Physics 213 Formula Sheet

Constants, Data, Definitions

- Absolute zero: $0\ \mathrm{K} = -273.15°\mathrm{C} = -459.67°\mathrm{F}$
- Avogadro's number: $N_A = 6.022 \times 10^{23}$ / mole
- Boltzmann constant: $k = 1.38 \times 10^{-23}\,\frac{J}{K} = 8.617 \times 10^{-5}\,\frac{eV}{K}$
- Universal gas constant: $R = k\,N_A = 8.314\,\frac{J}{mol\cdot K} = 8.206 \times 10^{-2}\,\frac{\ell\cdot atm}{mol\cdot K}$ (Universal gas const.)
- Planck's constant: $h = 6.626 \times 10^{-34}\ J\cdot s = 4.136 \times 10^{-15}\ eV.s$, $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\ J.s$,
- Magnetic moments: electron: $\mu_e = 9.2848 \times 10^{-24}\,\frac{J}{T}$, proton: $\mu_p = 1.4106 \times 10^{-26}\,\frac{J}{T}$
- Mass: electron: $m_e = 9.109 \times 10^{-31}\mathrm{kg}$, proton: $m_p = 1836\ m_e = 1.673 \times 10^{-27}\mathrm{kg}$
- STP: $T = 0°C$, $p = 1$ atm, V = 22.4 liters
- Stefan-Boltzmann constant: $\sigma_B = 5.670 \times \frac{10^{-8} W}{m^2 K^4}$
- $c = 2.998 \times 10^8\,\frac{m}{s}$, $1\ eV = 1.602 \times 10^{-19}\ J$, $g = 9.8\frac{m}{s^2}$,$1\ atm = 1.013 \times 10^5\ Pa$, $1\ liter = 10^{-3}\ m^3$

Particle	g/mol
N_2	28
O_2	32
He	4
Ar	40
CO_2	44
H_2	2
Si	28
Ge	73
Cu	64
Al	27
$1g = 10^{-3}kg$	

Classical Equipartition:

Per quadratic degree of freedom, $<\text{Energy}> = \frac{1}{2}kT$

Fundamental laws/ Principles

- **1st Law**: $dU = dQ + dW_{on} = dQ - dW_{by}$
- **2nd Law**: $\frac{d\sigma}{dt} \geq 0$
- $P_i \propto \Omega \equiv e^{\sigma_i}$

Entropy and Temperature:

$S \equiv k\sigma \equiv k \ln \Omega$, $\frac{1}{T} \equiv \left(\frac{\partial S}{\partial U}\right)_{V,N}$

Heat Capacities:

$C_V \equiv \left(\frac{\partial U}{\partial T}\right)_V$, $C_p \equiv \left(\frac{\partial (U+pV)}{\partial T}\right)_p$

Thermal processes

- $W_{by} = Q_H - Q_C$
- Carnot condition: $\frac{Q_H}{Q_C} = \frac{T_H}{T_C}$, $\epsilon_{Carnot} = 1 - \frac{T_c}{T_H}$
- $\Delta S = \Delta S_H + \Delta S_C = \frac{Q_H}{T_H} + \frac{Q_C}{T_C}$
- $dF = dU - T_{env}\,dS = -dW_{by}$, $dU = C_V dT$, $\Delta S = \int_{T_i}^{T_f}\left(\frac{C}{T}\right) dT$

Equilibrium with a reservoir

- Boltzmann statistics: $P_i = \frac{e^{-\frac{E_i}{kT}}}{Z}$, $Z \equiv \sum_i e^{-\frac{E_i}{kT}}$
- Free Energies: $F \equiv U - TS$, $G \equiv U - TS + pV$
- Chemical potential:

$$\mu \equiv \left(\frac{\partial F}{\partial N}\right)_{V,T} = \left(\frac{\partial G}{\partial N}\right)_{p,T}, \sum_i (\Delta N_i)\mu = 0 \text{ at equilib.}$$

$$\mu_i = kT \ln\left(\frac{n_i}{n_{Ti}}\right) - \Delta_i \text{ (ideal gas)}$$

Special properties of α −ideal gasses

- $U = \alpha NkT = \alpha nRt, \quad pV = nRT = NkT,$
- $C_V = \alpha Nk = \alpha nR, \; C_p = C_V + nR,$
- $\frac{C_p}{C_V} = \frac{\alpha+1}{\alpha} = \gamma$
- $S = C_V \ln\left(\frac{T_f}{T_i}\right) + Nk\ln\left(\frac{V_f}{V_i}\right)$

	α	γ
Monatomic gas	3/2	5/3
Diatomic gas	5/2	7/5
3D Harmonic oscillator	3	4/3

Process	**dU**	**dQ**	$\mathbf{dW_{on} = -dW_{by}}$
Isothermal, T= const.	0	$dQ = -dW_{on}$ $Q = -W_{on}$	$dW_{on} = -pdV,$ $W_{on} = nRT\ln(V_i/V_f)$
Adiabatic, Q=0 $PV^\gamma = \text{const.}$ $T^\alpha V = \text{const.}$	$dU = dW_{on}$ $= C_V dT$	0	$dW_{on} = -pdV,$ $W_{on} = \alpha(p_f V_f - p_i V_i)$ $= \alpha\, nR(T_f - T_i)$
Isobaric, p= const.	$dU = C_V dT$	$dQ = C_p dT$	$dW_{on} = -nRdT$
Isochoric, V= const.	$dU = C_V dT$	$dQ = C_V dT$	0

Heat conduction

- $J_x = \frac{\kappa\Delta T}{\Delta x}, \; H_X = J_x A = \frac{\Delta T}{R_{th}}, \; R_{th} = \frac{d}{\kappa A}, \; \frac{\Delta L}{L} = \alpha_T \Delta T$
- $T_A(t) = T_f + (T_A(0) - T_f)e^{-\frac{t}{\tau}}, \tau = R_{th}C_A$

Diffusion

- $D = \frac{\ell^2}{3\tau} = \frac{v\ell}{3}, \tau = \frac{\ell}{v}, < x^2 >= 2Dt, \langle r^2\rangle = 6Dt$

Spins

- $N = N_{up} + N_{down}, \; M = (N_{up} - N_{down})\mu = N\mu\tanh\left(\frac{\mu B}{kT}\right)$
- $\Omega(N, N_{up}) = \frac{N!}{N_{up}!N_{down}!}$
- $P(m) = \frac{\Omega(m)}{2^N}$

SHO

- $P_n = \left(1 - e^{-\frac{\epsilon}{kT}}\right)e^{-\frac{n\epsilon}{kT}},$
- $< E > = \frac{\epsilon}{e^{\epsilon/kT} - 1}$
- $\epsilon = hf, \; \Omega = \frac{(q+N_{osc}-1)!}{q!(N_{osc}-1)!}$

Counting, Bin Statistics, Entropy

Table for number of states Ω			
Kind of particles	Occupancy		
	Unlimited	**Single**	**Dilute**
Distinguishable	M^N	$\frac{M!}{(M-N)!}$	M^N
Indistinguishable	$\frac{(N+M-1)!}{N!\,(M-1)!}$	$\frac{M!}{N!\,(M-N)!}$	$\frac{M^N}{N!}$

Stirling's formula: $\ln N! \sim N\ln N - N$, for large N

Semiconductors:

- $n_h n_e = n_i^2, \; n_i = n_Q e^{-\frac{\Delta}{2kT}}$
- $n_Q = \left(\frac{2\pi mkT}{h^2}\right)^{3/2} = (10^{30} m^{-3})\left(\frac{m}{m_p}\right)^{\frac{3}{2}}\left(\frac{T}{300K}\right)^{3/2}$

Thermal Radiation

- $J = \sigma_B T^4$
- Power $= J \times$ (area)
- $\lambda_{max} T = 0.0029\ m \cdot K$