Delta V = 84$ kV
b. $\Delta V = 120$ kV
c. $\Delta V = 34$ kV
d. $\Delta V = 280$ kV
e. $\Delta V = 230$ kV

15) For the next two problems, consider Figure 2.

Figure 2



A slab of metal with no net charge(shaded region; left edge at x =9 cm, right edge at x = 11cm) is now placed between the two charged, infinite plates.

What are the surface charge densities for the left side of the slab (given by σ_L) and the right side of the slab (given by σ_R)?

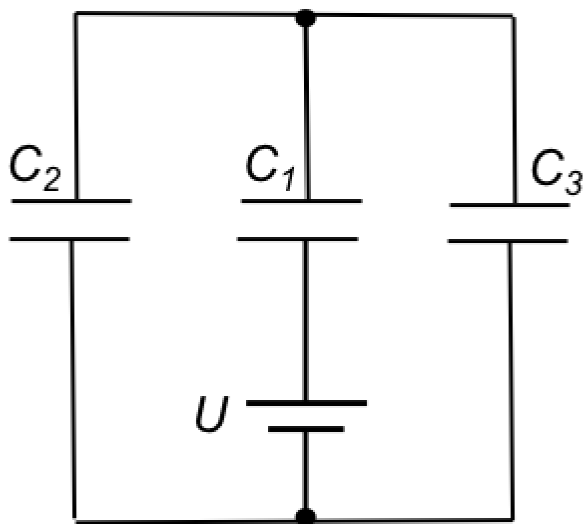
a. σ_L = -7 $\mu C/m^2$, σ_R =7 $\mu C/m^2$
b. σ_L = -2 $\mu C/m^2$, σ_R =2 $\mu C/m^2$
c. σ_L = 2 $\mu C/m^2$, σ_R =-2 $\mu C/m^2$
d. σ_L = 0 $\mu C/m^2$, σ_R = 0 $\mu C/m^2$
e. σ_L = 7 $\mu C/m^2$, σ_R =-7 $\mu C/m^2$

16) With the addition of the metal slab, what is the potential difference between point Q and point P?

a. ΔV = 100 kV
b. ΔV = 34 kV
c. ΔV = 0 kV
d. ΔV = 29 kV
e. ΔV = 120 kV

The next two questions pertain to the situation described below.

Three capacitors and an ideal battery are connected as shown in the figure below. The voltage of the battery is U= 9 V. The capacitances of the capacitors are $C_1 = 4\ \mu F$, $C_2 = 4\ \mu F$, and $C_3 = 6\ \mu F$.



17) Capacitors C_2 and C_3 are connected _______

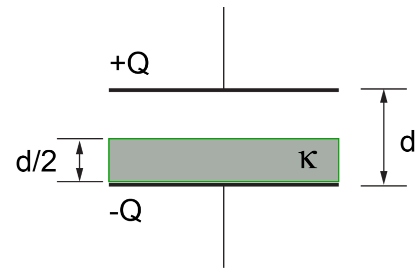
a. in series
b. in parallel
c. neither in parallel nor in series

18) What is the total energy stored in all three capacitors?

a. 120 μJ
b. 570 μJ
c. 26 μJ
d. 230 μJ
e. 1100 μJ

The next two questions pertain to the situation described below.

A parallel plate capacitor is constructed with a dielectric slab with $\kappa = 1.7$ inserted between the plates. The area of each plate is 6 cm^2, and the distance between the two plates is 2 mm. Assume the infinite plane approximation.



19) If we fix the charge on the capacitor to be $Q = 6\times10^{-11}$ C, what is the potential difference between the top and bottom plates? Note that in the region containing the dielectric medium, the electric field is $E = E_0 / \kappa$, where E_0 is the electric field in vacuum.

a. $\Delta V = 18$ V
b. $\Delta V = 6.6$ V
c. $\Delta V = 11$ V
d. $\Delta V = 23$ V
e. $\Delta V = 4.2$ V

20) If the dielectric slab is removed, the capacitance of the capacitor will

a. remain the same
b. decrease
c. increase