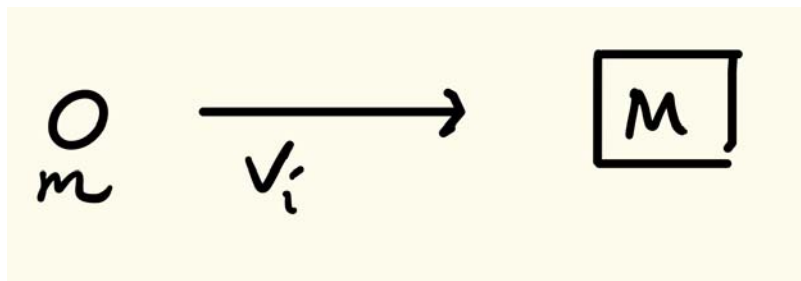In space (no gravity or friction), you throw a ball with mass 0.1 kg at a target with mass 1 kg. You throw the ball at a speed of 5 m/s. When the ball impacts the target, it sticks to it and they drift off together.


1) How much energy is in the translational energy of the block+ball? (Hint: Don't forget about momentum conservation.)

   a. 1.25 J
   b. 0.125 J
   c. 0.114 J
   d. 13.8 J
   e. 12.5 J

The next four questions pertain to the situation described below.

Consider a collection of atoms at 1.5×10^5 K, all with a single ground state and a triply-degenerate first-excited state (three states, which we label A, B, and C, all have the same energy). The energy gap between the ground and excited states is 10.2 eV.

2) What is the probability of finding an atom in the excited state C?
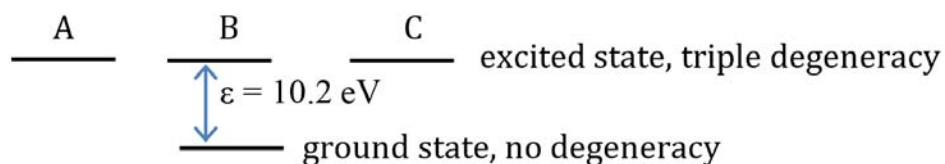
   a. 0.31
   b. 0.19
   c. 0.42
   d. 1.2
   e. 0.94

3) Now we apply a small magnetic field which splits the first-excited level into three (non-degenerate) states:

$E_A = \epsilon + \delta E$, $E_B = \epsilon$, $E_C = \epsilon - \delta E$, as shown. At 1.5×10^5 K we observe that the relative fraction of atoms in state A to those in state C is:

f = (number of atoms in A)/(number of atoms in C) = 1.8. What is δE ?

—— $E_A = \epsilon + \delta E$
—— $E_B = \epsilon$
—— $E_C = \epsilon - \delta E$

—— ground state

  a. 7.61 eV
  b. In thermal equilibrium it is impossible to have f > 1.
  c. 3.8 eV

4) As the temperature T becomes very large (kT >> ϵ), what is the dimensionless entropy for a system with N of these atoms (still with small magnetic field on)?

  a. N ln(3)
  b. N ln(2)
  c. N ln(4)

5) What is the heat capacity C and entropy S of a collection of N such atoms, as T→ 0 (consider only these internal energy states, not any translational or rotational kinetic energy of the entire atom) ?

  a. C∼4Nk and S∼Nk ln(4)
  b. None of the other options
  c. C∼Nk/4 and S∼Nk
  d. C ∼ Nk and S∼0
  e. C∼0 and S∼0

The next two questions pertain to the situation described below.

A sample consists of three simple harmonic oscillators that are thermally coupled to each other. In the sample, 5 quanta of energy are shared by the three oscillators.

6) By how much does the dimensionless entropy (*i.e.*, S/k) change if an additional oscillator is added, keeping the total number of quanta fixed at 5? Optional Hint: the number of microstates Ω available for a system of q quanta shared among N identical harmonic oscillators is given by: $\Omega = (q + N - 1)! / [q! (N-1)!]$

    a. -0.69
    b. 4.83
    c. 0
    d. -0.98
    e. 0.98

7) After the oscillators come into equilibrium, what is the average number of quanta in the last of the 4 oscillators?

    a. 5
    b. 1.25
    c. 0

A gas of N_2 is at temperature T near room temperature. At this temperature, vibrations of the N_2 molecule are not important (*i.e.*, they are not in equipartition).

8) How much energy is in the total translational kinetic energy of a single molecule?

    a. 5kT/2
    b. kT/2
    c. 3kT/2

The next three questions pertain to the situation described below.

Consider a proton in a magnetic field B and in contact with a thermal reservoir at temperature T. Like an electron, a proton can have two magnetic moment states, either aligned or anti-aligned with the magnetic field. The energy levels of the two spin states are $E_{align}= -\mu_P B$ and $E_{anti}= +\mu_P B$ where $\mu_P = 1.4 \times 10^{-26}$ J/Tesla is the magnetic moment of the proton. We can model this system as a two-state system.

9) Through measurements one finds that at a magnetic field $B = 12$ Tesla the aligned spin state is 5 times more probable than the anti-aligned state. What is the temperature?

    a. 31 mK
    b. 62 mK
    c. 15 mK
    d. 26 mK
    e. 7.5 mK

10) As the strength of the magnetic field is lowered to zero the entropy of the proton system approaches:

    a. kln(2)
    b. kln(1/2)
    c. 0
    d. kln(5)
    e. Infinity

11) Consider an identical system except with the proton replaced by a neutron. A neutron also has two possible magnetic moment states with energies $E_{align}= -\mu_N B$ and $E_{anti}= \mu_N B$ where $\mu_N = 9.7 \times 10^{-27}$ J/Tesla is the magnetic moment of the neutron.

How does the entropy of the proton system compare to the entropy of the neutron system at the same magnetic field B and temperature T?

    a. $S_{proton} = S_{neutron}$
    b. $S_{proton} < S_{neutron}$
    c. $S_{proton} > S_{neutron}$

The next two questions pertain to the situation described below.

Consider a diatomic molecule, and assume that we can treat the quantum energy levels of the molecule as a two-state system. The low-energy state E_L of each molecule has no rotational energy, and the high-energy state E_H has rotational energy. The energy difference $E_H - E_L = \Delta = 3.2 \times 10^{-22} J$.

12) At what temperature is the probability of finding the low-energy state equal to 0.75?

    a. 5.2 K
    b. 63 K
    c. 21 K
    d. 0.15 K
    e. 156 K

13) Assuming that we can freely adjust the temperature, what is the smallest probability we would ever find for the probability of the low-energy state?

    a. 0.50
    b. 0.19
    c. 0.75
    d. 0.72
    e. 0

14) You have a cup of coffee, at initial temperature of 90ºC. The mass of coffee is 0.25 kg. You would like to reduce the temperature to 60ºC. How much cream needs to be added to the cup of coffee? Assume that the heat capacity of cream is the same as that of coffee and that the temperature of your cream is 20ºC.

    a. 0.44 kg
    b. 0.19 kg
    c. 1.1 kg
    d. 0.11 kg
    e. 0.025 kg

The next two questions pertain to the situation described below.

A 1.5-liter box contains 2 moles of N_2 gas and 5 moles of He gas, all at temperature 22˚C. At this temperature, each He atom and nitrogen molecule has 3 and 5 quadratic degrees of freedom in equipartition, respectively.

15) What is the pressure inside the box?

    a. 161 atm
    b. 51.6 atm
    c. 113 atm

16) How much heat is required to raise the temperature of this gas mixture to 45˚C (at constant volume)?

    a. 1339 J
    b. 2390 J
    c. 3346 J

At extremely low temperatures, the molar heat capacity of silicon (Si) is temperature-dependent: $C(T) = A*T^3$, where $A = 7.55 \times 10^{-6}$ J/(mol-K^4).

Suppose that I have a sample of 0.1 kg of silicon, initially at 0.2 Kelvin.

17) If I heat the sample to 2 Kelvin, how much does its entropy change?

     a. $+7.18 \times 10^{-5}$ J/K
     b. -7.18×10^{-5} J/K
     c. 0 J/K
     d. -1.08×10^{-4} J/K
     e. $+1.08 \times 10^{-4}$ J/K