Lecture 4:

Classical Illustrations of Macroscopic Thermal Effects

- Heat capacity of solids & liquids
- Thermal conductivity

References for this Lecture:
Elements Ch 3,4A-C

Reference for Lecture 5:
Elements Ch 5

Last time: Heat capacity

Remember the 1st Law of Thermodynamics:

$Q = \Delta U + W_{by}$ (conservation of energy)

If we add heat to a system, it can do two things:

- Raise the temperature (internal energy increases)
- Do mechanical work (*e.g.*, expanding gas)

How much does the temperature rise?

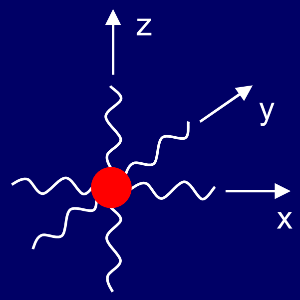
Define heat capacity to be the amount of heat required to raise the temperature by 1 K.	$C \equiv \dfrac{Q}{\Delta T}$

The heat capacity is proportional to the amount of material.
It can be measured either at constant volume (C_V) or constant pressure (C_P).

It depends on the material, and may also be a function of temperature.

Heat Capacity of a Solid

The atoms in a solid behave like little balls connected by springs. Here's one atom:
It has x, y, and z springs as well as x, y, and z motion.



If T is not too low, the equipartition theorem applies, and each kinetic and potential term contributes ½ kT to the internal energy:

$$U = 3(\tfrac{1}{2}kT) + 3(\tfrac{1}{2}kT) = 3kT$$

Equipartition often works near room temperature and above.

Therefore, a solid with N atoms has this heat capacity:

$$\boxed{C = 3Nk = 3nR}$$

Note: For solids (and most liquids), the volume doesn't change much, so $C_P \sim C_V$ (no work is done).

The temperature dependence of C is usually much larger in solids and liquids than in gases (because the forces between atoms are more important).

Heat Capacity and Specific Heat

Heat capacity: The heat energy required to raise the temperature of an object by 1K (=1° C).

It depends on the amount of material. Units: J / K

$$C = \frac{Q}{\Delta T}$$

Upper case "C"

Specific heat: The heat capacity normalized to a standard amount of material (mass or moles).

It only depends on the kind of material.

Normalize to mass: Units: J/kg·K → $c = \frac{C}{m}$

Lower case "c"

Normalize to moles: Units: J/mole·K → $c_{mol} = \frac{C}{n}$

"molar specific heat"

Normalize to volume: Units: J/m^3·K → $c_{vol} = \frac{C}{V}$

"volume specific heat"

Question: Which has the higher c, aluminum or lead?

Act 1

An m_1 = 485-gram brass block sits in boiling water (T_1 = 100° C). It is taken out of the boiling water and placed in a cup containing m_2 = 485 grams of ice water (T_2 = 0° C). What is the final temperature, T_F, of the system (*i.e.*, when the two objects have the same T)? (c_{brass} = 380 J/kg·K; c_{water} = 4184 J/kg·K)

a. T_F < 50° C

b. T_F = 50° C

c. T_F > 50° C

Solution

An m_1 = 485-gram brass block sits in boiling water (T_1 = 100° C). It is taken out of the boiling water and placed in a cup containing m_2 = 485 grams of ice water (T_2 = 0° C). What is the final temperature, T_F, of the system (*i.e.*, when the two objects have the same T)? (c_{brass} = 380 J/kg·K; c_{water} = 4184 J/kg·K)

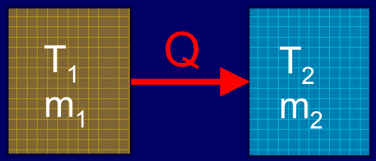
a. $T_F < 50°$ C

b. $T_F = 50°$ C

c. $T_F > 50°$ C

Solution:

Heat flows from the brass to the water. No work is done, and we assume that no energy is lost to the environment.



Remember: $Q = C\Delta T = mc\Delta T$

Brass (heat flows out): $Q_1 = \Delta U_1 = m_1c_1(T_F - T_1)$

Water (heat flows in): $Q_2 = \Delta U_2 = m_2c_2(T_F - T_2)$

Energy is conserved: $Q_1 + Q_2 = 0$

Solve for T_F:

$$T_F = (m_1c_1T_1 + m_2c_2T_2) / (m_1c_1 + m_2c_2)$$
$$= (c_1T_1 + c_2T_2) / (c_1 + c_2) = 8.3° \text{ C}$$

We measured T_F = _____ ° C.

Home Exercises: Cooking

While cooking a turkey in a microwave oven that puts out 500 W of power, you notice that the temperature probe in the turkey shows a 1°C temperature increase every 30 seconds. If you assume that the turkey has roughly the same specific heat as water (c= 4184 J/kg-K), what is your estimate for the mass of the turkey?

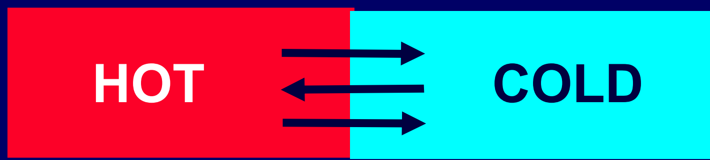
3.6 kg

You place a copper ladle of mass m_L=0.15 kg (c_L = 386 J/kg-K) - initially at room temperature, T_{room}= 20° C - into a pot containing 0.6 kg of hot cider (c_c = 4184 J/kg-K), initially at 90° C. If you forget about the ladle while watching a football game on TV, roughly what is its temperature when you try to pick it up after a few minutes?

88° C = 190° F

Heat Conduction

Thermal energy randomly diffuses equally in all directions, like gas particles (next lecture). More energy diffuses out of a high T region than out of a low T region, implying net energy flow from HOT to COLD.



The heat current, H, depends on the gradient of temperature,

For a continuous change of T along x: $H \propto dT / dx$

For a sharp interface between hot and cold: $H \propto \Delta T$

Heat Conduction (2)

Heat current density J is the heat flow per unit area through a material.
Units: Watts/m^2

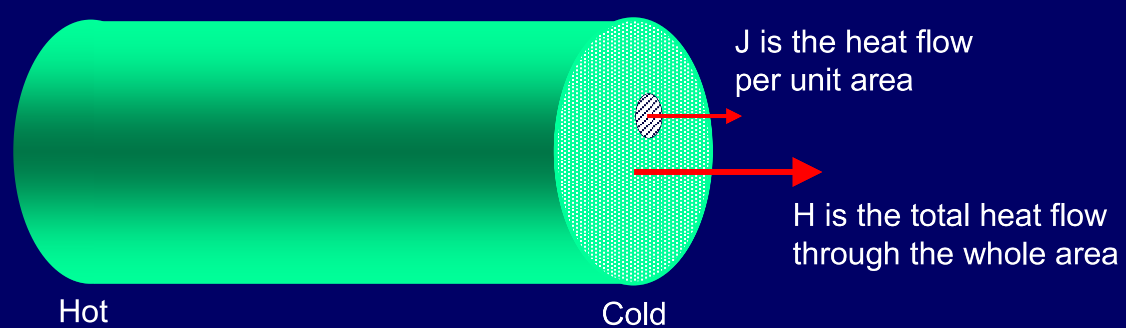
$$\boxed{J = -\kappa \, dT / dx}$$

(- sign because heat flows toward cold)

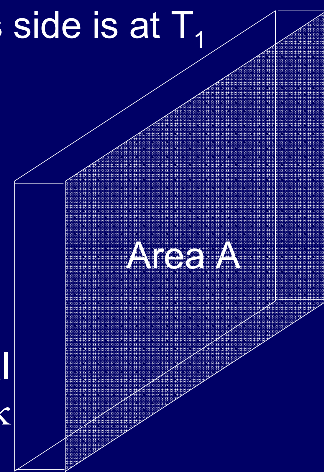
Thermal conductivity κ is the proportionality constant, a property of the material.
Units: Watts/m·K

Total heat current H is the total heat flow through the material.
Units: Watts

$$\boxed{H = J \cdot A}$$



Typical Problem in Heat Conduction



$$J = -\kappa \frac{dT}{dx} = -\kappa \frac{T_1 - T_2}{d} = \kappa \frac{\Delta T}{d}$$

$$H = J \cdot A = \left(\frac{\kappa A}{d}\right) \Delta T = \frac{\Delta T}{R_{thermal}},$$

$$\text{where } R_{thermal} \equiv \frac{d}{\kappa A}$$

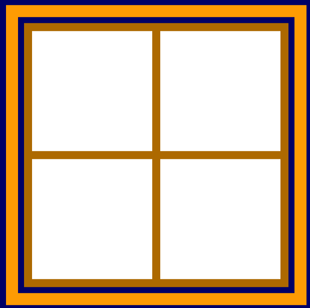
Thermal resistance is defined to make the similarity to electrical current flow (Ohm's Law) clear:

$$I = \frac{\Delta V}{R}$$

$\Delta V = V_1 - V_2$
= voltage drop

Exercise: Heat Loss Through Window

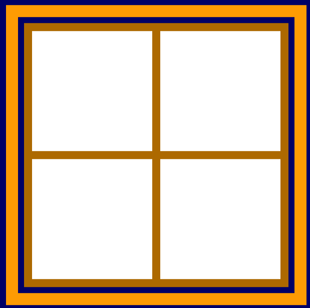
If it's 22°C inside, and 0°C outside, what is the heat flow through a glass window of area 0.3 m^2 and thickness 0.5 cm ?



The thermal conductivity of glass is about 1 W/m·K.

Solution

If it's 22°C inside, and 0°C outside, what is the heat flow through a glass window of area 0.3 m^2 and thickness 0.5 cm ?



The thermal conductivity of glass is about 1 W/m·K.

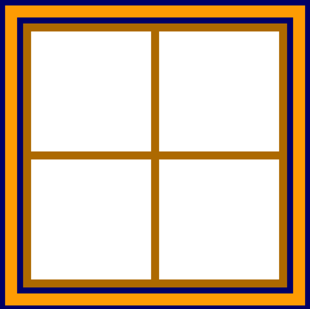
$$H = J \cdot A = \left(\frac{\kappa A}{d}\right)\Delta T$$

$$H = \left(1\frac{W}{mK}\right)\left(\frac{0.3m^2}{5\times 10^{-3}m}\right)(22K) = 1320\ W$$

That's a lot! Windows are a major cause of high heating bills.

ACT 2

How much heat is lost through a double-pane version of that window, with an 0.5-cm air gap?
The thermal conductivity of air is about 0.03 W/(m K).



Hint: Ignore the glass, which has a much higher conductivity than air.
H is limited by the high resistance air gap.

A) 20 W B) 40 W C) 1320 W d) 44,000 W

Solution

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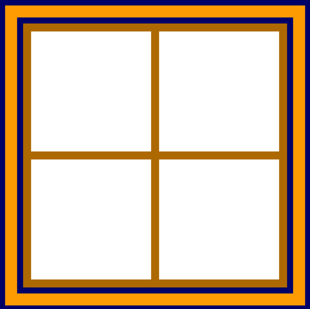


Hint: Ignore the glass, which has a much higher conductivity than air.
H is limited by the high resistance air gap.

A) 20 W   B) 40 W   C) 1320 W   d) 44,000 W

$$H = \underbrace{\left(0.03\frac{W}{mK}\right)}_{\text{air}}\left(\frac{0.3m^2}{5\times10^{-3}m}\right)(22K) = 39.6\ W$$

<< 1320 W

Note: Large air gaps don't always work, due to convection currents.

Thermal conductivities (κ at 300 K):

air	0.03 W/m-K
wood	0.1 W/mK
glass	1 W/m-K
aluminum	240 W/m-K
copper	400 W/m-K

How small can κ be ???

Aerogel 8×10^{-5} W/m-K

What's aerogel?

Aerogel

An artificial substance formed by specially drying a wet silica gel, resulting in a solid mesh of microscopic strands.

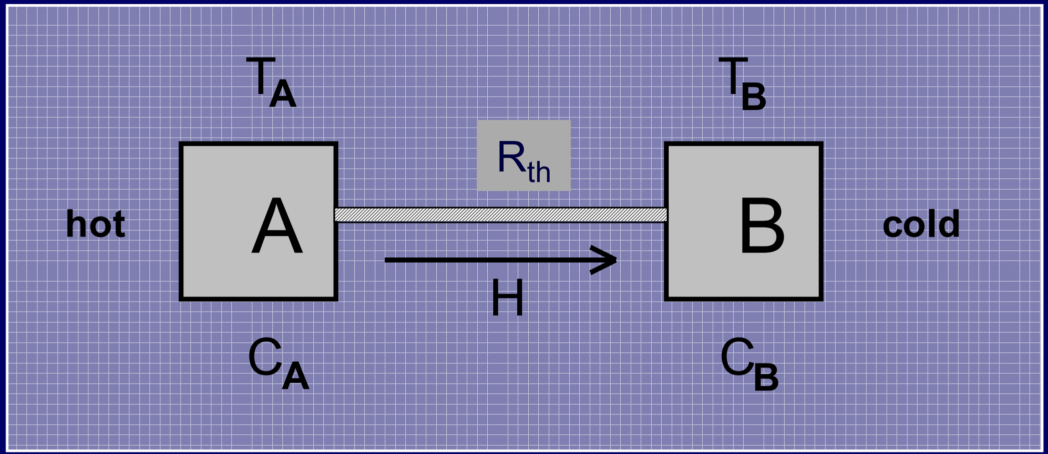
Used on space missions to catch comet dust

The least dense solid material known (ρ = 1.9 mg/cm^3. ρ_{air} = 1.2 mg/cm^3). 98% porous, but nevertheless, quite rigid:

2.5 kg brick 2 g aerogel

κ = 8 x 10^{-5} W/m-K

How Long Does Heat Conduction Take?



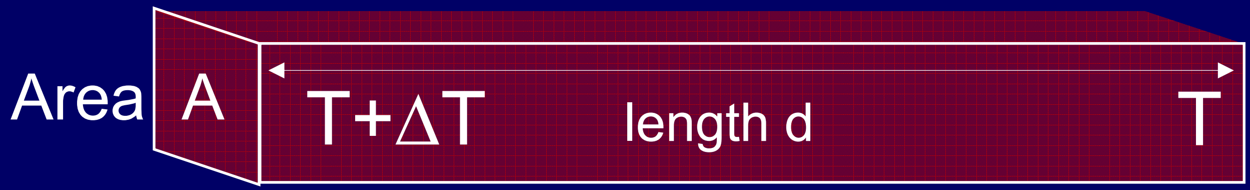
The heat current *H* depends on the temperature difference between the two samples, and the thermal resistance: $R_{th} = d/A\kappa$.

Assume that all the heat leaving A enters B.

The temperature of the samples depends on their initial temperatures, the amount of heat flowing into (out of) them, and their heat capacities.

In general, the time to reach thermal equilibrium is a nontrivial problem, but we can estimate the time it will take.

How Long to Equilibrate a Rod?



The rate at which heat flows from hot to cold is about

$H = \kappa A \Delta T / d.$

The heat capacity is $C = cm = c\,\rho\,(\text{vol}) = c\,\rho\,(dA)$ ρ = density

So the rate at which ΔT is reduced is about

$$H = \frac{dQ}{dt} = \mathrm{C}\frac{d(\Delta T)}{dt} = \frac{\Delta T\,(t)}{\mathrm{R_{th}}}$$

$$\frac{d(\Delta T)}{dt} = \frac{\Delta T\,(t)}{\mathrm{R_{th}C}} = \frac{\Delta T\,(t)}{\tau}$$

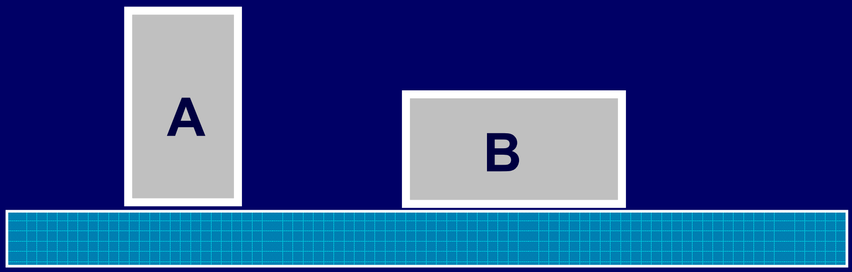
$$\tau = \mathrm{R_{th}C} = (\mathrm{d}/\kappa \mathrm{A})(\mathrm{c}\rho \mathrm{dA}) \propto \mathrm{d}^2$$

Alternatively, the typical distance that the thermal energy has *diffused* varies with the square root of the time: $d \propto \sqrt{t}$. This fact is a result of the random nature of heat flow, which we'll discuss more next lecture.

ACT 3: Cooling pots

You are cooking with two pots that have the same volume. Pot B has half the height, but twice the area as Pot A. Initially the pots are both full of boiling water (e.g., 100 °C). You set them each on the bottom of your metal sink. Which cools faster?

A) Pot A
B) Pot B
C) They cool at same rate



Solution

You are cooking with two pots that have the same volume. Pot B has half the height, but twice the area as Pot A. Initially the pots are both full of boiling water (e.g., 100 °C). You set them each on the bottom of your metal sink. Which cools faster?

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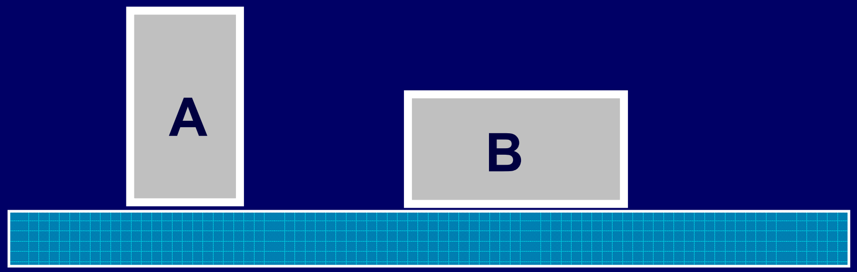
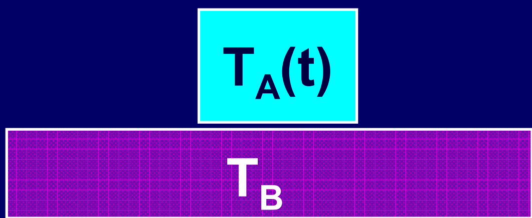
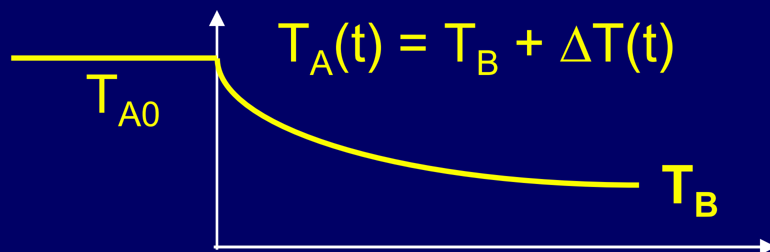


The rate of cooling is determined by the time constant $\tau = R_{th}C$.
The pots have the same amount of water, so they have the same heat capacity.
The thermal resistance $R_{th} = d/\kappa A$.
Assume that the thickness, d, of the pot bottoms is the same.
Pot B has a larger area, so it will have a smaller R_{th}, and therefore a shorter τ
→ it will cool faster.

Heat conduction – How long does it take?

- For simplicity we assume that system B is really big (a “thermal reservoir”), so that it’s temperature is always T_B.





$$C = \frac{\Delta U}{\Delta T}$$

- Heat “into” A:

$$H_A(\mathrm{t}) = -\frac{T_A(t) - T_B}{\mathrm{R_{th}}} = \boxed{-\frac{\Delta T\,(t)}{\mathrm{R_{th}}}} = \frac{Q_A}{dt} = \frac{dU_A}{dt} = \boxed{C_A \frac{d(\Delta T\,(t))}{dt}}$$

$$\frac{d(\Delta T\,(t))}{dt} = -\frac{\Delta T\,(t)}{\mathrm{R_{th}} C_A}$$

- Guess solution: $\Delta T(t) = \Delta T(t{=}0)\, e^{-t/\tau}$, or $T_A(t) = T_B + (T_{A0} - T_B)\, e^{-t/\tau}$

- Plug into above DiffEQ: $\tau = R_{th}C_A$ (like a discharging capacitor!)

Act 4: Exponential Cooling

A hot steel bearing (at T = 200 C) is dropped into a large vat of cold water at 10 C. Compare the time it takes the bearing to cool from 200 to 190 C to the time it takes to cool from 100 to 90 C.(Assume the specific heat of steel is ~constant over this temperature range.)

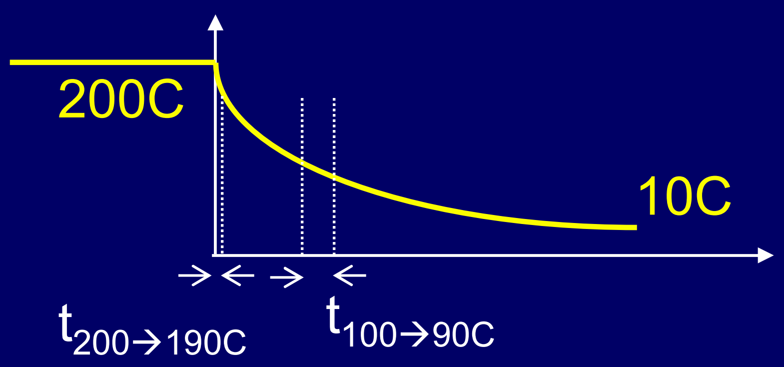
a. $t_{200\rightarrow 190C} > t_{100\rightarrow 90C}$ b. $t_{200\rightarrow 190C} = t_{100\rightarrow 90C}$ c. $t_{200\rightarrow 190C} < t_{100\rightarrow 90C}$

Solution

A hot steel bearing (at T = 200 C) is dropped into a large vat of cold water at 10 C. Compare the time it takes the bearing to cool from 200 to 190 C to the time it takes to cool from 100 to 90 C.(Assume the specific heat of steel is ~constant over this temperature range.)

a. $t_{200\rightarrow190C} > t_{100\rightarrow90C}$ b. $t_{200\rightarrow190C} = t_{100\rightarrow90C}$ c. $\boxed{t_{200\rightarrow190C} < t_{100\rightarrow90C}}$

However, the *rate* of heat flow out of the bearing depends on $T_{bearing}(t) - T_{water}$, and is different (~190 C and ~90 C) for the two cases. Because more heat flows at the outset, the initial temperature drop is faster.



FYI: Thermal Diffusion and Heat Conduction

How is it that random motion can give heat flow in a particular direction?

Thermal energy randomly diffuses around, spreading out. However, the heat flow out of a region is proportional to the amount of energy that is there at that time.

Look at region 2. More heat will randomly diffuse in from a high T region than from low T:

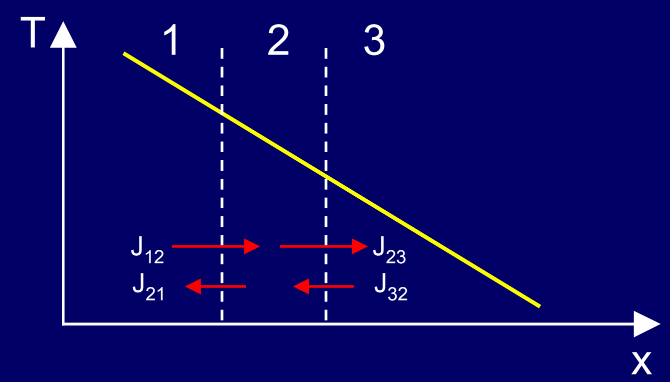
$J_{12} > J_{21}$, and $J_{23} > J_{32}$.

So there will be a net flow of heat in the direction of decreasing T.

In 1-D the heat current density is:

$$\boxed{J_H = -\kappa \frac{dT}{dx}}$$

where κ is the thermal conductivity, a property of the material.



Next Time

- Random Walk and Particle Diffusion
- Counting and Probability
- Microstates and Macrostates
- The meaning of equilibrium