

1. A circular rod of length  $L = 2$  meter and diameter  $d=2$  cm is composed of a material with a heat conductivity of  $\kappa = 6$  W/m-K. One end is held at  $100$  °C, the other at  $0$  °C. At what rate does the bar conduct heat?

- a) 0.09 W
- b) 0.18 W
- c) 300 W
- d) 0.0009 W
- e) 0.015 W

2. Estimate the amount of heat required to raise the temperature of 4 kg of Aluminum from  $20$ °C to  $30$ °C

- a. 3.7 J
- b. 37 J
- c. 370 J
- d. 3700 J
- e. 37000 J

3. Given 1000 spins that can be 'up' or 'down' and a very high temperature  $T \rightarrow \infty$ , what is the probability that 600 are up and 400 are down ?

- a  $5 \times 10^{-11}$
- b 0.40
- c 0.60
- d  $10^{-301}$
- e. 0.0

4. Two indistinguishable particles are confined to two *single occupancy* cells. The particles are now allowed to expand into 8 additional *unlimited occupancy* cells, (now 10 cells in all) by how much does the dimensionless entropy increase?

*Hint: Count the total number of microstates in three cases: one in which two particles are in the original cells, one in which there is one particle in an original cell, and another in which there are no particles in the original cells.*

- a)  $\Delta\sigma = 6.91$
- b)  $\Delta\sigma = 3.45$
- c)  $\Delta\sigma = 3.97$
- d)  $\Delta\sigma = 3.58$
- e)  $\Delta\sigma = 2.30$

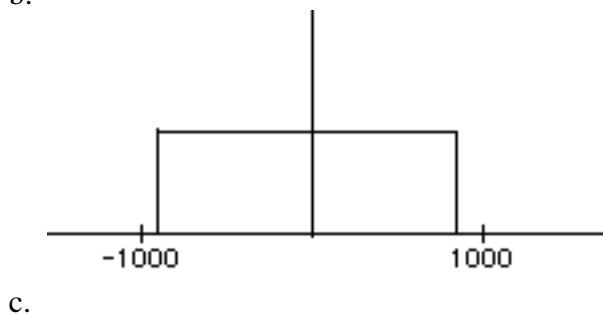
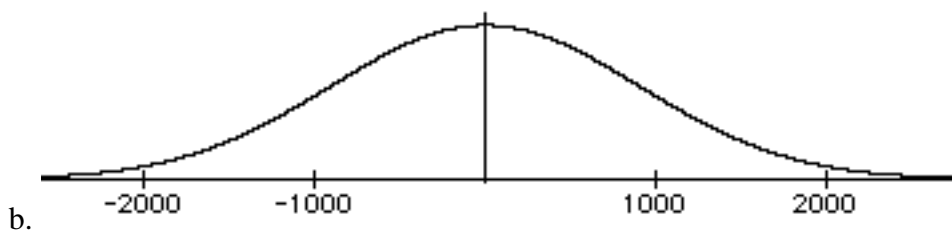
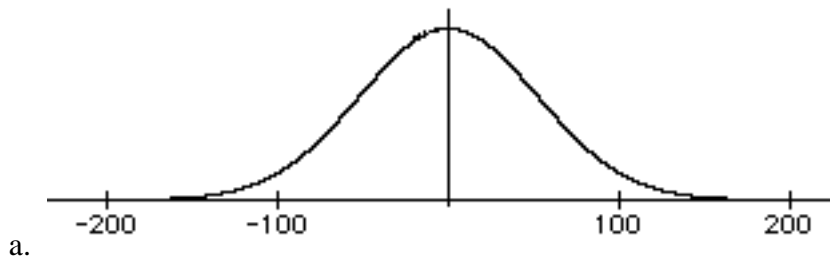
5. A spherical black body of radius 1 meter generates heat (due to some internal chemical reaction perhaps) at a rate of 100 kWatt. What is its steady-state surface temperature?

- a 339 °C
- b 612 °C
- c 877 °C

6. The temperature of intergalactic space is 2.7 K. What is the wavelength of its peak electromagnetic radiation intensity?

- a. 1 nm
- b. 1 micron
- c. 1 mm
- d. 1 meter
- e. 10 meters

7. 1000 particles move in one dimension. They all start at the origin. Each second they each move 3 cm in a random direction. After 300 seconds, which of these plots most correctly describes the distribution of the number density of these particles?



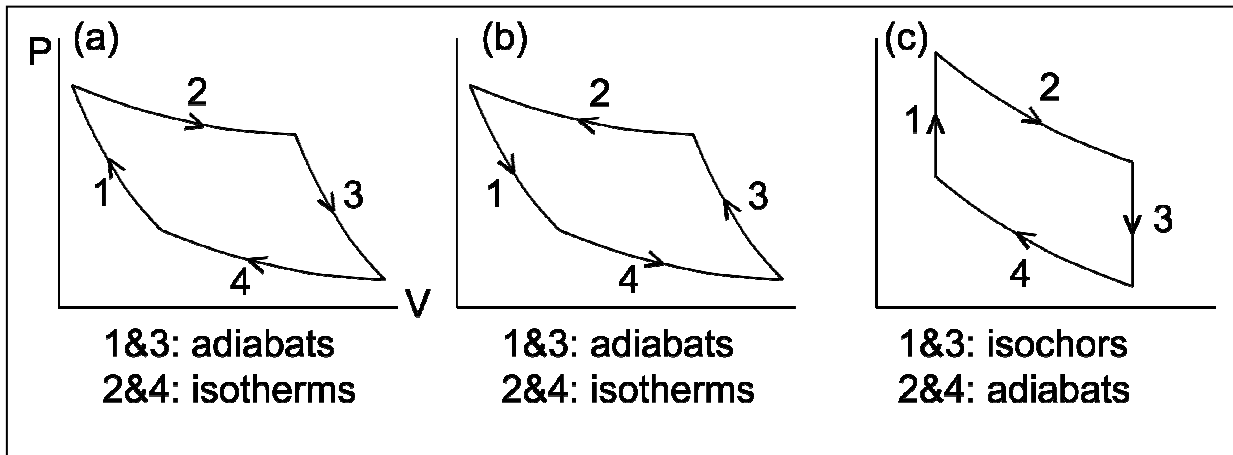
8. A certain liquid is observed to boil at standard pressure ( $1.01 \times 10^5 \text{ Pa}$ ) at a temperature of  $42^\circ\text{C}$ . When the pressure is lowered to  $0.1 \times 10^5 \text{ Pa}$ , it boils at  $38^\circ\text{C}$ . What is its latent heat per molecule?

- a.  $7.8 \times 10^{-19} \text{ J}$
- b.  $5.1 \times 10^{-20} \text{ J}$
- c.  $5.1 \times 10^{20} \text{ J}$
- d.  $4.1 \times 10^{-21} \text{ J}$
- e.  $6.2 \times 10^{-21} \text{ J}$

*The next two questions pertain to the following situation:*

9. Suppose that the heat flow out of your  $20^\circ\text{C}$  house is  $2 \text{ kW}$ . If the temperature outside is  $-5^\circ\text{C}$ , how much power would an ideal heat pump require to keep the inside of your house at  $20^\circ\text{C}$ ?

- a.  $12.4 \text{ W}$
- b.  $36.6 \text{ W}$
- c.  $171 \text{ W}$
- d.  $2 \text{ kW}$
- e.  $3.7 \text{ kW}$



10. Which of the above diagrams corresponds to the ideal heat pump of the previous problem?

- a
- b
- c

11. If 0.28 moles of an ideal gas is isothermally compressed to half its initial volume, what is the change in entropy?

- a.  $\Delta S = -1.6 \text{ J/K}$
- b.  $\Delta S = -0.5 \text{ J/K}$
- c.  $\Delta S = 1.3 \text{ J/K}$
- d.  $\Delta S = 2.7 \text{ J/K}$
- e. not enough information is given

12. A block of a material has a temperature-dependent heat capacity given by  $C = B T^3$ , where  $B = 3 \times 10^{-6} \text{ J/K}^4$ . What is the change in entropy if the temperature is increased from 200 K to 250 K?

- a.  $\Delta S = 2.54 \text{ J/K}$
- b.  $\Delta S = 7.63 \text{ J/K}$
- c.  $\Delta S = 10.23 \text{ J/K}$
- d.  $\Delta S = 15.27 \text{ J/K}$
- e.  $\Delta S = 28.94 \text{ J/K}$

A closed cycle heat engine is used to extract work from two reservoirs with fixed temperature  $T_H = 450 \text{ K}$  and  $T_C = 320 \text{ K}$ .

13. If the efficiency of the engine is less than the Carnot efficiency, which of the following statements is true regarding the change in the entropies?

Define:

$\Delta S_H$  = change in entropy of the hot reservoir in one cycle.  
 $\Delta S_C$  = change in entropy of the cold reservoir in one cycle.  
 $\Delta S_E$  = change in entropy of the engine in one cycle.

- a.  $|\Delta S_E| = 0, |\Delta S_H| > |\Delta S_C|$
- b.  $|\Delta S_E| = 0, |\Delta S_H| < |\Delta S_C|$
- c.  $|\Delta S_E| > 0, |\Delta S_H| > |\Delta S_C|$
- d.  $|\Delta S_E| > 0, |\Delta S_H| < |\Delta S_C|$
- e.  $|\Delta S_E| > 0, |\Delta S_H| = |\Delta S_C|$

***The next two questions pertain to the following situation:***

A brick with constant heat capacity  $C = 3 \text{ J/K}$  is heated to  $120^\circ\text{C}$  and connected to one side of a Carnot engine; the other side is kept at a constant temperature of  $20^\circ\text{C}$ .

14. How much total work can be extracted from the brick?

- a.  $W = 41.9 \text{ J}$
- a.  $W = 54.6 \text{ J}$
- a.  $W = 75.2 \text{ J}$
- a.  $W = 103.5 \text{ J}$
- a.  $W = 134.2 \text{ J}$

15. Which of these statements is true regarding the change in the total entropy?

- a.  $\Delta S_{\text{TOT}} > 0$
- b.  $\Delta S_{\text{TOT}} = 0$
- b.  $\Delta S_{\text{TOT}} < 0$

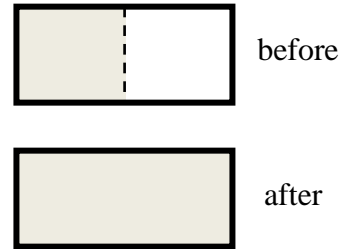
16. In thermal equilibrium, the free energy of a small system in contact with a reservoir is minimized. Which of these statements is an equivalent description of thermal equilibrium?

- a. The entropy of the small system is maximized.
- b. The internal energy of the small system is minimized.
- c. The total entropy of the reservoir plus small system is maximized.

17. The Boltzmann factor tells us that the probability a small system is in a macrostate of energy  $E$  is proportional to  $e^{-E/kT}$ . The reason that the macrostates are not equally probable is:

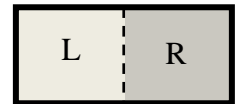
- a. Conservation of energy requires this probability distribution.
- b. We need to take into account the degeneracy at each value of  $E$ .
- c. As the small system's energy increases, the entropy of the reservoir decreases.

18. One mole of  $O_2$  (an ideal gas) is allowed to free expand from half of a 2-liter bottle into the entire bottle (*i.e.*,  $V_i = 1$  liter, see the figure). The initial temperature is  $T_i = 300$  K. How much does the chemical potential,  $\mu$ , of the gas change during this process?



- a.  $\Delta\mu = 0$  J
- b.  $\Delta\mu = 2.87 \times 10^{-21}$  J
- c.  $\Delta\mu = -2.87 \times 10^{-21}$  J
- d.  $\Delta\mu = 1729$  J
- e.  $\Delta\mu = -1729$  J

19. Consider two materials made up of different atoms (call them A and B). The materials are put in contact, and the atoms are free to move within the entire volume of the A+B object. Which of the following equations is valid in thermal equilibrium? The subscripts A & B denote atom type, while L & R denote the side (left or right) of the object. For example,  $\mu_{AL}$  denotes the chemical potential of atoms A on the left.



- a.  $\mu_{AL} = \mu_{BR}$
- b.  $F_{AL} = F_{BR}$
- c.  $\mu_{AL} + \mu_{AR} = \mu_{BL} + \mu_{BR}$
- d.  $F_{AL} + F_{BL} = F_{AR} + F_{BR}$
- e.  $\mu_{AL} + \mu_{BL} = \mu_{AR} + \mu_{BR}$

**The next two problems refer to this situation:**

At a temperature of 300 K, the density of electrons in the conduction band of a particular intrinsic semiconductor is  $n_e = 10^{10}/\text{m}^3$ . The energy gap in this semiconductor is 1.0 eV.

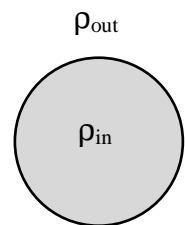
20. Suppose we dope the semiconductor so that  $n_e = 10^9$ . What is the density,  $n_h$ , of holes after we do this?

- a.  $n_h = 1.00 \times 10^9 / \text{m}^3$
- b.  $n_h = 3.16 \times 10^{10} / \text{m}^3$
- c.  $n_h = 1.00 \times 10^{10} / \text{m}^3$
- d.  $n_h = 3.16 \times 10^{10} / \text{m}^3$
- e.  $n_h = 1.00 \times 10^{11} / \text{m}^3$

21. Go back to the undoped situation. What is the density of electrons at the same temperature in an intrinsic semiconductor that has a 1.1 eV energy gap?

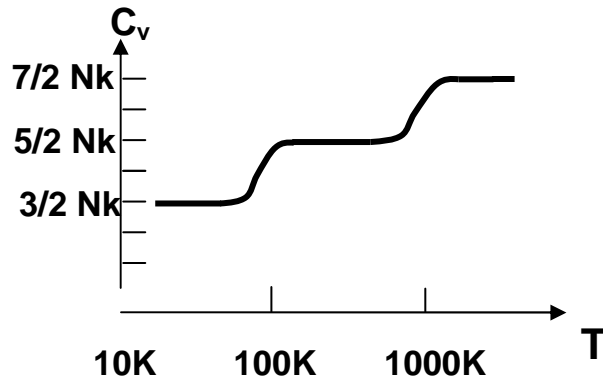
- a.  $n_e = 2.1 \times 10^8 / \text{m}^3$
- b.  $n_e = 9.0 \times 10^9 / \text{m}^3$
- c.  $n_e = 1.0 \times 10^{10} / \text{m}^3$
- d.  $n_e = 1.1 \times 10^{10} / \text{m}^3$
- e.  $n_e = 4.8 \times 10^{11} / \text{m}^3$

22. In order to live, cells must maintain concentration differences across their membranes. Consider the cell in the figure. Its volume is  $10^{-18} \text{ m}^3$ , and the temperature is 300 K. The density of potassium ions inside the cell is maintained at  $\rho_{\text{in}} = 1.0 \text{ mole}/\text{m}^3$ , while the density outside (*i.e.*, the environment) is  $\rho_{\text{out}} = 0.01 \text{ mole}/\text{m}^3$ . If the cell does nothing, potassium ions will leak out at a rate of one per millisecond. How much power,  $P$ , (*i.e.*, Watts) must the cell expend to maintain the required  $\rho_{\text{in}}$ ?



- a.  $P = 1 \times 10^{-18} \text{ W}$
- b.  $P = 2 \times 10^{-17} \text{ W}$
- c.  $P = 4 \times 10^{-16} \text{ W}$
- d.  $P = 1 \times 10^{-3} \text{ W}$
- e.  $P = 1000 \text{ W}$

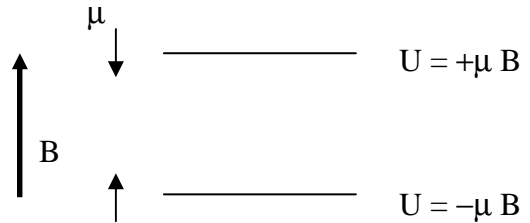
23. Use the following diagram of the heat capacity of a gas of a particular diatomic molecule to estimate the frequency  $f$  of the first excited *vibrational* state



- $f = 2.1 \times 10^{12}$  Hz
- $f = 3.2 \times 10^{12}$  Hz
- $f = 5.3 \times 10^{12}$  Hz
- $f = 2.1 \times 10^{13}$  Hz
- Cannot be determined.

**The next two problems are related:**

Consider a collection of  $N$  nuclear spins, each with its associated magnetic moment  $\mu$ . They are placed in the earth's magnetic field  $B$ , leading to the energy-level diagram shown here:



24. If the earth's magnetic field magnitude is  $0.5 \times 10^{-4}$  Tesla (0.5 Gauss), and the magnetic moment is  $\mu = 1.4 \times 10^{-26}$  J/T, what is the largest temperature that will allow the probability  $P_{\text{ground}}$  that a given spin will be in the lowest energy state to be 10%?

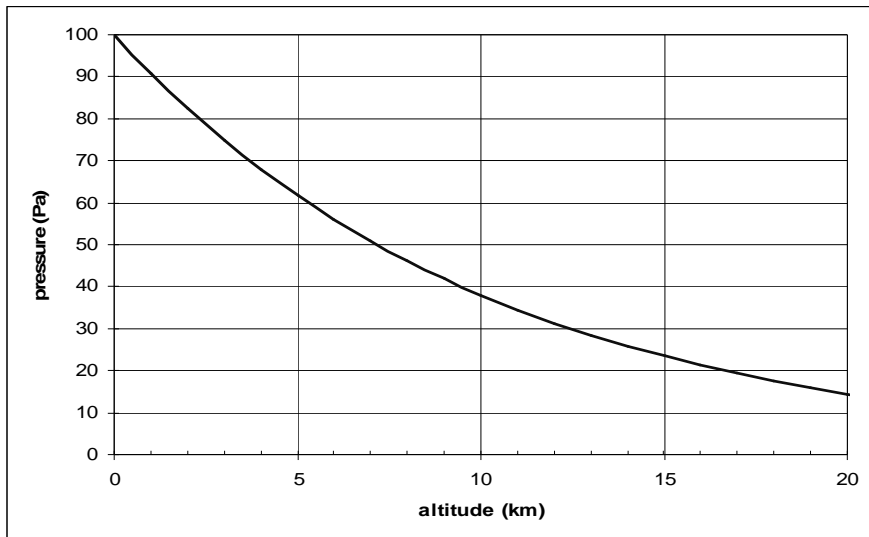
- $T < 5$  nK
- $T < 50$  nK
- $T < 500$  nK
- This will not occur at any temperature.
- Cannot be determined from the information given.

25. What is the limiting value,  $S_0$ , of the entropy of an ensemble of  $N$  such spins, as the temperature  $T \rightarrow \infty$ ?

- $S_0 = N k$
- $S_0 = N k \ln(2)$
- $S_0 = 2 N k$



26. An exploratory satellite visiting a distant planet measures the following pressure-versus-altitude plot.



Assuming a gravitational acceleration of  $1 \text{ m/s}^2$ , and a uniform day-time temperature of  $50\text{K}$ , what *single* substance might we conclude makes up the atmosphere (at these altitudes) ?

- a. H
- b.  $\text{H}_2$
- c. He
- d.  $\text{N}_2$
- e. Ar

27. Compare the chemical potential of the substance at 5 km with the substance at 10 km.

- a.  $\mu_{5\text{km}} > \mu_{10\text{km}}$
- b.  $\mu_{5\text{km}} = \mu_{10\text{km}}$
- c.  $\mu_{5\text{km}} < \mu_{10\text{km}}$

28. 3 moles of  $H_2$  gas are in a 2-liter volume, at 300K. Compare the final temperature in the following three circumstances:

1. the gas is compressed under constant pressure to half its original volume
2. the gas is compressed under constant temperature to half its original volume
3. the gas is compressed to half its original volume without any heat added or removed.

- a.  $T_1 = T_2 = T_3$
- b.  $T_1 < T_2 < T_3$
- c.  $T_2 = T_3 < T_1$
- d.  $T_3 < T_2 < T_1$
- e.  $T_3 < T_2 = T_1$

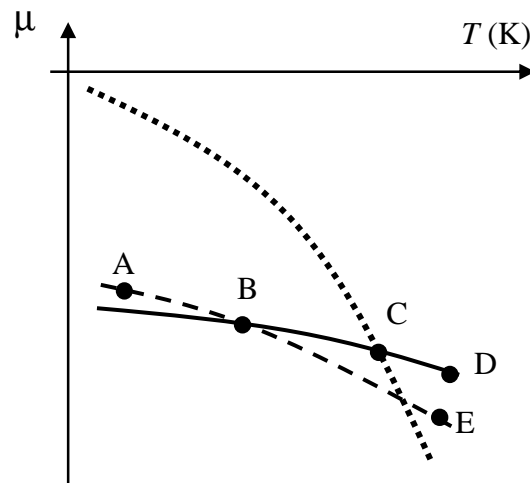
***The next two problems are related:***

29. You observe that when a glass of water is left out on the kitchen counter, it's empty a few days later. Which of the following explanations is incorrect?

- a. The fluctuations in the local temperature occasionally exceed the boiling temperature, thereby giving the molecules enough energy to become unbound.
- b. The molecules are always leaving and rejoining the water in the glass, but the in-bound process is less likely.
- c. The local partial pressure of water vapor just above the surface of the water is less than the vapor pressure of the water at room temperature.

30. Which point on the diagram best represents the situation (values of  $T$  and  $\mu$ ) described above, just after the glass of water is placed on the counter?

- a. A
- b. B
- c. C
- d. D
- e. E

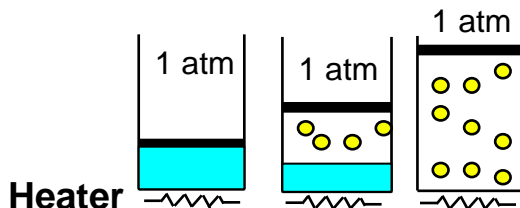


31. Consider the following reaction:  $\text{N}_2\text{O}_4 \leftrightarrow 2\text{NO}_2$ . Assume we start – in equilibrium – with 3 moles of  $\text{N}_2\text{O}_4$  and 2 moles of  $\text{NO}_2$ . We then add an additional mole of  $\text{NO}_2$  (at constant temperature), and let the system re-equilibrate. What will happen to the final relative concentration of  $\text{N}_2\text{O}_4$ , i.e., the concentration relative to that of  $\text{NO}_2$ ?

- a.  $n(\text{N}_2\text{O}_4)/n(\text{NO}_2)$  will increase.
- b.  $n(\text{N}_2\text{O}_4)/n(\text{NO}_2)$  will stay the same.
- c.  $n(\text{N}_2\text{O}_4)/n(\text{NO}_2)$  will decrease.

***The next two problems are related:***

A small amount of water (5 g) initially at  $20^\circ\text{C}$  is placed into a sealed container, with a movable piston. The air above the (massless) piston is at 1-atm pressure. Assume the heat of vaporization of water is  $2260 \text{ kJ/kg}$ , and the specific heat of water is  $4.2 \text{ kJ/kg}\cdot\text{K}$ .



32. Estimate the total change in entropy of the water as the temperature of the container is raised from  $20^\circ\text{C}$  to  $100.01^\circ\text{C}$  (i.e., just hot enough to boil the water). The three pictures represent the system as it is being heated but before boiling, while it is boiling, and after the boiling is completed.

- a.  $\Delta S = 5.07 \text{ J/K}$
- b.  $\Delta S = 38.5 \text{ J/K}$
- c.  $\Delta S = 43.6 \text{ J/K}$
- d.  $\Delta S = 113 \text{ J/K}$
- e.  $\Delta S$  cannot be determined from the information given.

33. Estimate how much work is done in this process.

- a. No work is done, since both the pressure and temperature are constant.
- b. 340 J
- c. 860 J

34. Carbon dioxide ( $\text{CO}_2$ ) has a latent heat of fusion of 184 kJ/kg. Approximately by what factor is the number of microstates for *each*  $\text{CO}_2$  molecule increased by melting a quantity of carbon dioxide from solid to liquid at the melting temperature  $-78^\circ\text{C}$ ?

- a. 5
- b. 50
- c. 150
- d. 250
- e. 500