

*The next two questions pertain to the following situation:*

Consider the following two systems:

A: three interacting harmonic oscillators with total energy  $6\epsilon$ .

B: two interacting harmonic oscillators, with total energy  $4\epsilon$ .

1. What is the ratio of entropies for the two systems?

a.  $\sigma_A/\sigma_B = 0$

b.  $\sigma_A/\sigma_B = 0.86$

c.  $\sigma_A/\sigma_B = 1$

d.  $\sigma_A/\sigma_B = 2.07$

e.  $\sigma_A/\sigma_B = 3.5$

Count microstates and take the logarithm:

Put  $N$  identical things (energy packets) into  $M$  bins (# oscillators):

System A:  $\frac{(N+M-1)!}{N!(M-1)!} = \frac{8!}{6!2!} = 28$ .  $\sigma = 3.33$

System B:  $\frac{(N+M-1)!}{N!(M-1)!} = \frac{5!}{4!1!} = 5$ .  $\sigma = 1.61$ .

Ratio = 2.07

2. Now the systems are brought into contact and allowed to reach equilibrium. The increase in entropy due to the process is

a. 0

b. 1.97

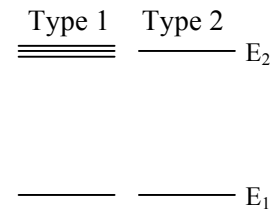
c. 6.41

Calculate  $\sigma_{TOT}$ :  $N = 10, M = 5$ .  $\frac{(N+M-1)!}{N!(M-1)!} = \frac{14!}{10!4!} = 1001$ .  $\sigma = 6.91$ .

$$\Delta\sigma = 6.91 - (3.33+1.61) = 1.97$$

COMMENT: These two questions are too closely coupled for my exam style.

3. Two types of atoms have energy levels as shown in the figure. All of the excited states are at the same energy,  $E_2$ , but atoms of type 1 have three states at that energy, while atoms of type 2 have only one. A box of gas contains both types of atoms in equilibrium at temperature,  $T$ . Compare the probability that an atom of type 1 is in its ground state (*i.e.*, has energy  $E_1$ ) with the probability that an atom of type 2 is in its ground state.



- a. Atoms of type 1 are more likely to be in the ground state.
- b. Atoms of type 2 are more likely to be in the ground state.
- c. Atoms of the two types are equally likely to be in the ground state.

Because the Type 1 atom has three states at  $E_2$ , the probability that it has that energy is three times larger than the simple Boltzmann factor. Therefore, the probability that it is in the ground state is somewhat smaller than it would be if there were only one state at  $E_2$ .

**The next three questions are related:**

Consider a large collection of spins with dipole moment  $\mu = 1 \times 10^{-23}$  J/Tesla.

4. If  $B = 1$  Tesla, for what temperature will twice as many spins be pointing up (along the field) as down?

a.  $T = 2.1$  K

b.  $T = 5.2$  K

c.  $T = 14.5$  K

d.  $T = 23.7$  K

e.  $T = 88.4$  K

We want the ratio of Boltzmann factors to be 2:

$$2 = e^{\Delta E/kT} = e^{2\mu B/kT}$$

Solve for T:

$$T = \frac{2\mu B}{k \ln(2)} = 2.1 \text{ K}$$

5. Assuming there are  $N$  spins, what is the heat capacity in the limit of high temperature?

**\*Note: This problem may be longer/more difficult; therefore, you may want to do it last.\***

a.  $C_V = \frac{\mu^2 B^2}{kT^2}$

b.  $C_V = \frac{Nk^2 T^2}{\mu B}$

c.  $C_V = \frac{N\mu B}{kT^2}$

d.  $C_V = \frac{N\mu^2 B^2}{kT^2}$

e.  $C_V = \frac{N\mu B}{T}$

$$U(T) = -MB = -\left( N\mu \frac{e^{\mu B/kT} - e^{-\mu B/kT}}{e^{\mu B/kT} + e^{-\mu B/kT}} \right) B = -N\mu B \tanh\left(\frac{\mu B}{kT}\right)$$

$$C_V = \frac{\partial U}{\partial T} = -N\mu B \left( \frac{-\mu B}{kT^2} \right) \text{sech}^2\left(\frac{\mu B}{kT}\right)$$

At large  $T$ ,  $\text{sech}(1/T) \rightarrow 1$ , so

$$C_V = \frac{N\mu^2 B^2}{kT^2}$$

**COMMENTS:**

- You could avoid hyperbolic functions by taking the  $T \rightarrow \infty$  limit in the first expression for  $U$ , but you must keep the first two terms in the Taylor series. This is slightly too mathematical for my taste.
- The no-math solution:** You can eliminate b, c, and e, because  $C_V$  must be positive, even if  $\mu < 0$  or  $B < 0$ . You can eliminate a, because  $C_V$  must be proportional to  $N$ .

6. Now let there be  $10^6$  spins and let B go to zero. After the spins have completely randomized, approximately what is the probability of observing  $N_{\text{up}} - N_{\text{down}} = 1000$ ?

- a.  $P = 5 \times 10^{-2}$
- b.  $P = 5 \times 10^{-3}$
- c.  $P = 5 \times 10^{-4}$
- d.  $P = 5 \times 10^{-5}$
- e.  $P = 5 \times 10^{-6}$

For these large numbers, use the Gaussian distribution (on the formula sheet):

$$P(m) = \sqrt{\frac{2}{\pi N}} e^{-m^2 / 2N}$$

Here,  $m = 1000$  and  $N = 10^6$ . So,  $P = 4.8 \times 10^{-4}$ .

**The next two questions pertain to the following situation:**

7. A box of total volume  $V$  initially has an insulating partition, which separates  $N_{Ar}$  Argon atoms (monatomic, each with mass  $m_{Ar}$ ) at initial temperature  $T_{Ar}$  from  $N_{N_2}$  nitrogen molecules (diatomic, each with mass  $m_{N_2}$ ) at initial temperature  $T_{N_2}$ . The partition is suddenly removed, and the gases allowed to equilibrate. The final temperature  $T_f$  is

a.  $T_f = \frac{3N_{Ar}T_{Ar} - 5N_{N_2}T_{N_2}}{3N_{Ar} + 5N_{N_2}}$

Use  $C_V$ , not  $C_P$ , because no work is being done.

b.  $T_f = \frac{N_{Ar}T_{Ar} - N_{N_2}T_{N_2}}{N_{Ar} + N_{N_2}}$

$C_{VAr} = 3/2 N_{Ar}k$ .  $C_{VN_2} = 5/2 N_{N_2}k$ .

c.  $T_f = \frac{5N_{Ar}T_{Ar} + 3N_{N_2}T_{N_2}}{5N_{Ar} + 3N_{N_2}}$

$T_f$  is the weighted average of the initial temps:

d.  $T_f = \frac{3N_{Ar}T_{Ar} + 5N_{N_2}T_{N_2}}{3N_{Ar} + 5N_{N_2}}$

$$T_f = \frac{C_{VAr}T_{Ar} + C_{VN_2}T_{N_2}}{C_{VAr} + C_{VN_2}} = \frac{\frac{3}{2}N_{Ar}kT_{Ar} + \frac{5}{2}N_{N_2}kT_{N_2}}{\frac{3}{2}N_{Ar}k + \frac{5}{2}N_{N_2}k}$$

e.  $T_f = \frac{N_{Ar}T_{Ar} + N_{N_2}T_{N_2}}{N_{Ar} + N_{N_2}}$

8. If  $U'_{Ar}$  and  $U'_{N_2}$  are the final energies of each component, which of the following is true in equilibrium:

a.  $\frac{U'_{Ar}}{N_{Ar}} = \frac{U'_{N_2}}{N_{N_2}}$

b.  $U'_{Ar} = U'_{N_2}$

c.  $m_{Ar} \langle v_{Ar}^2 \rangle = m_{N_2} \langle v_{N_2}^2 \rangle$

**It's not a**, because the internal energy per molecule depends on rotation, which is not the same for  $N_2$  and Ar.

**It's not b**, because if you have a ton of Ar and an ounce of  $N_2$ , the internal energies are obviously not the same.

**It's c**, because each molecule has, on average, the same CM kinetic energy.

9. The vibrational mode of the  $N_2$  molecule acts like a harmonic oscillator with energy spacing,  $\epsilon = 0.292$  eV. Estimate the probability that a molecule of  $N_2$  in equilibrium at room temperature ( $T = 300$  K) is in the **first excited vibrational state** (not the ground state).

- a. Probability =  $1.54 \times 10^{-10}$
- b. Probability =  $1.24 \times 10^{-5}$
- c. Probability = 0.0885
- d. Probability = 0.292
- e. Probability = 0.99999

Use the Boltzmann factor:  $e^{-\epsilon/kT} = 1.24 \times 10^{-5}$ . This is small enough that you don't have to worry about the normalization factor:  $Z = \sum_i e^{-E_i/kT} \approx 1$ .

10. As absolute temperature goes to zero, the specific heat of  $N$  one-dimensional harmonic oscillators approaches

- a. 0
- b.  $\frac{1}{2}Nk$
- c.  $Nk$

As  $T \rightarrow 0$ , the probability that an oscillator is in an excited state becomes exponentially small, due to the Boltzmann factor. So,  $U \approx N\epsilon e^{-\epsilon/kT}$ .

$$C = \frac{dU}{dT} = \frac{N\epsilon^2}{kT^2} e^{-\epsilon/kT} \rightarrow 0.$$

***The next two problems are related:***

11. Consider a 2-dimensional gas, in which particles are allowed to move only in a plane. In this case, the probability density is given by  $P(E) = (kT)^{-1} e^{-E/kT}$ . If the particles have mass of  $10^{-20}$  kg, at  $T = 300$  K approximately what fraction of the particles have an energy between 0.030 and 0.031 eV?

- a. 0.001
- b. 0.012**
- c. 0.24
- d. 0.33
- e. 0.5

The energy interval is small, so:

$$P(E, \Delta E) \approx \left( \frac{1}{kT} e^{-E/kT} \right) \Delta E = \left( \frac{1}{0.0258 \text{ eV}} e^{-0.030/0.0258} \right) 0.001 \text{ eV} = 0.0121$$

It would be slightly more accurate to use the middle of the energy interval in the exponential. ( $E + \Delta E/2$  rather than  $E$ )

12. What is the root-mean-square velocity of the particles in this 2-d gas?

- a.  $v_{rms} = 0$
- b.  $v_{rms} = \sqrt{\frac{2kT}{m}}$**
- c.  $v_{rms} = \sqrt{\frac{3kT}{m}}$

Equipartition tells us that  $\frac{1}{2} m \langle v^2 \rangle = \frac{2}{2} kT$ . Remember that  $v_{rms} \equiv \sqrt{\langle v^2 \rangle}$ .

Notice: **2/2, not 3/2**, because it's a 2-d gas.

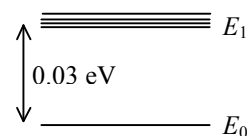
***The next two problems are related:***

13. A molecule has one electronic state with energy  $E_0$ , and four with energy  $E_1 = E_0 + 0.03$  eV. At  $T = 273$  K, what is the relative number of molecules at each energy level?

a.  $N_1 < N_0$

b.  $N_1 > N_0$

c. The answer cannot be determined from the information given.



You need to include both the Boltzmann factor and the degeneracy (# of states) at  $E_1$ :

$$4e^{-0.03\text{eV}/kT} = 4 \times 0.279 = 1.12 .$$

Compare this with  $1e^0 = 1$  for the  $E_0$  state.

14. What is the average energy of a molecule in the previous case (assume  $E_0 = 0$ )?

a. 0.0065 eV

b. 0.016 eV

c. 0.021 eV

$$\langle E \rangle = P_0 E_0 + P_1 E_1 = (1 \times 0 + 1.12 \times 0.03) / (1 + 1.12) = 0.0158$$

COMMENT: These two problems are too tightly coupled for an exam.



***The next two problems are related:***

A sphere of radius 1m is released into deep space (away from the sun), with an initial temperature of 72 °C.

15. What is the wavelength of the peak in the ‘blackbody radiation’ from the sphere?
- a. 830 nm
  - b. 8.4 micrometers
  - c. 40 micrometers

Use  $\lambda_{\text{max}} T = 0.0029 \text{ m-K}$  (equation sheet).

Don’t forget that  $72^\circ\text{C} = 345 \text{ K}$ .

16. Approximating the sphere as a perfect “black body”, and assuming its heat capacity is  $10^4 \text{ J/K}$ , about how long does the sphere take to cool to 335 K?
- a. 0.01 s
  - b. 0.1 s
  - c. 1 s
  - d. 10 s
  - e. 100 s

It radiates energy at a rate:  $J = \sigma_b T^4 = 5.67 \times 10^{-8} \times 345^4 = 803 \text{ W/m}^2$ .

The surface area is  $A = 4\pi R^2 = 12.56 \text{ m}^2$ .

It must lose  $10^5 \text{ J}$ , which takes:  $10^5 / (803 \times 12.56) = 9.92 \text{ sec}$ .

17. Brick A has mass  $m_A = 1$  kg, with specific heat  $c_A = 1000 \text{ J/kg-K}$ , initially at temperature  $T_A = 100$  K. Brick B has mass  $m_B = 2$  kg, with specific heat  $c_B = 2000 \text{ J/kg-K}$ , initially at temperature  $T_B = 200$  K. The bricks are put in thermal contact with each other (but are isolated from the rest of the world). After the two-brick system reaches thermal equilibrium, by how much,  $\Delta S_{\text{tot}}$ , has their total entropy changed?

- a.  $\Delta S_{\text{tot}} = -81 \text{ J/K}$
- b.  $\Delta S_{\text{tot}} = 0.0 \text{ J/K}$
- c.  $\Delta S_{\text{tot}} = 81 \text{ J/K}$
- d.  $\Delta S_{\text{tot}} = 166 \text{ J/K}$
- e.  $\Delta S_{\text{tot}} = 1009 \text{ J/K}$

$$dS = \frac{dQ}{T}.$$

Both bricks have constant  $c$ , so (assuming their volumes do not change):

$$\Delta S_A = m_A c_A \int_{T_A}^{T_f} \frac{dT}{T} = m_A c_A \ln \left( \frac{T_f}{T_A} \right)$$

$$\Delta S_B = m_B c_B \int_{T_B}^{T_f} \frac{dT}{T} = m_B c_B \ln \left( \frac{T_f}{T_B} \right)$$

$$\Delta S_{\text{tot}} = \Delta S_A + \Delta S_B$$

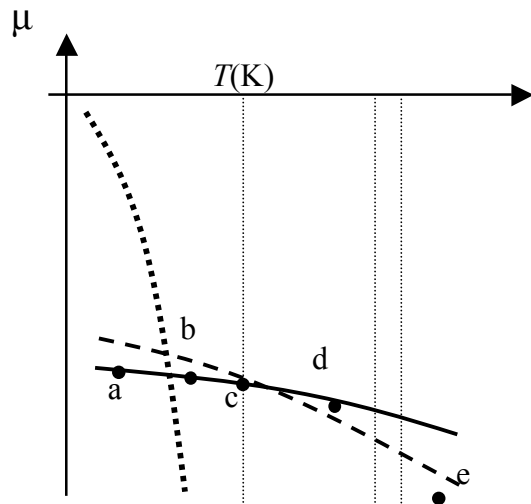
We need  $T_f$ . It is the weighted average:

$$T_f = \frac{m_A c_A T_A + m_B c_B T_B}{m_A c_A + m_B c_B} = 180 \text{ K}$$

So,  $\Delta S_{\text{tot}} = 166.3 \text{ J/K}$ .

*The next two problems are related:*

A substance has the following chemical potential vs T diagram at a particular pressure.



18. Which of the points corresponds to a substance that is in thermal equilibrium?

- a. In equilibrium,  $\mu$  is minimized.
- b. Only point **a** has the smallest possible  $\mu$  for the specified temperature.
- c.
- d.
- e.

19. Which of the points corresponds to a substance that is about to sublime?

- a. The dotted, dashed, and solid lines tell us  $\mu$  for the
- b. gas, liquid, and solid phases respectively. (Why?)
- c. Point **b** shows the material in the solid phase, with
- d. only the gaseous  $\mu$  below it. So, it will sublime
- e. (go directly from solid to gas phase).

20. Calculate the chemical potential,  $\mu$ , of Argon gas at room temperature,  $T = 300 \text{ K}$ , and atmospheric pressure,  $p = 1.01 \times 10^5 \text{ Pa}$ . Ignore the effect of external forces, such as gravity.

- a.  $\mu = -2.49 \times 10^{-9} \text{ eV}$
- b.  $\mu = -0.42 \text{ eV}$
- c.  $\mu = 0 \text{ eV}$
- d.  $\mu = +0.42 \text{ eV}$
- e.  $\mu = +2.49 \times 10^{-9} \text{ eV}$

There is no potential energy, so  $\mu = kT \ln \left( \frac{n}{n_Q} \right)$ .

The argon atom is 40 times as massive as the proton, so

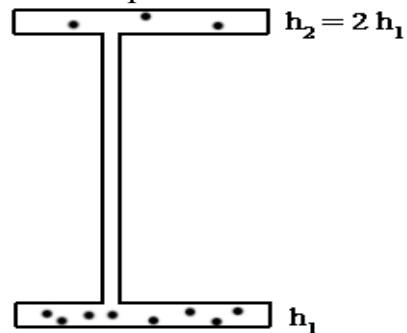
$$n_Q = 10^{30} \text{ m}^{-3} \times 40^{3/2} \times 1^{3/2} = 2.53 \times 10^{32} \text{ m}^{-3}$$

For an ideal gas,  $n = \frac{p}{kT} = 2.44 \times 10^{25}$

Thus  $\mu = -0.418 \text{ eV}$

21. A container holds  $N_{\text{total}}$  gas molecules in equilibrium at two different heights as pictured. What is the relationship between the chemical potentials of the molecules at height  $h_1$  and  $h_2 = 2h_1$ ?

- a)  $\mu_1 = \mu_2/4$
- b)  $\mu_1 = \mu_2/2$
- c)  $\mu_1 = \mu_2$
- d)  $\mu_1^2 = \mu_2$
- e)  $\mu_1 = \mu_2^2$



In equilibrium, objects that can exchange particles must have the same chemical potential. (Why?)

22. The gas molecules are 80%  $N_2$  and 20%  $O_2$  at sea level, i.e.,  $n_{N_2}/n_{O_2} = 4$ . Neglecting the effects of thermal mixing, at what height is the ratio  $n_{N_2}/n_{O_2} = 1$ ? (Assume a constant temperature  $T = 260$  K.)
- a. 33 km
  - b. 76 km
  - c. There is no altitude above sea level where the ratio is 1.

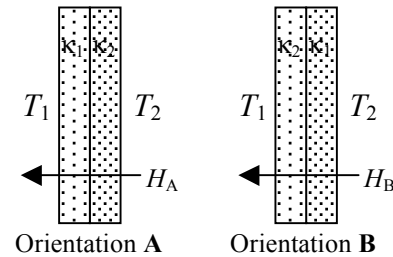
We want the ratio of Boltzmann factors to be 4:

$$\begin{aligned}e^{-m_{O_2}gh/kT} &= 4e^{-m_{N_2}gh/kT} \\ -m_{O_2}gh/kT &= \ln 4 - m_{N_2}gh/kT \\ h &= \frac{kT \ln 4}{g(m_{N_2} - m_{O_2})}\end{aligned}$$

Oxygen is more massive than nitrogen, so  $h < 0$ .

*The next two problems are related:*

23. A window is made of two layers of material that have different thermal conductivities,  $\kappa_2 > \kappa_1$ . The temperature on one side of the window is higher than the other:  $T_2 > T_1$ . Which orientation of the window (see the figure) gives more heat flow through the window?



- a.  $H_A > H_B$  (**A** has more heat flow.)
- b.  $H_A = H_B$  (The heat flows are equal.)
- c.  $H_A < H_B$  (**B** has more heat flow.)

In an electric circuit, the current through two resistors in series does not depend on the resistor ordering. The same is true here. Solve these equations for H:

$$H = K_2 (T_2 - T_M) = K_1 (T_M - T_1)$$

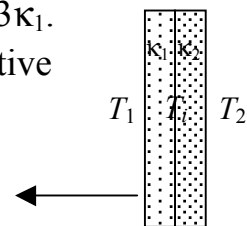
$$T_M = \frac{K_1 T_1 + K_2 T_2}{K_1 + K_2}$$

$$H = \left( \frac{K_1 K_2}{K_1 + K_2} \right) (T_2 - T_1)$$

This is symmetrical. The answer is unchanged if we swap  $K_1$  and  $K_2$ .

24. Now consider that the layers have equal thickness, and  $\kappa_2 = 3\kappa_1$ .

The temperatures  $T_1 = 10^\circ\text{C}$  and  $T_2 = 20^\circ\text{C}$ . What is the relative temperature  $T_i$  at the interface between the two layers?



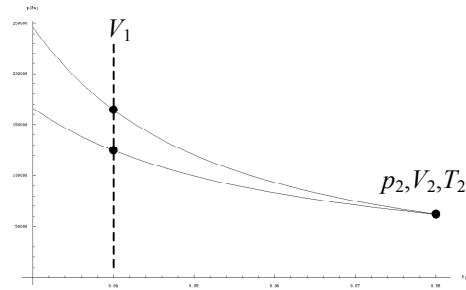
- a.  $12.5^\circ\text{C}$
- b.  $15^\circ\text{C}$
- c.  $17.5^\circ\text{C}$
- d.  $18^\circ\text{C}$
- e.  $20^\circ\text{C}$

$$J = \kappa \Delta T / \Delta x$$

For a given heat flow, the temperature drop is inversely proportional to the conductivity. Because the heat flow is the same in both layers, drop #1 is three times as big as drop #2.

Therefore,  $T_i = 17.5^\circ\text{C}$ .

25. Compare the isothermal and adiabatic expansions from  $V_1 = 0.04 \text{ m}^3$  to  $V_2 = 0.08 \text{ m}^3$  of two moles of an ideal diatomic gas, as shown in the figure. Both processes end up at the same state,  $T_2 = 300 \text{ K}$ , and  $p_2 = 62.3 \text{ kPa}$ . Calculate the ratio,  $W_I/W_A$ , of the work done by the two processes (isothermal divided by adiabatic).



- a.  $W_I/W_A = 0.40$
- b.  $W_I/W_A = 0.87$
- c.  $W_I/W_A = 1.40$
- d.  $W_I/W_A = 2.00$
- e.  $W_I/W_A = 3.46$

**Isothermal:**

$$W_{by} = \int_{V_1}^{V_2} P dV = NkT_2 \ln \frac{V_2}{V_1} = P_2 V_2 \ln \frac{V_2}{V_1} = 3455 \text{ J}$$

**Adiabatic:** Because  $Q = 0$ ,

$$W_{by} = -\Delta U = \alpha Nk(T_1 - T_2)$$

We don't know  $T_1$ . Use

$$VT^\alpha = \text{const} \Rightarrow T_1 = T_2 \left( \frac{V_2}{V_1} \right)^{\frac{1}{\alpha}} :$$

$$W = \alpha NkT_2 \left( \left( \frac{V_2}{V_1} \right)^{\frac{1}{\alpha}} - 1 \right) = 3981 \text{ J}$$

26. A Carnot heat engine achieves 33.3% efficiency when operating between temperatures  $T_h$  and  $T_c$ . If it is operated as a refrigerator operating between the same two reservoirs, how much work,  $W$ , must we supply in order to remove 1 kJ of heat from the cold reservoir?

- a.  $W = 500 \text{ J}$
- b.  $W = 666 \text{ J}$
- c.  $W = 1 \text{ kJ}$
- d.  $W = 2 \text{ kJ}$
- e.  $W = 3 \text{ kJ}$

33.3% efficiency means  $W = 1/3 Q_H$  and  $Q_C = 2/3 Q_H$ . So,  $W = 1/2 Q_C$ . When we run it backwards, the ratios remain unchanged.

27. The following chemical reaction occurs:  $2\text{N}_2\text{O}_5 \rightarrow 4\text{NO}_2 + \text{O}_2$ . Which of the following correctly expresses the relationship between the chemical potentials of all three species?

- a)  $2\mu_{\text{N}_2\text{O}_5} = 4\mu_{\text{NO}_2} + \mu_{\text{O}_2}$
- b)  $2\mu_{\text{N}_2\text{O}_5} = \mu_{\text{NO}_2} + 4\mu_{\text{O}_2}$
- c)  $\mu^2_{\text{N}_2\text{O}_5} = \mu^4_{\text{NO}_2} + \mu_{\text{O}_2}$
- d)  $\mu_{\text{N}_2\text{O}_5} = \mu_{\text{NO}_2} + \mu_{\text{O}_2}$
- e)  $2\mu_{\text{N}_2\text{O}_5} = 4\mu_{\text{NO}_2} = \mu_{\text{O}_2}$

It should say, “in equilibrium”.

Suppose two molecules of  $\text{N}_2\text{O}_5$  dissociate. That change of free energy due to that is  $-2\mu_{\text{N}_2\text{O}_5}$ . This creates 4 molecules of  $\text{NO}_2$  and one molecule of  $\text{O}_2$ . That changes the free energy by  $4\mu_{\text{NO}_2} + \mu_{\text{O}_2}$ . In equilibrium, the sum must be zero.



28. Two identical blocks each have heat capacity 100 J/K. One block is at temperature 500K and the other is at temperature 100K. Which block has the higher free energy relative to the environment at 300 K?

- a) the cold block
- b) the hot block
- c) they have the same free energy

$$dF = dU - T_E dS = dU - \frac{T_E}{T} dQ.$$

For each brick:

$$\begin{aligned}\Delta F &= \Delta U - CT_E \int_{T_i}^{T_f} \frac{dT}{T} = C(T_f - T_i) - CT_E \ln\left(\frac{T_f}{T_i}\right) \\ &= 100(-200) - 100 \times 300 \times \ln(100 / 300) = 12958 \text{ J (cold)} \\ &= 100(+200) - 100 \times 300 \times \ln(500 / 300) = 4675 \text{ J (hot)}\end{aligned}$$

29. A block of material has a temperature-dependent heat capacity given by  $C_v(T) = 5 \text{ J/K} + T \times (2 \text{ J/K}^2)$ . How much does the entropy of this object change as its temperature is increased from 10°C to 40°C at constant volume?

- a) -66.9 J/K
- b) -60.5 J/K
- c) 0.50 J/K
- d) 60.5 J/K
- e) 66.9 J/K

$$\begin{aligned}
 \Delta S &= \int_{T_1}^{T_2} \frac{dQ}{T} \\
 &= \int_{T_1}^{T_2} \frac{C_V dT}{T} \\
 &= \int_{283}^{313} \frac{(5 + 2T) dT}{T} \\
 &= 5 \ln \left( \frac{313}{283} \right) + 60 = 60.5 \text{ J/K}
 \end{aligned}$$

30. The star Rigel (in the constellation Orion) is a blue supergiant. It is very large ( $R_{\text{Rigel}} = 70R_{\text{Sun}}$ ) and very bright, i.e., has very high total radiated power ( $P_{\text{Rigel}}/P_{\text{Sun}} = 80,000$ ). How much hotter is it than the sun? That is, what is the ratio,  $T_{\text{Rigel}}/T_{\text{Sun}}$ ?

- a.  $T_{\text{Rigel}}/T_{\text{Sun}} = 2$
- b.  $T_{\text{Rigel}}/T_{\text{Sun}} = 16$
- c.  $T_{\text{Rigel}}/T_{\text{Sun}} = 70$
- d.  $T_{\text{Rigel}}/T_{\text{Sun}} = 4,900$
- e.  $T_{\text{Rigel}}/T_{\text{Sun}} = 78,000$

The total radiated power is given by:  $P = A\sigma_B T^4$ .

Compare Rigel to the Sun:

$$\frac{P_R}{P_S} = \frac{A_R}{A_S} \frac{T_R^4}{T_S^4} \Rightarrow \frac{T_R}{T_S} = \left( \frac{P_R}{P_S} \frac{A_S}{A_R} \right)^{1/4} = \left( \frac{P_R}{P_S} \frac{R_S^2}{R_R^2} \right)^{1/4} = 2.01$$

31. The semiconductor silicon (Si) has a band gap of 1.14 eV. The quantum density associated with its valence and conduction bands is  $1.72 \times 10^{25}/\text{m}^3$ . How many free holes are there in the valence band at 300K if a density of  $10^{20}/\text{m}^3$  phosphorous atoms (electron donors) is added to the crystal?

- a)  $n_h = 2.1 \times 10^7/\text{m}^3$
- b)  $n_h = 2.1 \times 10^9/\text{m}^3$
- c)  $n_h = 2.1 \times 10^{11}/\text{m}^3$

$$n_e n_h = n_Q^2 e^{-\Delta/kT} = 2.08 \times 10^{31} \text{ m}^{-6}$$

$$\text{So, } n_h = 2.08 \times 10^{11} \text{ m}^{-3}$$

32. A 1-liter (non-elastic) balloon is filled with pure helium gas. The balloon is in a room 10x10x10m at 300 K and 1 atm. (Assume the air in the room is all nitrogen and oxygen molecules.) Now we pop the balloon, allowing the He to diffuse throughout the room. By what amount does the Free energy of the He gas change?

- a) -1382 J
- b) -691 J
- c) 0
- d) + 691 J
- e) +1382 J

The temperature is constant, so  $\Delta U = 0$ .

$$\Delta F = -T \Delta S = -NkT \ln \frac{V_f}{V_i} = -P_i V_i \ln \frac{V_f}{V_i} = -1395 \text{ J}$$

Note that  $\Delta F$  must be negative, so choose **a and b** if you can't do the problem.

The next two problems are related:

33. Argon (molecular weight 40 g/mole) is a monatomic compound. If liquid argon is confined to a container and held at a constant temperature of 80.5 K, what is the approximate vapor pressure of gaseous argon, assuming the liquid has no entropy and a binding energy of 0.1 eV? [Note: At 1 atm, the boiling point is 87.3 K.]

a) 2 atm  $\mu_{\text{Ar}} = kT \ln \left( \frac{n}{n_Q} \right) \text{ and } \mu_{\text{H}_2\text{O}} = -\Delta.$

b) 0.2 atm Set them equal:  $kT \ln \left( \frac{n}{n_Q} \right) = kT \ln \left( \frac{p}{p_Q} \right) = -\Delta$

c) 0.02 atm  $\frac{n}{n_Q} = e^{-\frac{\Delta}{kT}} \Rightarrow p = nkT = n_Q kT e^{-\frac{\Delta}{kT}}$

d) 0.002 atm For Argon at 80.5 K:

e) 0.0002 atm  $n_Q = (10^{30} \text{ m}^{-3}) (40)^{3/2} \left( \frac{80.5 \text{ K}}{300 \text{ K}} \right)^{3/2} = 3.52 \times 10^{31} \text{ m}^{-3}$

$p = 2.14 \times 10^4 \text{ Pa} = 0.21 \text{ Atm}$

34. The measured value of the latent heat of vaporization of argon (at 1 atm) is 6.43 kJ/mol. Use this to estimate the binding energy.

a) 0.027 eV The latent heat is:  $H = \Delta U + p\Delta V$ . For one mole of  
b) 0.059 eV argon at  $T = 87.3 \text{ K}$ ,  $\Delta V = RT/p$  (ignoring the  
c) 0.067 eV volume of the liquid). So,  
d) 0.11 eV  $\Delta U = 6.43 \text{ kJ} - 0.73 \text{ kJ} = 5.70 \text{ kJ}$  per mole.  
e) 0.4 eV Convert to eV ( $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ) and divide by  
Avogadro's number:  $\Delta U = 0.059 \text{ eV}$  per atom.

The latent heat is  $H = \Delta U + p\Delta V$ , because you must both break the bonds and do work to turn the liquid into gas.

***The next two questions pertain to the following situation:***

In a hydrogen atom, the electron (e) is electrostatically bound to the proton (p) with an energy  $-13.6$  eV. Inside a star, the density is  $\sim 10^{24}/\text{m}^3$ , and the temperature is  $7000$  K.

**35. \*Note: This problem may be longer/more difficult; therefore, you may want to do it last.\***

What is the density of free electrons (and protons)? You may assume that  $n_H = 1 \times 10^{24} \text{ m}^{-3}$  and that  $m_H = m_p = 1836 m_e$ .

- a.  $6.0 \times 10^{13} \text{ m}^{-3}$       We are looking for the equilibrium of this reaction:  
 b.  $2.7 \times 10^{15} \text{ m}^{-3}$        $H \leftrightarrow p + e$ . This problem is done in lecture 13, page 7.  
 c.  $5.1 \times 10^{17} \text{ m}^{-3}$        $\mu_p + \mu_e = \mu_H \Rightarrow kT \ln\left(\frac{n_p}{n_{Qp}}\right) + kT \ln\left(\frac{n_e}{n_{Qe}}\right) = kT \ln\left(\frac{n_H}{n_{QH}}\right) - \Delta$ , or  
 d.  $2.1 \times 10^{18} \text{ m}^{-3}$        $\left(\frac{n_p}{n_{Qp}}\right)\left(\frac{n_e}{n_{Qe}}\right) = \left(\frac{n_H}{n_{QH}}\right) e^{-\Delta/kT}$ .  
 e.  $4.7 \times 10^{20} \text{ m}^{-3}$       Note that  $n_p = n_e$ . Also,  $n_{Qp} = n_{QH}$ .

$$\text{So, } n_p^2 = n_{Qe} n_H e^{-\Delta/kT}. \quad n_{Qe} = 10^{30} \text{ m}^{-3} \left(\frac{1}{1836}\right)^{3/2} \left(\frac{7000}{300}\right)^{3/2} = 1.43 \times 10^{27} \text{ m}^{-3}.$$

$$\text{I get } n_p = 4.8 \times 10^{20} \text{ m}^{-3}.$$

**36.** Which of the following would *increase* the fraction of unbound protons?

- a. increase the temperature.
- b. increase the pressure.
- c. increasing the binding energy  $\Delta$ .

**NOTE: Not choice b.**  $n_p$  increases, but  $n_p/n_H$  decreases.