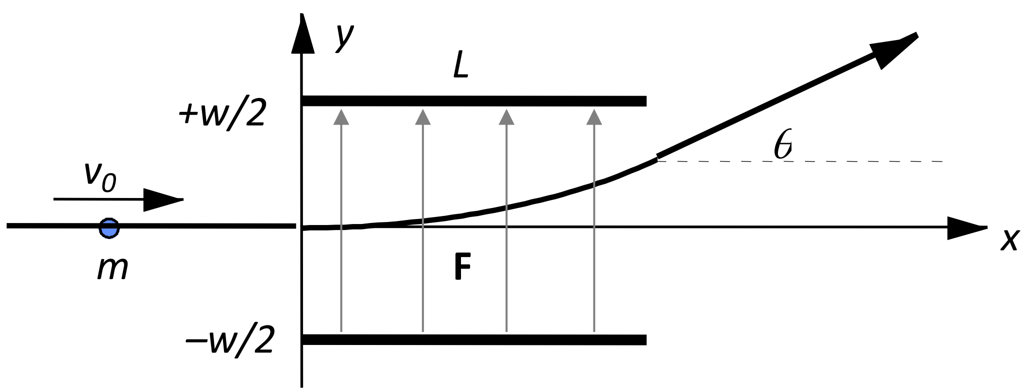
Discussion Problem 1A
Physics 212, Week 1
Physics 211 Review: 2-D Motion with Uniform Force

The mathematics and physics of the problem below are similar to problems you will encounter in P212, where the force is due to the action of an electric field on a charged particle.

A point particle of mass *m* travels freely in the *x*-direction with uniform velocity v_0. At $x = 0$, it enters a region between two plates oriented perpendicular to the *y*-axis and extended to infinity along the z-axis; the plate spacing is *w*, and the plate length in the *x*-direction is *L*. The particle enters on the mid-plane $y = 0$. While between the plates, it experiences a constant, spatially uniform force *F* in the +*y*-direction. After exiting the plates the particle again moves freely.



(a) Our first task will be to obtain an expression for the ***y*-coordinate** of the point at which the particle **exits** the plates. We will assume that the plate spacing is wide enough that the particle never strikes either plate. But before we start, consider these possible solutions:

(1) $y = \dfrac{F}{m(v_0 + L)}$ (2) $y = \dfrac{FL}{mv_0}$ (3) $y = \dfrac{1}{2}\dfrac{F}{m}\left(\dfrac{L}{v_0}\right)^2$

Could any of them be correct? Why or why not? Remember, **units** and **limiting behavior**! In fact, from *only* those two considerations, you can write down the correct answer to within a factor of 2 without using any formulas at all. (This procedure is called *dimensional analysis* and physicists use it all the time, especially when developing new theories.)

(b) Derive the correct expression for y.

(c) Suppose the initial velocity had a non-zero component in the z-direction; that is, suppose $\vec{v}_0 = v_0\,\hat{x} + v_z\,\hat{z}$. How will this change in velocity alter your calculation in part (b)?

In parts (d) and (e), assume again that the initial velocity is $\vec{v}_0 = v_0\,\hat{x}$.

(d) Find an expression for the **maximum value** of the force *F* for which the particle passes through the force region without striking either plate.

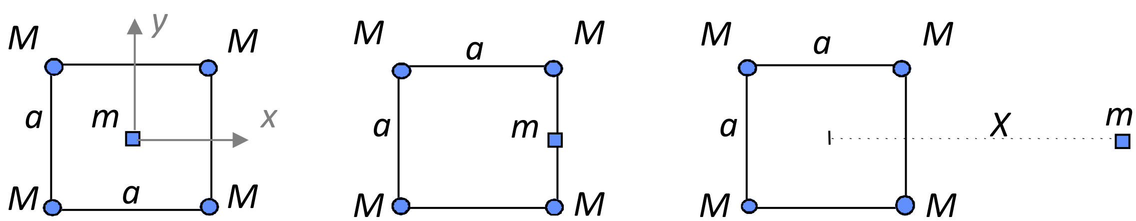
(e) For the maximum force found in part (d), find an expression for $\tan\theta$ where θ is the **angle of deflection** at which the particle exits the force region. (Did you draw a sketch? Did you check your units?)

Discussion Problem 1B
Physics 212, Week 1
Physics 211 Review: Gravitational Forces and Superposition

$$\mathbf{F}_{1\to 2} = -G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12}\,, \quad 1 \xrightarrow{\quad\mathbf{r}_{12}\quad} 2 \qquad \xrightarrow{\hat{\mathbf{r}}_{12}}$$

This problem develops skills you will need in P212 in finding the electric fields created by sets of point charges.

Four particles of equal mass M are fixed at the corners of a square with sides of length a. A fifth particle has mass m and moves under the gravitational forces of the other four.



(a) Find the ***x*- and *y*-components** of the net gravitational force on m due to the other four masses when m is located at the center of the square (left-hand figure).

> Hint: <u>**Draw a sketch!**</u> Use superposition and draw a *vector diagram* consisting of four vectors, each representing the force exerted by one of the corner particles on m. For ease of reference, label the four (equal) corner masses M_1, M_2, M_3, and M_4. Label the corresponding force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, etc. With the vector diagram in hand, it is vastly easier to calculate the requested components of the total force.

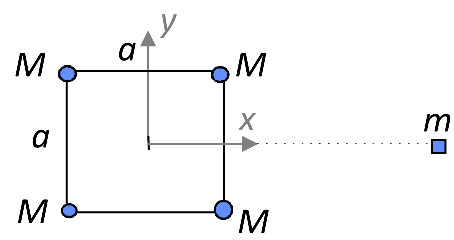
(b) Symmetry principles play an important role in studying electricity and magnetism. Suppose the configuration in part (a) is generalized to a system consisting of n point particles of mass M located at the vertices of a regular n-gon and a point particle of mass m located at the center of the n-gon. Name a few geometric symmetries present in this system. Use a symmetry argument to deduce the net gravitational force on m.

For parts (c) and (d), assume the square configuration again.

(c) Find the ***x*- and *y*-components** of the net gravitational force on m when it is located at the center point of the right-hand side of the square (middle figure). Use the same solution procedure that was recommended above for part (a).

(d) When mass m is located on the x-axis at distance X, large compared to a (right-hand figure), one can use a *simple physical argument* to see that the net force on m due to the other four particles is approximately $F_x = CGMmX^{-2}$ and $F_y = 0$ where C is a numerical constant. (This is a long-distance approximation: it gets better and better as X increases.) What is the applicable **physical argument**? Use it to find the value of the dimensionless **constant** C.

Discussion Problem 1C
Physics 212, Week 1
Physics 211 Review: Potential Energy



Four particles of equal mass M are fixed at the corners of a square, centered at the origin and with sides of length a. A fifth particle has mass m and moves under the gravitational forces of the other four.

Consider the gravitational potential energy U of the mass m as a function of its position (x,y). This *potential energy map* $U(x,y)$ is an extremely useful way of representing the effect of the gravitational force on the mass m:

- If m is held at rest at some point (x_1,y_1) and you let it go, it will always move toward a point of *lower* potential energy. You can therefore think of the function $U(x,y)$ as a topographical map: the particle m will always **"roll downhill"**.
- If the mass m moves from point (x_1,y_1) to point (x_2,y_2), the **potential energy difference** $U(x_1,y_1) - U(x_2,y_2)$ tells you exactly how much work was done by gravity, and exactly how much kinetic energy the particle gained as a result.

The master formula for the gravitational potential energy between two masses m_1 and m_2 separated by a distance r_{12} is given by:

$$U_{12} = -G\frac{m_1 m_2}{r_{12}}$$

(a) Using the above formula, compute net **gravitational potential energy** of particle m when it's located at infinity and at the origin.

Hint: The equation above gives the gravitational potential energy of mass m_1 in the presence of mass m_2, or vice versa. Use it and superposition to find the net potential energy of mass m in the presence of the four fixed masses M. As long as the masses M remained fixed, it is not necessary to consider their mutual potential energies.

(b) If the particle m is released at rest from infinity and passes through the origin, calculate the **magnitude of its velocity** there.

(c) Calculate the **net potential energy** of m when it is at the mid-point of the right-hand side of the square.

(d) If the particle *m* is released at rest from the mid-point of the right-hand side of the square, does it **reach the origin**? Support your answers with physical arguments.

(e) The potential energy along the x-axis is $-\frac{2GMm}{\sqrt{\left(x-\frac{a}{2}\right)^2+\frac{a^2}{4}}}-\frac{2GMm}{\sqrt{\left(x+\frac{a}{2}\right)^2+\frac{a^2}{4}}}$. For small x, this potential energy is approximated by $-\frac{4\sqrt{2}GMm}{a}-\frac{2\sqrt{2}GMm}{a^3}x^2+O(x^4)$, which is a concave down function. Using your answers to part (a,c,d), plot the potential energy function $U(x)$ for points along the x-axis.