Lecture 13
Heat Engines

- Thermodynamic processes and entropy
- Thermodynamic cycles
- Extracting work from heat
  - How do we define engine efficiency?
  - Carnot cycle: the best possible efficiency

Reading for this Lecture:
Elements Ch 4D-F

Reading for Lecture 14:
Elements Ch 10

A Review of Some Thermodynamic Processes of an Ideal Gas

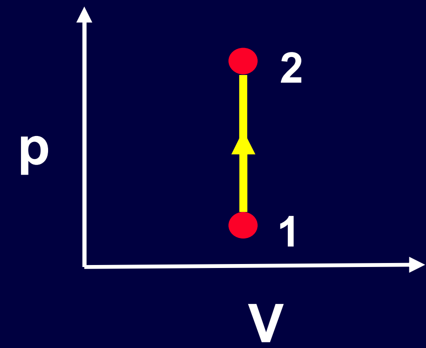
We will assume ideal gases in our treatment of heat engines, because that simplifies the calculations.

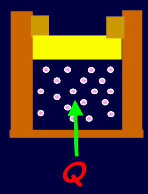
Isochoric (constant volume)

$$W_{by} = \int p dV = 0$$

$$\Delta U = \alpha N k \Delta T = \alpha V \Delta p$$

$$Q = \Delta U$$





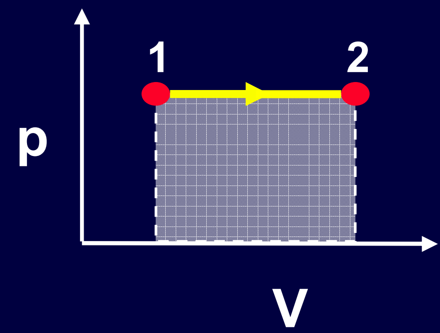
Temperature changes

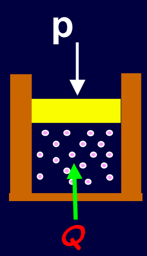
Isobaric (constant pressure)

$$W_{by} = \int p dV = p \Delta V$$

$$\Delta U = \alpha N k \Delta T = \alpha p \Delta V$$

$$Q = \Delta U + W_{by} = (\alpha + 1) p \Delta V$$





Temperature and volume change

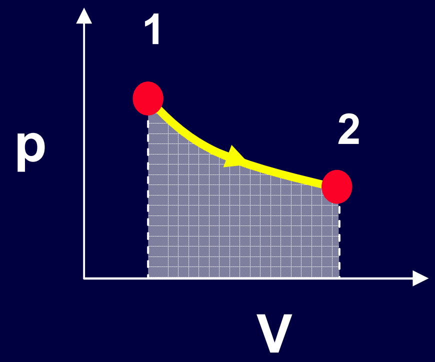
Review (2)

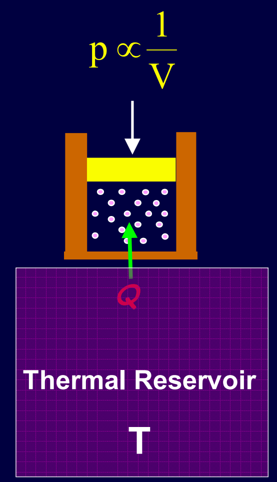
Isothermal (constant temperature)

$$W_{by} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{NkT}{V}\,dV = NkT\ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta U = 0$$

$$Q = W_{by}$$





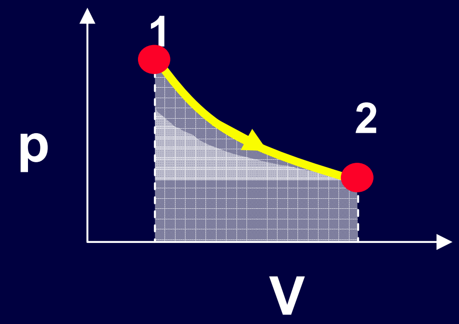
Volume and pressure change

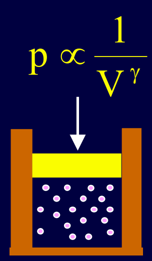
Adiabatic (no heat flow)

$$W_{by} = -\Delta U$$

$$\Delta U = \alpha Nk\left(T_2 - T_1\right) = \alpha\left(p_2V_2 - p_1V_1\right)$$

$$Q = 0$$



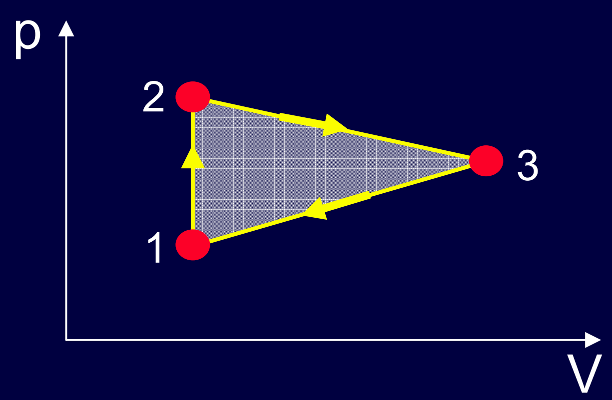


Volume, pressure and temperature change

Closed Thermodynamic Cycles

Closed cycles will form the basis of our heat engine discussion.

A closed cycle is one in which the system returns to the initial state. (same *p*, *V*, and *T*) For example:



- U is a state function. Therefore: $\Delta U = 0$
- The net work is the enclosed area. $W = \oint p dV \neq 0$
- Energy is conserved (1st Law): $Q = W \neq 0$

This is the reason that neither W nor Q is a state function. It makes no sense to talk about a state having a certain amount of W.

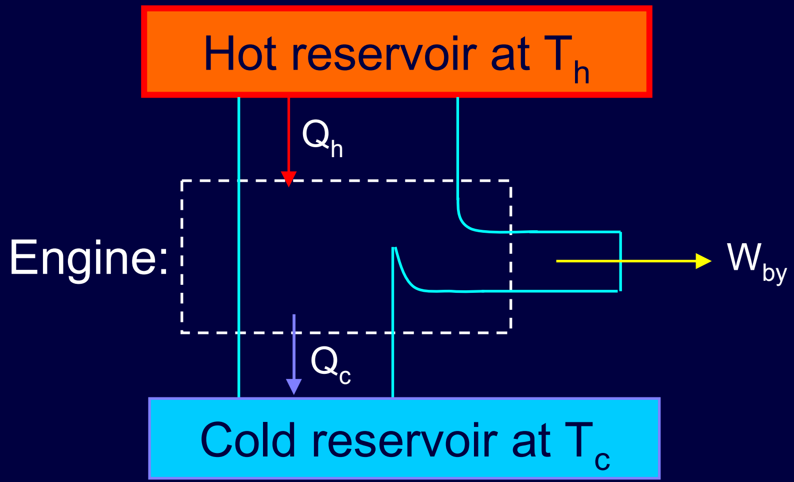
Though of course we could use *any* curves to make a closed cycle, here we will consider isochoric, isobaric, isothermal, and adiabatic processes.

Introduction to Heat Engines

One of the primary applications of thermodynamics is to Turn heat into work.

The standard heat engine works on a cyclic process:

1) extract heat from a hot reservoir,
2) perform work, using some of the extracted heat,
3) dump unused heat into a cold reservoir (often the environment).
4) repeat over and over. We represent this process with a diagram:



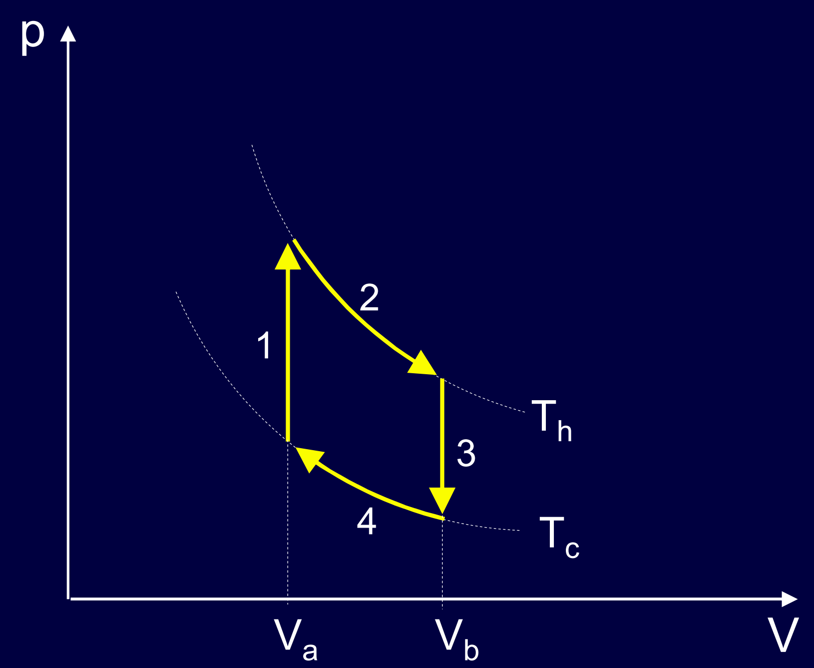
A "reservoir" is a large body whose temperature doesn't change when it absorbs or gives up heat

For heat engines we will define Q_h, Q_c, and W_{by} as positive.

Energy is conserved: $Q_h = Q_c + W_{by}$

A Simple Heat Engine:

the Stirling Cycle

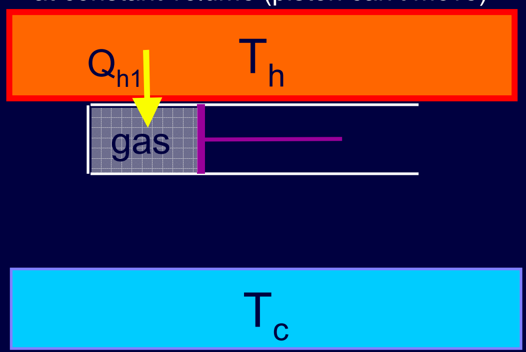


Two reservoirs: T_h and T_c.
Four processes: Two isotherms and two isochors
The net work during one cycle: The area of the "parallelogram".

The Stirling Cycle (2)

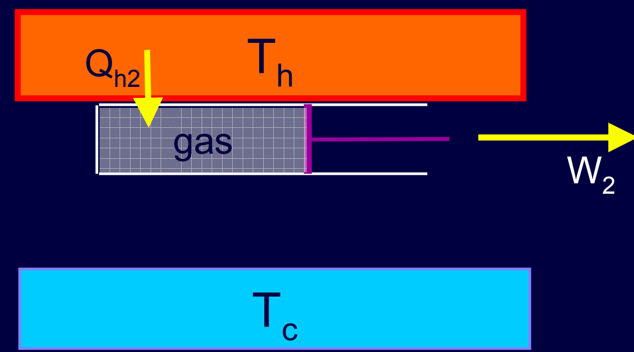
1) Isochoric

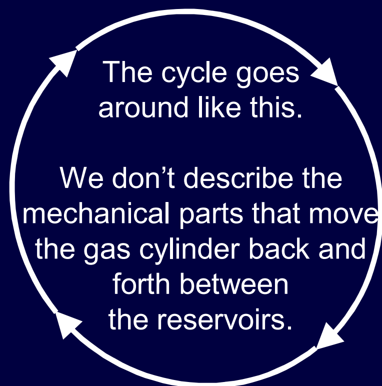
Gas temperature increases
at constant volume (piston can't move)



2) Isothermal

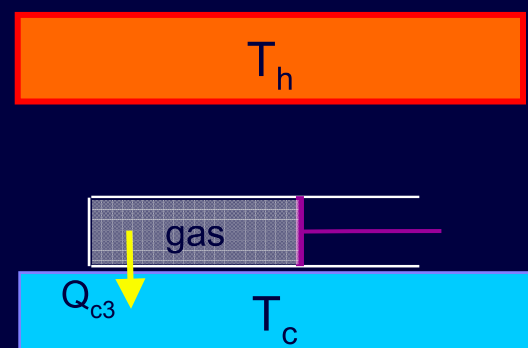
Gas expands at constant T_h





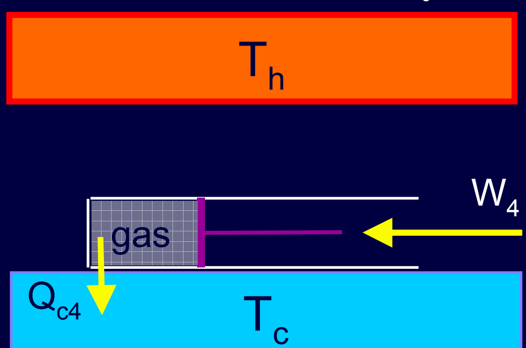
3) Isochoric

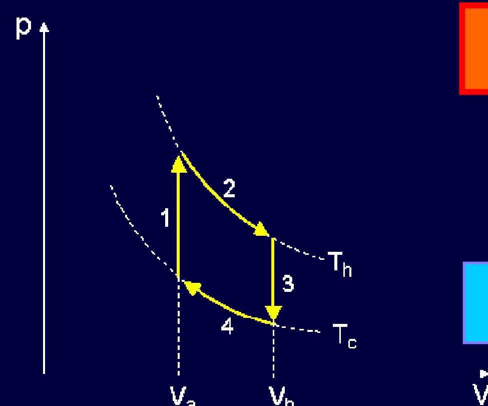
Gas temperature decreases
at constant volume (piston can't move)



4) Isothermal

Gas is compressed at constant T_c





How Does this Engine Do Work?

Look at the two isothermal processes (2 and 4) on the previous slide

Process 2: expanding gas does W_2 on the piston, as it expands from V_a to V_b.

Process 4: contracting gas is done W_4 by the piston, as it contracts from V_b to V_a.

If $W_2 > W_4$, the net work is positive.

This is true, because the contracting gas is colder ($\Rightarrow$lower pressure).

During one cycle:

- The hot reservoir has lost some energy ($Q_h=Q_{h1}+Q_{h2}$).
- The cold reservoir has gained some energy ($Q_c=Q_{c3}+Q_{c4}$).
- The engine (the gas cylinder) has neither gained nor lost energy.

The energy to do work comes from the hot reservoir, not from the engine itself.

The net work done by the engine is:

$$W_{by} = W_2 - W_4 = Q_h - Q_c = Q_{h2} - Q_{c4}$$

Not all of the energy taken from the hot reservoir becomes useful work.

Some is lost into the cold reservoir. We would like to make Q_c as small as possible.

Act 1

On the last slide, why did we write $Q_h - Q_c = Q_{h2} - Q_{c4}$?
What happened to Q_{h1} and Q_{c3}? (since $Q_h = Q_{h1} + Q_{h2}$, and $Q_c = Q_{c1} + Q_{c2}$)

a) They both = 0.
b) They are not = 0, but they cancel.
c) They average to zero over many cycles.

Solution

On the last slide, why did we write $Q_h - Q_c = Q_{h2} - Q_{c4}$?
What happened to Q_{h1} and Q_{c3}? (since $Q_h = Q_{h1} + Q_{h2}$, and $Q_c = Q_{c1} + Q_{c2}$)

a) They both = 0.
b) They are not = 0, but they cancel.
c) They average to zero over many cycles.

They cancel, because the amount of heat needed to raise the temperature from T_c to T_h at constant volume equals the amount of heat needed to lower the temperature from T_h to T_c at constant volume*, even though $V_a \neq V_b$.

*Note: This is only valid for ideal gases.
C_V is only independent of V for an ideal gas.

Heat Engine Efficiency

We pay for the heat input, Q_H, so:

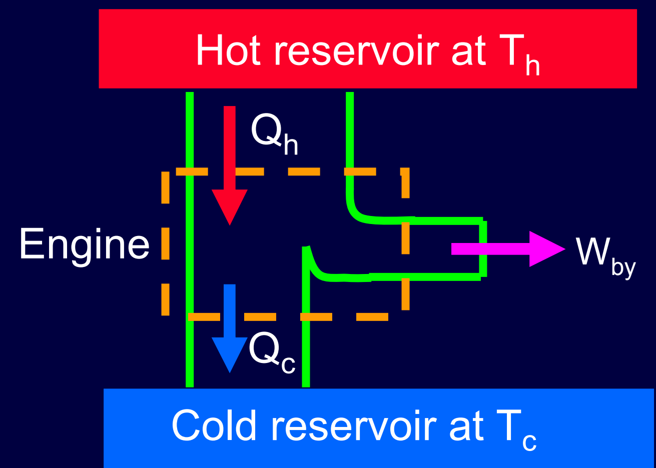
Define the efficiency

$$\varepsilon \equiv \frac{\text{work done by the engine}}{\text{heat extracted from reservoir}} = \frac{\text{results}}{\text{cost}}$$

$$\varepsilon \equiv \frac{W_{by}}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

Valid for all heat engines.
(Conservation of energy)

Cartoon picture of a heat engine:



Remember:

We define Q_h and Q_c as *positive*.
The arrows define direction of flow.

What's the best we can do?

The Second Law will tell us.

Review

Entropy in Macroscopic Systems

Traditional thermodynamic entropy: $S = k \ln\Omega = k\sigma$

We want to calculate S from macrostate information (p, V, T, U, N, *etc.*)
Start with the definition of temperature in terms of entropy:

$$\frac{1}{kT} \equiv \left(\frac{\partial \sigma}{\partial U}\right)_{V,N}, \text{ or } \frac{1}{T} \equiv \left(\frac{\partial S}{\partial U}\right)_{V,N}$$

The entropy changes when T changes: (We're keeping V and N fixed.)

$$dS = \frac{dU}{T} = \frac{C_V dT}{T} \Rightarrow \Delta S = \int_{T_1}^{T_2} \frac{C_V dT}{T}$$

If C_V is constant: $$= C_V \int_{T_1}^{T_2} \frac{dT}{T} = C_V \ln\left(\frac{T_2}{T_1}\right)$$

Entropy in Quasi-static Heat Flow

When V is constant: $dS \equiv dU/T = dQ/T$ ⇐ W = 0, so dU = dQ

In fact, dS = dQ/T during any reversible (quasi-static) process, even if V changes.

The reason: In a reversible process, S_{tot} (system plus environment) doesn't change:

$$0 = dS_{sys} + dS_E$$ ΔS_{tot} = 0 if process is reversible.

$$= dS_{sys} + \frac{dU_E}{T}$$ The reservoir is supplying (or absorbing) heat.

$$= dS_{sys} - \frac{dQ}{T}$$ The reservoir's energy gain is the system's heat loss. That's how they interact.

$$dS = \frac{dQ}{T}, \text{ or } \Delta S = \int_{init}^{final} \frac{dQ}{T}$$ for any reversible process

ΔS in Isothermal Processes

Suppose V & p change but T doesn't.

Work is done ($dW_{by} = pdV$).
Heat must enter to keep T constant: $dQ = dU + dW_{by}$.
So:

$$dS = \frac{dQ}{T} = \frac{dU + dW_{by}}{T} = \frac{dU + pdV}{T}$$

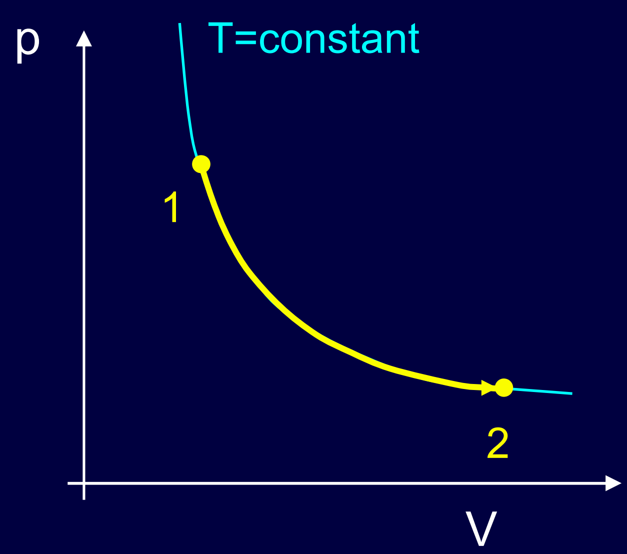
Remember: This holds for quasi-static processes, in which the system remains near thermal equilibrium at all times.

Special case, ideal gas:

For an ideal gas, if $dT = 0$, then $dU = 0$.

$$dS = \frac{pdV}{T} = \frac{NkTdV}{VT} = \frac{NkdV}{V}$$

$$\Delta S = \int_{V_1}^{V_2} \frac{NkdV}{V} = Nk \ln\left(\frac{V_2}{V_1}\right)$$



Quasi-static Adiabatic Processes

Q = 0 (definition of an adiabatic process)

V and T both change as the applied pressure changes.

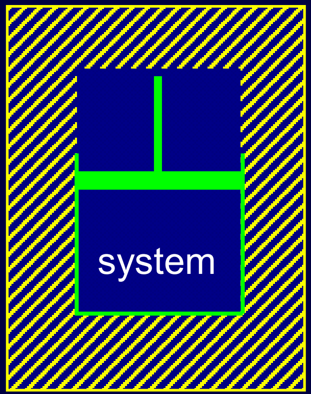
For example, if p increases (compress the system):

- V decreases, and the associated S also decreases.
- T increases, and the associated S also increases.

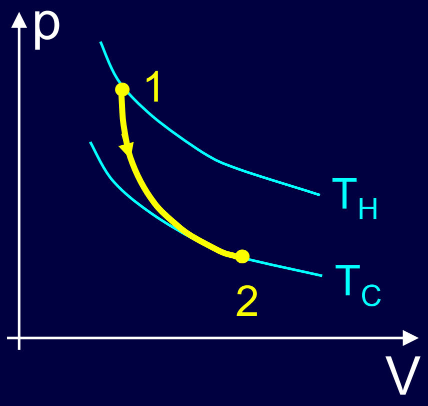
These two effects must exactly cancel !!

Why? Because:

- This is a reversible process, so $S_{tot} = 0$.
- No other entropy is changing.
  ($Q = 0$, and W_{by} just moves the piston.)



Insulated walls



So, in a quasi-static adiabatic process, $\Delta S = 0$.

Note: We did not assume that the system is an ideal gas. This is a general result.

Heat Engine Efficiency

We pay for the heat input, Q_H, so:

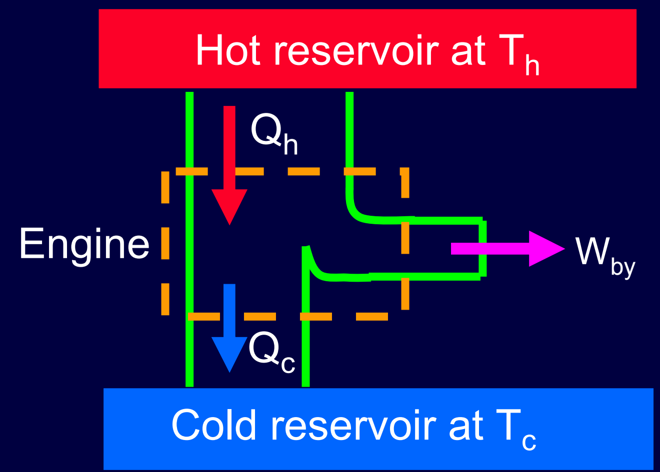
Define the efficiency

$$\varepsilon \equiv \frac{\text{work done by the engine}}{\text{heat extracted from reservoir}} = \frac{\text{results}}{\text{cost}}$$

$$\varepsilon \equiv \frac{W_{by}}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

Valid for all heat engines.
(Conservation of energy)

Cartoon picture of a heat engine:



Remember:

We define Q_h and Q_c as *positive*.
The arrows define direction of flow.

What's the best we can do?

The Second Law will tell us.

The 2^{nd} Law Sets the Maximum Efficiency (1)

$$\Delta S_{tot} \geq 0$$

How to calculate ΔS_{tot}?

Over one cycle:

$$\Delta S_{tot} = \Delta S_{engine} + \Delta S_{hot} + \Delta S_{cold}$$

ΔS_{engine} = 0

Remember:

Q_h is the heat taken from the hot reservoir, so $\Delta S_{hot} = -Q_h/T_h$.

Q_c is the heat added to the cold reservoir, so $\Delta S_{cold} = +Q_c/T_c$.

From the definition of T

$$\Delta S_{tot} = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} \geq 0$$ 2^{nd} Law

$$\Rightarrow \frac{Q_C}{Q_H} \geq \frac{T_C}{T_H}$$

Q_c cannot be zero.
Some energy is always lost.

The 2nd Law Sets the Maximum Efficiency (2)

$$efficiency = \varepsilon \equiv \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

$$\frac{Q_C}{Q_H} \geq \frac{T_C}{T_H}$$ from the 2nd Law

$$\Rightarrow \boxed{\varepsilon \leq 1 - \frac{T_C}{T_H}}$$

This is a universal law !! (equivalent to the 2nd law)

It is valid for any procedure that converts thermal energy into work.
We did not assume any special properties (*e.g.*, ideal gas) of the material in the derivation.

The maximum possible efficiency, $\varepsilon_{Carnot} = 1 - T_c/T_h$, is called the Carnot efficiency.
The statement that heat engines have a maximum efficiency was the first expression of the 2nd law, by Sadi Carnot in 1824.

How Efficient Can an Engine be?

Consider an engine that uses steam (T_h = 100° C) as the hot reservoir and ice (T_c = 0° C) as the cold reservoir. How efficient can this engine be?

The Carnot efficiency is $\varepsilon_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{273\text{ K}}{373\text{ K}} = 0.27$

Therefore, an engine that operates between these two temperatures can, at best, turn only 27% of the steam's heat energy into useful work.

Question: How might we design an engine that has higher efficiency?

Answer: By increasing T_h. (That's more practical than lowering T_c.)

Electrical power plants and race cars obtain better performance by operating at a much higher T_h.

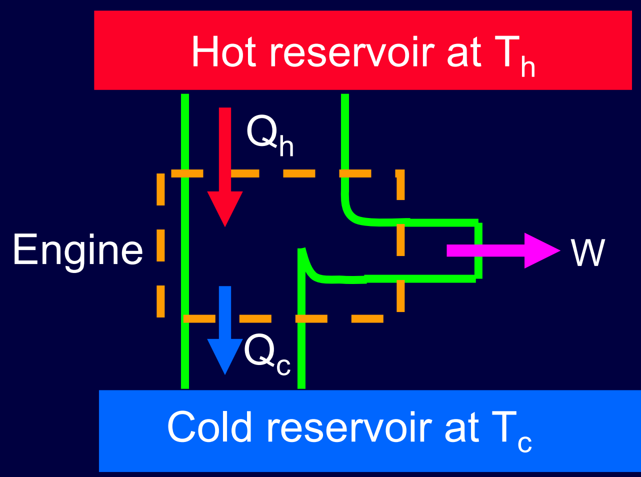
When Is ε Less than ε_{Carnot}?

We can write the efficiency loss in terms of the change of total entropy:

$$\Delta S_{tot} = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = -\frac{Q_H}{T_H} + \frac{Q_H - W}{T_C}$$

$$\Rightarrow W = Q_H\left(1 - \frac{T_C}{T_H}\right) - T_C \Delta S_{tot}$$

$$\varepsilon \equiv \frac{W}{Q_H} = \underbrace{\left(1 - \frac{T_C}{T_H}\right)}_{\varepsilon_{Carnot}} - \frac{T_C \Delta S_{tot}}{Q_H}$$



Lesson: Avoid irreversible processes.
(ones that increase S_{tot}).

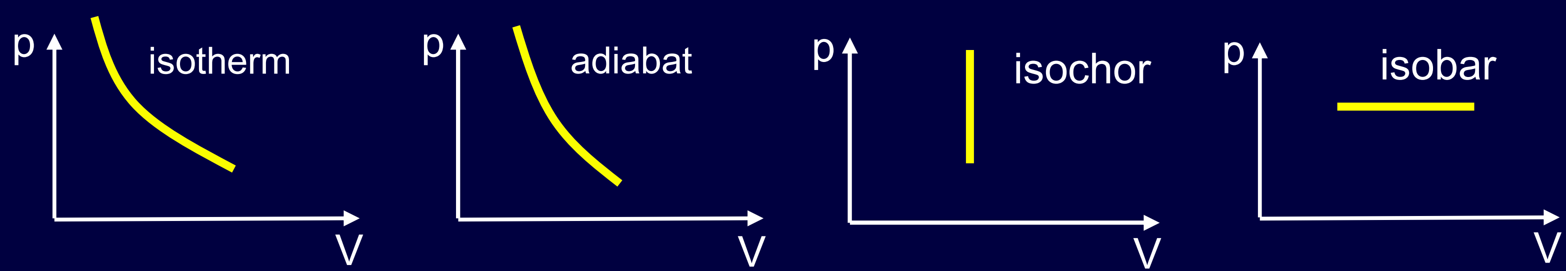
- direct heat flow from hot to cold
- free expansion (far from equilibrium)
- sliding friction

Irreversible Processes

Entropy-increasing processes are irreversible,
because the reverse processes would reduce entropy.
Examples:

- Free-expansion (actually, any particle flow between regions of different density)
- Heat flow between two systems with different temperatures.

Consider the four processes of interest here:
Isothermal: Heat flow but no T difference. Reversible
Adiabatic: $Q = 0$. No heat flow at all. Reversible
Isochoric & Isobaric: Heat flow between different T's. Irreversible
(Assuming that there are only two reservoirs.)



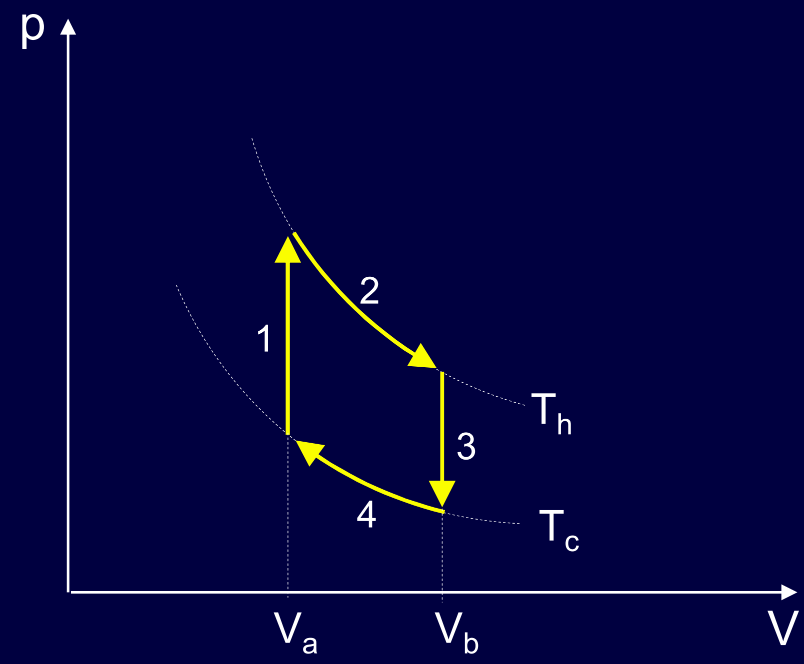
reversible reversible irreversible irreversible

Act 2: Stirling Efficiency

Will our Stirling engine achieve Carnot efficiency?

a) Yes

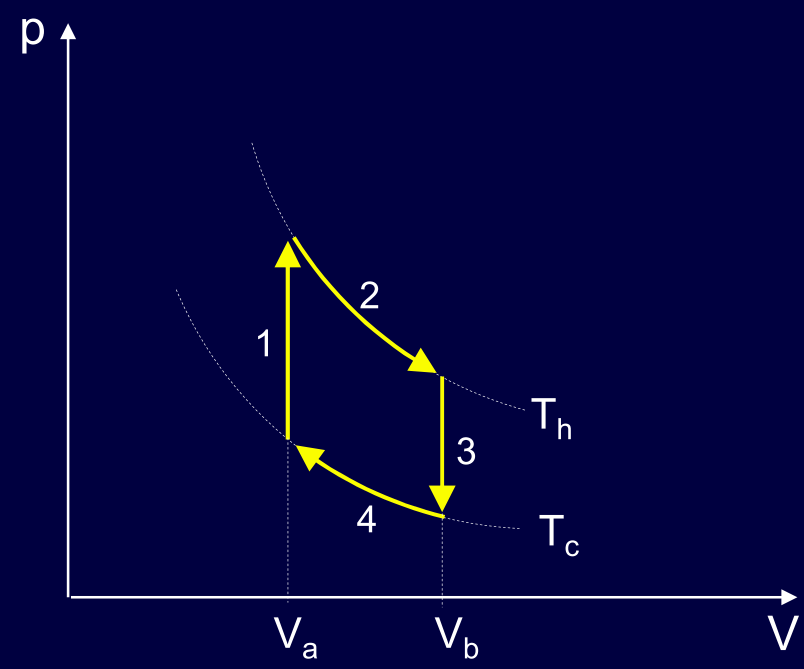
b) No



Solution

Will our Stirling engine achieve Carnot efficiency?

a) Yes   b) No



Processes 1 and 3 are irreversible.
(isochoric heating and cooling)

1: Cold gas touches hot reservoir.
3: Hot gas touches cold reservoir.

How to Achieve Carnot Efficiency

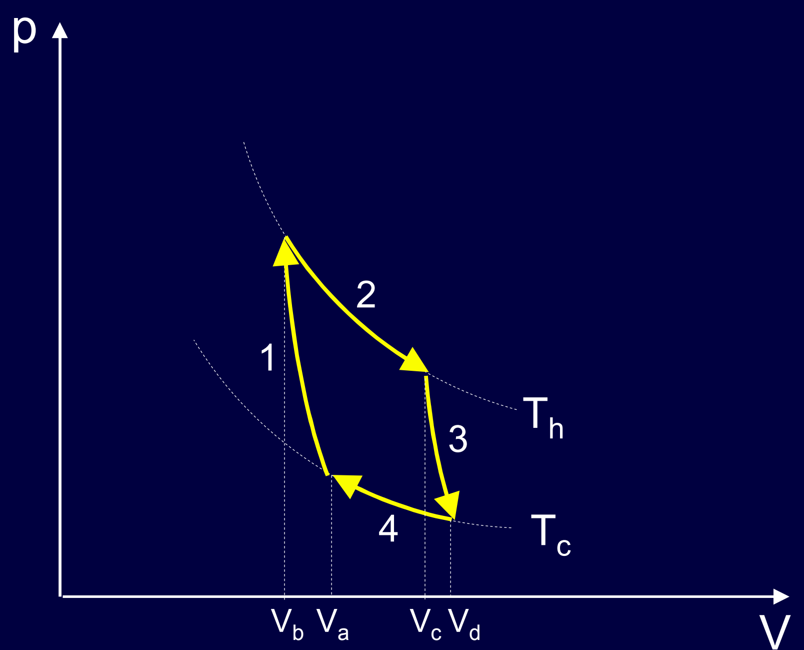
To achieve Carnot efficiency, we must replace the isochors (irreversible) with reversible processes. Let's use adiabatic processes, as shown:

Processes 1 and 3 are now adiabatic.
Processes 2 and 4 are still isothermal.

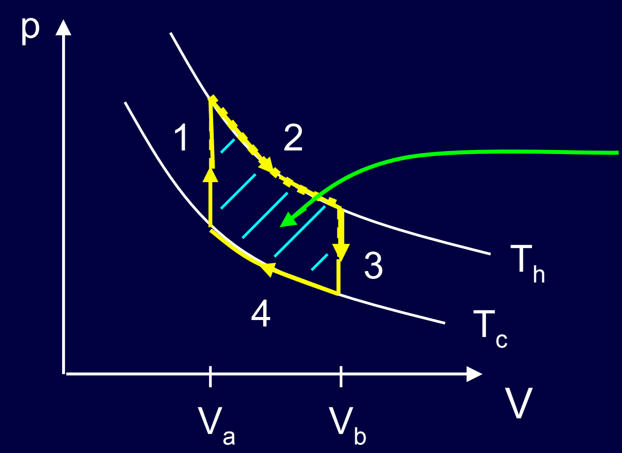
This cycle is reversible, which means:

S_{tot} remains constant: $\Rightarrow \varepsilon = \varepsilon_{Carnot}$.

This thermal cycle is called the Carnot cycle, and an engine that implements it is called a Carnot heat engine.



Example: Efficiency of Stirling Cycle



Area enclosed:
$W_{by} = \oint p\,dV$

Total work done by the gas is the sum of steps 2 and 4:

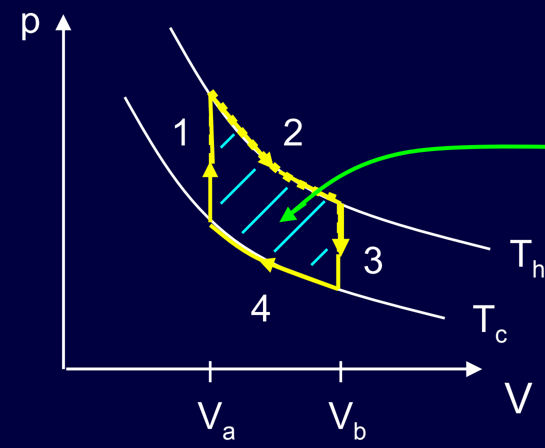
$$W_2 = \int_{V_a}^{V_b} p\,dV = NkT_h \int_{V_a}^{V_b} \frac{dV}{V} = NkT_h \ln\left(\frac{V_b}{V_a}\right)$$

$$W_4 = \int_{V_b}^{V_a} p\,dV = NkT_c \int_{V_b}^{V_a} \frac{dV}{V} = -NkT_c \ln\left(\frac{V_b}{V_a}\right)$$

$$W_{by} = W_2 + W_4 = Nk\left(T_h - T_c\right)\ln\left(\frac{V_b}{V_a}\right)$$

We need a temperature difference if we want to get work out of the engine.

Solution



Area enclosed:
$$W_{by} = \oint p\,dV$$

Heat extracted from the hot reservoir, exhausted to cold reservoir:

$$Q_h = Q_1 + Q_2 = \alpha Nk(T_h - T_c) + NkT_h \ln\left(\frac{V_b}{V_a}\right)$$

$$-Q_c = Q_3 + Q_4 = -\alpha Nk(T_h - T_c) - NkT_c \ln\left(\frac{V_b}{V_a}\right)$$

Solution

Let's put in some numbers:

$V_b = 2V_a$
$\alpha = 3/2$ (monatomic gas)
$T_h = 373K$ (boiling water)
$T_c = 273K$ (ice water)

$$\varepsilon \equiv \frac{W_{by}}{Q_h} = \frac{(T_h - T_c)\ln\left(\frac{V_b}{V_a}\right)}{\frac{3}{2}(T_h - T_c) + T_h \ln\left(\frac{V_b}{V_a}\right)} \qquad (\ell n\, 2 = 0.69)$$

$$= \frac{100(0.69)}{150 + 373(0.69)} = 16.9\%$$

For comparison:

$\varepsilon_{carnot} = 1 - 273/373 = 26.8\%$

Heat Engine Summary

For all cycles: $\varepsilon = 1 - \frac{Q_c}{Q_h}$ Some energy is dumped into the cold reservoir.

For the Carnot cycle: $\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$ Q_c cannot be reduced to zero.

Carnot (best) efficiency: $\varepsilon_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$ Only for reversible cycles.

Carnot engines are an idealization - impossible to realize.
They require very slow processes, and perfect insulation.
When there's a net entropy increase, the efficiency is reduced:

$$\varepsilon = \varepsilon_{\text{Carnot}} - \frac{T_C \Delta S_{tot}}{Q_H}$$

Next Time

Heat Pumps

Refrigerators,

Available Work and Free Energy