Last Name:________________First Name
NetID____________________________
Discussion Section:__Discussion TA Name:________________________________

Instructions—

This is a closed book exam. You have ninety (90) minutes to complete it.

1. Use a #2 pencil. Do not use a mechanical pencil or pen. Darken each circle completely, but stay within the boundary. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner. Be especially careful that your mark covers the **center** of its circle.

2. You may find the version of **this Exam Booklet at the top of page 2**. Mark the **version** circle in the **TEST FORM** box near the bottom right of your answer sheet. **DO THIS NOW!**

3. Print your **NETWORK ID** in the designated spaces at the *right* side of the answer sheet, starting in the left most column, then **mark the corresponding circle** below each character. If there is a letter "o" in your NetID, be sure to mark the "o" circle and not the circle for the digit zero. If and only if there is a hyphen "-" in your NetID, mark the hyphen circle at the bottom of the column. When you have finished marking the circles corresponding to your NetID, check particularly that you have not marked two circles in any one of the columns.

4. Print **YOUR LAST NAME** in the designated spaces at the *left* side of the answer sheet, then mark the corresponding circle below each letter. Do the same for your **FIRST NAME INITIAL**.

5. Print your UIN# in the **STUDENT NUMBER** designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION box.

6. Sign your name **(DO NOT PRINT)** on the **STUDENT SIGNATURE** *line*.

7. On the **SECTION** *line*, print your **DISCUSSION SECTION**. You need not fill in the COURSE or INSTRUCTOR lines.

*Before starting work, check to make sure that your test booklet is complete. You should have **9 numbered pages** plus a Formula Sheet at the end.*

Academic Integrity—**Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including expulsion.**

This Exam Booklet is Version A. Mark the **A** circle in the **Test Form** box near the bottom right of your answer sheet. **DO THIS NOW!**

Exam Grading Policy—

The exam is worth a total of 114 points, composed of two types of questions. For each type of question, you should **choose the best answer available**.

MC5: *multiple-choice-five-answer questions, each worth 6 points.*
Partial credit will be granted as follows.

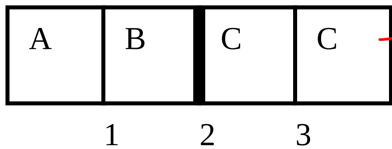
(a) If you mark only one answer and it is the correct answer,
you earn **6** points.
(b) If you mark *two* answers, one of which is the correct answer,
you earn **3** points.
(c) If you mark *three* answers, one of which is the correct answer,
you earn **2** points.
(d) If you mark no answers, or more than *three*, you earn 0 points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*
No partial credit.

(a) If you mark only one answer and it is the correct answer,
you earn **3** points.
(b) If you mark a wrong answer or no answers, you earn **0** points.

The next two questions pertain to the following situation

Consider a box having 4 bins, divided into two parts by a moveable partition. The left part has 2 distinguishable particles, A and B, and the right part has 2 indistinguishable particles, each called C. A microstate of the system is specified by identifying which particles are in which bins. Allow multiple occupancy in each side. One particular microstate is illustrated, and particles are free to move between bins on their respective sides of the partition.



1. With the partition as shown, what is the total entropy σ_T?

$\sigma = \ln \Omega$

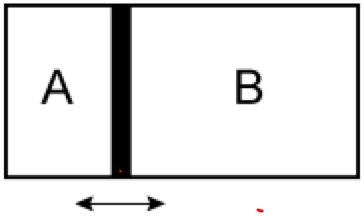
$\Omega_T = \Omega_L \times \Omega_R$

$4 \times 3 = 12$

a. $\sigma_T = 1.52$
~~b. $\sigma_T = 1.94$~~
c. $\sigma_T = 2.48$

2. If we now remove the partition and allow the particles to move freely in all four bins, what is the new total entropy? (Hint: The total number of microstates is the product of the microstates for the different type of particles, e.g., A, B, C.)

$\Omega_A = 4$, $\Omega_B = 4$



a. $\sigma_T = 2.97$
b. $\sigma_T = 3.56$
c. $\sigma_T = 4.16$
d. $\sigma_T = 5.08$
e. $\sigma_T = 5.54$

$$\Omega_C = \frac{(N+M-1)!}{N!(M-1)!} = \frac{(2+4-1)!}{2!(4-1)!} = 10$$

$$\Omega_{tot} = \Omega_A \Omega_B \Omega_C = 160$$

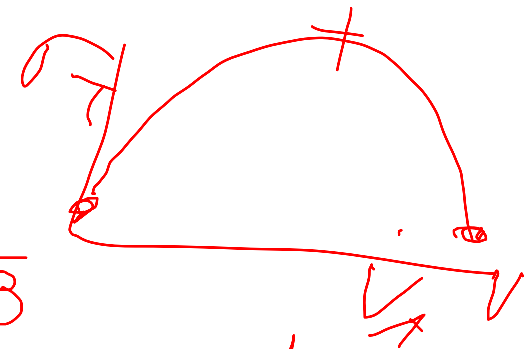
3. Let two different (and not necessarily ideal) gases (A and B) at fixed equal temperature be confined to opposite sides of a fixed-volume cylinder with a sliding piston in between. In thermal equilibrium, which of these equations holds (with the usual definitions of the symbols):

a. $\sigma_A=\sigma_B$
b. $d\Omega_A/dV_A=d\Omega_B/dV_B$
c. $d\sigma_A/dV_A=d\sigma_B/dV_B$

Max total number of microstate
" " entropy

$\sigma_T = \sigma_A + \sigma_B$

$\frac{d\sigma_T}{dV_A} = 0 = \frac{d\sigma_A}{dV_A} + \frac{d\sigma_B}{dV_A} = \frac{d\sigma_A}{dV_A} - \frac{d\sigma_B}{dV_B}$



$dV_A = -dV_B$

$\Omega = n^N$
$= n_T V^N$
$\sigma = N \ln V + \text{const}$
$d\sigma = \frac{N}{\ldots}$

$pV = NkT$

$\frac{N_A}{V_A} = \frac{N_B}{V_B}$

$\frac{N}{V} = \frac{p}{\ldots}$

$\sigma_A = \ln \Omega_A$

The next two questions pertain to the following situation:

An isolated system in internal thermal equilibrium can be in either macrostate A with entropy $\boldsymbol{\sigma_A}$ **= 27** or macrostate B with entropy $\boldsymbol{\sigma_B}$ **=20.** A particular microstate X is part of the A macrostate, and a particular microstate Y is part of the B macrostate.

4. What is the ratio of the probabilities, P_A/P_B, that the system will be found in the two macrostates?

a. $P_A/P_B = 1097$
b. $P_A/P_B = 1.0$
c. $P_A/P_B = 1.3$
d. $P_A/P_B = 148$
e. $P_A/P_B = 0.2$

$P_A = \dfrac{\Omega_A}{\Omega_{tot}}$

$P_B = \dfrac{\Omega_B}{\Omega_{tot}}$

$\dfrac{P_A}{P_B} = \dfrac{\Omega_A}{\Omega_B} = \dfrac{e^{\sigma_A}}{e^{\sigma_B}} = e^{\sigma_A - \sigma_B} = e^{(27-20)} = e^7$

5. What is the ratio of the probabilities, P_X/P_Y, that the system will be found in the two particular microstates?

a. $P_X/P_Y = 1.3$
b. $P_X/P_Y = 0.2$
c. $P_X/P_Y = 1097$
d. $P_X/P_Y = 1.0$
e. $P_X/P_Y = 148$

All microstates equally likely

$P_x = P_y$

q quanta

$q=3$

The next two questions pertain to the following situation

A total of 3 energy quanta are distributed in equilibrium within a collection of 2 oscillators, each with the same energy level spacing.

$$\frac{(3+2-1)!}{3!\,(2-1)!} = \frac{4!}{3!} = 4$$

6. How many microstates are available?

a. Ω=3
b. Ω=6
c. Ω=4
d. Ω=8
e. Ω=9

BAD Problem

3 oscillators $q = 3$ $N = 3$

7. What is the ratio of the average number of quanta to the most likely number of quanta in the first oscillator?
<q> = average number of quanta in the first oscillator.
q_{ml} = most likely number of quanta in the first oscillator.

$\frac{3 \text{ quanta}}{3 \text{ oscillators}}$ $\langle N \rangle = 3$

a. $\langle q \rangle / q_{ml} = 0$
b. $\langle q \rangle / q_{ml} = \infty$
c. $\langle q \rangle / q_{ml} = 1$

$\frac{q \text{ in } 1:}{}$ Ω_1, Ω_2 $\Omega_{tot} = \Omega_1 \times \Omega_2$

0

1

2

3

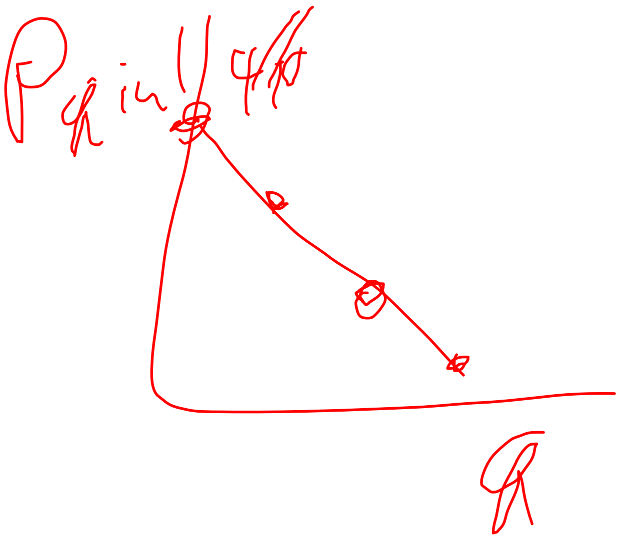
All equally likely

$\langle N \rangle = \frac{1}{4}[0+1+2+3] = \frac{6}{4} = 1.5$

3 quanta

$\langle N \rangle = 1 \quad \left(= \frac{3}{3}\right)$

Most likely = 0



q in 1	Ω_1	Ω_{23}	Ω_{tot}
0	1	4	4
1	1	3	3
2	1	2	2
3	1	1	1
			10

$$\frac{(q+N-1)!}{q!\,(N-1)!} = \frac{(3+2-1)!}{3!\,(2-1)!}$$

The next two questions pertain to the following situation

One end of a cylinder containing 2 moles of O_2 gas at 25 °C is sealed by a massless piston. The cylinder also contains a small piece of paper. You want to ignite the paper by quickly compressing the piston and adiabatically heating the O_2 gas. (Recall the fire starter demo from class.) Paper burns at 232 °C.

No heat flow $\boxed{Q = 0}$ $Q = \Delta U + W_{by}$

8. What is the smallest fractional change in the position of the piston to cause the paper to burn?

a. ΔL/L = 0.73
b. ΔL/L = 0.55
c. ΔL/L = 0.63
d. ΔL/L = 0.27
e. ΔL/L = 0.94

$$\frac{A(L-\Delta L)}{AL} = \frac{V_f}{V_i} = \left(\frac{T_i}{T_f}\right)^{5/2} = \left(\frac{25+273}{232+273}\right)^{5/2} = 0.267$$

$\Delta U = Q + W_{on}$

$W_{on} = -W_{by}$

$$1 - \frac{\Delta L}{L} = 0.267$$

9. How much work is done to compress the gas to 232 °C?

a. 8.6 × 10^3 J
b. 1.2 × 10^4 J
c. 5.2 × 10^3 J

$U = \alpha nRT$

$\Delta U = W_{on}$

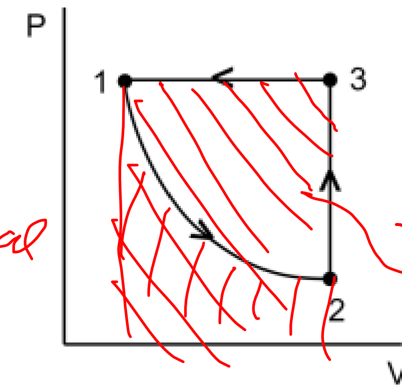
$$\Delta U = \frac{5}{2}(2)(8.314)\boxed{207}$$

$232 - 25$

Adiabatic

Isothermal

$W_A > W_{isothermal}$



oscillation

$+|W_{on}| = -W_{by,3\to1}$

$dW = p\,dV$

$\Delta W = \int p\,dV$

10. 1 mole of N_2 gas expands isothermally at 300 K from 1 liter to 2 liters (1→2). The gas is then compressed back to 1 liter along the path shown (2→3→1). Calculate the work done by the gas on the closed path (1→2→3→1).

a. 0 J
b. 324 J
c. -1245 J
d. -1547 J
e. -765.4 J

$W_{31} = p_1 \Delta V$ $V_3 - V_1$ $= 1\ell$

$+|W_{by}|_{1\to2}$

$= \frac{nRT}{V_1}(V_3 - V_1)$

$W_{tot} = nRT\left[\ln\frac{V_2}{V_1} - \frac{1}{V_1}(V_3 - V_1)\right]$ (300)

$W_{12} = \int p\,dV$

$= nRT \int \frac{dV}{V}$

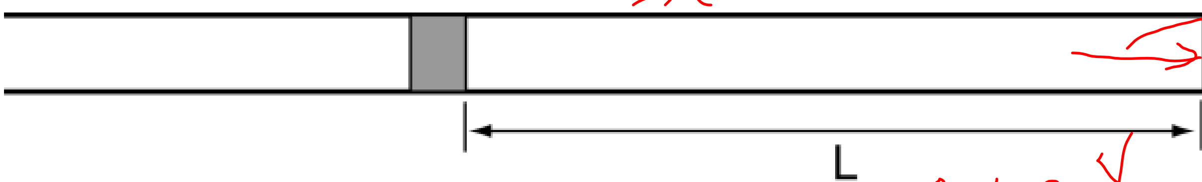
$= nRT \ln\frac{V_2}{V_1}$

The next two questions pertain to the following situation

$+\frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} kT$

Suppose 1 mole of He is confined inside a long cylindrical vessel. The cross section of the vessel is so small that the He atoms can only move along the long axis of the cylinder. Thus, their motion is one dimensional. One end of the vessel is sealed with a movable piston.

$\rightarrow x$



$p_i = m v_x$

$p_f = -m v_x$

$\Delta p = 2 m v_x$

momentum

11. If the He gas is at 300 K, estimate the average force acting on the piston when the piston is located a distance L = 3 m from the closed end of the cylinder.

a. <F> = 830 N
b. <F> = 0.83 N
c. <F> = 8.3 N
d. <F> = 8300 N
e. <F> = 83 N

$F = \frac{dp}{dt} = \frac{\Delta p_{atom}}{\Delta t} N = \frac{2 m v_x}{2L/v_x} N = \frac{v_x^2 m N}{L} = \frac{N}{L} kT = \frac{6\times10^{23} \cdot 300 \cdot k}{3}$

Equipartition:

$\cancel{\tfrac{1}{2}} m v_x^2 = \cancel{\tfrac{1}{2}} kT$

$F_x = P_x \cdot A = \left(\frac{nRT}{V}\right) A = \left(\frac{nRT}{\cancel{A} \cdot L}\right) \cancel{A}$

$\cancel{\frac{3}{2}kT} = \frac{1}{2}m\langle v_x^2\rangle$

12. If we substitute the He gas in the cylinder with 1 mole of N_2, how does the average force change?

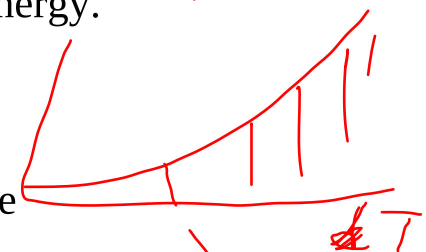
a. Increases, because N_2 is a diatomic molecule and therefore has a larger average thermal energy than He.
b. Decreases, because N_2 has more degrees of freedom than He and therefore has less energy available for translational degrees of freedom.

false

c. Stays the same, because N_2 and He have the same average translational kinetic energy.

$\langle KE_{trans}\rangle = \frac{3}{2}kT$

13. The heat capacity of a particular solid has the following form $C(T) = \alpha T^3$, where $\alpha = 3\times10^{-4}$ J K^{-4}. Calculate the internal energy of the solid at T = 100 K.



a. 5.0×10^3 J
b. 9.4×10^3 J
c. 7.5×10^3 J
d. 3.2×10^4 J
e. 1.3×10^4 J

$C = \frac{dQ}{dT} = \frac{dU}{dT}$

$dQ = dU + \cancel{dW_{by}}$

$dU = C\,dT$

$\Delta U = \int_0^T C\,dT = \alpha\int_0^T T^3\,dT = \alpha\frac{T^4}{4}\Big|_0^T$

$= \frac{\alpha T^4}{4} = \frac{3\times10^{-4}(100)^4}{4}$

$Q = U + W_{by}$

New question: $S(T=100K)$

$$\frac{1}{T} = \left.\frac{dS}{dU}\right|_V$$

$$dS = \frac{dU}{T} = \frac{C\,dT}{T} = \frac{(aT^3)\,dT}{T}$$

"Third Law"

$$S(T\to 0) \to 0$$

$$S = \int dS =$$

$$dS = aT^2\,dT$$

$$\int_0 dS = a\int T^2\,dT$$

$$= a\left.\frac{T^3}{3}\right|_0^{100}$$

$$= \frac{aT^3}{3}$$

Alternate: All 3-R? C_4?

$C_4 = \frac{dU}{dT} = \frac{d}{dT}[N3kT] = 3Nk = 3nR$

$= RC$

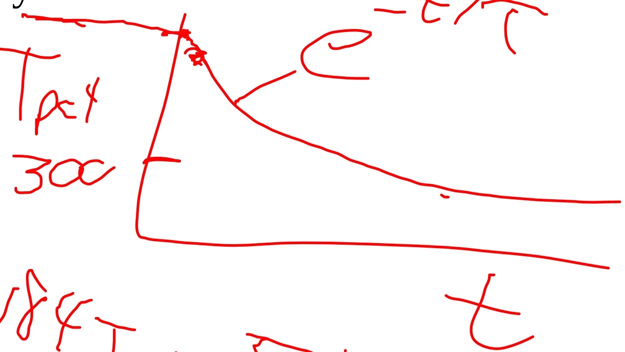
~~325~~

14. A pot containing 3 kg of water is heated to 350 K and placed on a countertop at 300 K. Here, we treat the countertop as an infinite heat reservoir. The specific heat of water is 4184 J kg^{-1}K^{-1}. The pot is made from copper with $\kappa = 400$ W m^{-1} K^{-1}. The contact area between the pot and the countertop is 0.09 m^2, and the copper on the bottom portion of the pot is 2-mm thick. Neglect the heat capacity of the pot. How long does it take for the temperature of the water to cool by 5 K?

a. 0.700 s
b. 4.720 s
c. 7.000 s
d. 0.074 s
e. 2.450 s

How much heat removed?

T_{at} 300



$Q = C\,dT = (3\,kg)(4184\,\frac{J}{kg\,K}) \cdot 5\,K$

$H = \frac{\Delta T}{R_{th}} = \frac{kA\,\Delta T}{d}$ $\Delta T \Rightarrow 50\,K$

$R_{th} = \frac{d}{kA}$

$= [H] = \text{Watts} = \frac{dQ}{dt} = \frac{C\,dT}{dt} = \frac{\Delta T}{R_{th}}$

$\Delta T(t) = \Delta T(t=0)\, e^{-t/\tau}$

$45 = 50\, e^{-t/\tau}$

$dt = R_{th}\, C\, dT$

$\Delta T = \frac{d}{kA}\, mc \frac{5K}{50K}$

$= 0.0697\,s$

$10\,\mu m^2/s$

$\frac{R^2}{6D} = \frac{(1\,\mu m)^2}{6 \cdot 10\,\mu m^2/s} = 0.01625 \approx 0.02\,s$

The next three problems are related

15. Cells (e.g., animal cells) are complex; survival requires transport of various components throughout the cell. For small cells (e.g., bacteria), this occurs via simple diffusion. If the diffusion constant for a protein in the cell is 10 μm^2/s, approximately how long will it take a protein to diffuse from the center of the cell to any point on the cell wall (assume a spherical cell), assuming the cell radius is 1 μm?

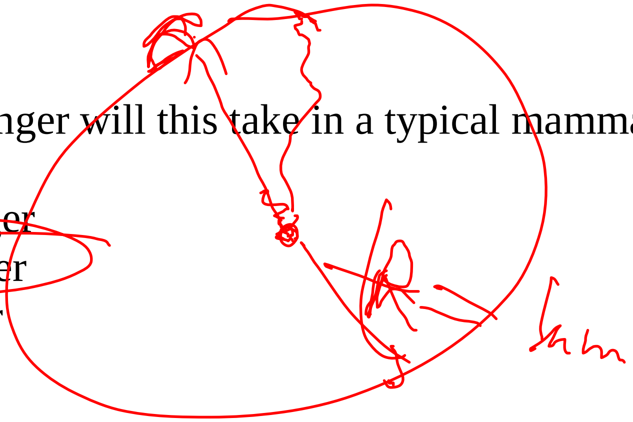
a. 20 milliseconds
b. 500 milliseconds
c. 0.5 milliseconds
d. 50 milliseconds
e. 200 milliseconds

$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$

16. How much longer will this take in a typical mammalian cell (radius ~10 μm)?

$= 6Dt$

a. ~~3.16 times longer~~
b. 100 times longer
c. 10 times longer



$t =$

$t \sim R^2$

$$\frac{t(10\,\mu m)}{t(1\,\mu m)} = 100$$

17. Assuming the diffusion is driven simply by thermal random motion, by what factor would the diffusion time across the bacteria change if we were to raise the temperature from 23°C to 30°C (and assuming the mean free path of the protein is the same)?

a. The time becomes longer by a factor of 1.034.
b. The time becomes shorter by a factor of 0.967.
c. The time becomes longer by a factor of 1.017.
d. The time becomes shorter by a factor of 0.876.
e. The time becomes shorter by a factor of 0.988.

$$t \sim \frac{r^2}{6D} \propto \frac{1}{D} \propto \frac{1}{v} \propto \frac{1}{\sqrt{T}}$$

$$D = \frac{\ell v}{3}$$

$$\frac{1}{2} m v^2 = \frac{3}{2} kT$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

$$\frac{t_{30}}{t_{23}} = \sqrt{\frac{293}{303}}$$

$$\approx 0.988$$

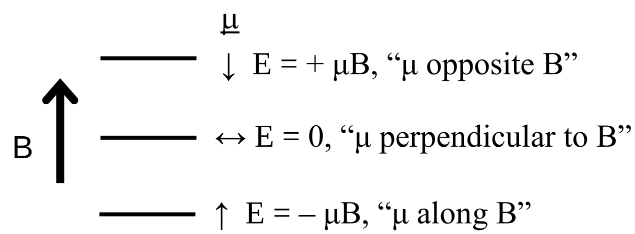
The next four questions are related

18. Consider a collection of atoms, each of which, according to quantum mechanics, can have precisely three values of the z-component of angular momentum ($-h/2\pi$, 0, and $+h/2\pi$), and thus can have a magnetic moment with the three possible values, $m\mu_z$, where m = +1, 0, and –1. In a magnetic field (along the z-direction), these have an energy given by $-m\mu_z B$, as shown in the diagram.

Assuming $\mu_z = 9.27 \times 10^{-24}$ J/T, and B = 1 Tesla, at T = 2 K what is the likelihood P(0) to find an atom in the <u>middle</u> state (i.e., with m = 0)?

$$\frac{\mu B}{kT} \approx 0.33$$

a. 0.003
b. 0.03
c. 0.32
d. 0.5
e. 0.67



$$Z = e^{-\mu B/kT} + e^{0} + e^{\mu B/kT}$$

$$P(0) = \frac{e^{-0}}{Z} = \frac{1}{e^{0.33} + 1 + e^{-0.33}} \approx 0.32$$

19. Now we increase the magnetic field to 10 T. What happens to P(0)?

a. P(0) decreases
b. P(0) increases
c. P(0) stays the same

~ All ~~are~~ spins align with field

$P(-\mu B) \Rightarrow 1$

$\therefore P(0) \Rightarrow 0$

20. Now (with the field at 10 T) we increase the temperature T (a lot). In the limit as the temperature goes to infinity, what happens to P(0)?

a. P(0) → 1/3
b. P(0) → 1/2
c. P(0) → 1
d. P(0) → 0
e. Cannot tell from the information given

$$P(0) \Rightarrow \frac{1}{e^{+\mu B/kT} + 1 + e^{-\mu B/kT}}$$

$$\Rightarrow \frac{1}{1+1+1} = \frac{1}{3}$$

21. Consider N such atoms (located on a lattice). What is the ratio of the entropy of these atoms at high temperature, to the entropy at low temperature?

$S = k \ln \Omega$

$\Omega(T \to \infty) = 3^N$

$S = kN \ln 3$

$T \to 0$

$\Omega \Rightarrow 1$ (all in lowest state)

$S \Rightarrow 0$

a. $S(T\to\infty)/S(T\to 0) = \infty$

b. $S(T\to\infty)/S(T\to 0) = 0$

c. $S(T\to\infty)/S(T\to 0) = Nk\ln 3$

$\downarrow \quad +\mu B$

$- \quad 0$

$\uparrow \quad -\mu B$

$$\frac{C(T \to \infty)}{C(T \to 0)} \qquad C = \frac{dU}{dT} \Rightarrow 0$$

all in lowest

$n \approx N/L$

$n(h) \propto Prob(h)$

$n(h=0) \propto Prob(0)$

The next three questions are related

22. We want to design an experiment to demonstrate the law of atmospheres. We will do this by creating tiny water droplets of a fixed size, letting them fly around in a tall tube, and observing the decrease in their density (how many droplets per m^3) as a function of height in the tube.

What should be the approximate mass of each water droplet, such that the density of the water droplets at 1-meter height is half of what it is at 0 height, assuming 25°C? $= 298$

$E_h = mgh$

$$\frac{n(h)}{n(0)} = \frac{Prob(h)}{Prob(0)} = \frac{e^{-mgh/k_BT}}{e^{0}}$$

a. 1×10^{-25} kg
b. 3×10^{-24} kg
c. 2×10^{-23} kg
d. 3×10^{-22} kg
e. There is no choice of mass for which this is possible.

$$e^{-mgh/k_BT} = \frac{1}{2} \qquad m = \frac{k_BT}{gh} \ln\left(\frac{1}{2}\right)$$

23. Now we add some heat into the container, and allow it to re-equilibrate (at a higher average temperature). What will happen to the density of the water droplets near the top of the tube?

a. the density will decrease
b. the density will stay the same
c. the density will increase

24. Compare the temperature near the top of the tube to that near the bottom.

a. hotter at the top
. hotter at the bottom
c. same temperature

$$h_c = \frac{kT}{mg}$$