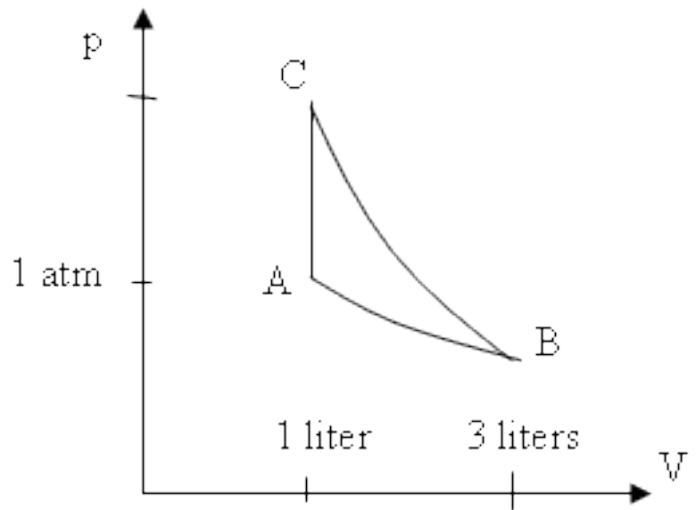


**Physics 213**  
**Spring 2009 Midterm exam**  
**Review Lecture**

**The next two questions pertain to the following situation.**

A container of air (primarily nitrogen and oxygen molecules) is initially at 300 K and atmospheric pressure in a 1 liter container (state A on the figure).



1. The container is **isothermally** expanded to 3 liters (state B). How much work  $W_{by}$  is done by the gas in going from state A to state B?

- (a)  $W_{by} = 1.1$  l-atm      (b)  $W_{by} = 1.4$  l-atm      (c)  $W_{by} = 2.7$  l-atm

$$W_{by} = \int_{V_1}^{V_2} p dV = NkT \ln\left(\frac{V_2}{V_1}\right) = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = 1 \text{ l-atm} \ln(3) = 1.1 \text{ l-atm}$$

2. From state B, the container is **adiabatically** compressed to volume  $V = 1$  liter (state C), and finally returned to the initial conditions (state A). How much heat  $Q$  must be removed from the container to return it to  $T = 300$  K (i.e to state A, with  $V = 1$  liter)?

- (a)  $Q = 1.1$  l-atm      (b)  $Q = 1.4$  l-atm      (c)  $Q = 2.7$  l-atm  
 (d)  $Q = 14$  l-atm      (e)  $Q = 32$  l-atm

The C-A transition is constant volume, so no work is done. Therefore,  $Q = \Delta U = U_A - U_C$ . For an ideal diatomic gas,  $U = 5/2 NkT = 5/2 pV$ . To calculate  $\Delta U$ , we need to know  $p_C$  or  $T_C$ . I'll do  $p$ .

For an adiabatic process,  $pV^\gamma = \text{const}$ , so  $\frac{p_C}{p_B} = \left(\frac{V_B}{V_C}\right)^\gamma$ .

Now,  $p_B = 1/3 p_A$ . Thus,  $p_C = \frac{p_A}{3} \left(\frac{V_B}{V_C}\right)^\gamma = \frac{1 \text{ atm}}{3} 3^{7/5} = 1.55 \text{ atm}$ .

Finally,  $Q = -5/2(0.55 \text{ atm})(1 \text{ liter}) = -1.4 \text{ l-atm}$ . Heat flows out.

***The next two questions pertain to the following situation.***

A scuba tank is filled with air (78% N<sub>2</sub> and 22% O<sub>2</sub>) at a temperature of 300 K.

3. What is the average thermal energy per molecule of the N<sub>2</sub> in the container?

- (a)  $4.14 \times 10^{-21}$  J      (b)  $6.21 \times 10^{-21}$  J      (c)  $1.04 \times 10^{-20}$  J

Diatomic molecule: five quadratic modes:  $U = 5/2 kT = 1.04 \times 10^{-20}$  J.

4. What is the ratio of the root-mean-square speed of O<sub>2</sub> molecules to the root-mean-square speed of the N<sub>2</sub> molecules in the container

( $R = v_{rms}(O_2) / v_{rms}(N_2)$ )?

- (a)  $R = 0.88$       (b)  $R = 0.94$       (c)  $R = 1.07$   
(d)  $R = 1.14$       (e)  $R = 1.25$

$KE_{CM} = \frac{1}{2}mv^2$  is the same for both kinds of molecules ( $3/2 kT$ ), so the ratio of the speeds is the inverse of the square root of the mass ratio:

$$R = \frac{v_{O_2}}{v_{N_2}} = \sqrt{\frac{m_{N_2}}{m_{O_2}}} = \sqrt{\frac{28}{32}} = 0.94$$

*The next two questions pertain to the following situation.*

A balloon is filled with  $\text{H}_2$  gas at  $T = 300 \text{ K}$ ,  $P = 1 \text{ atm}$ , and  $V = 5 \text{ liters}$ .

5. What is the total **rotational energy** of  $\text{H}_2$  molecules in the balloon?

- (a)  $2.5 \times 10^2 \text{ J}$    (b)  $5.0 \times 10^2 \text{ J}$    (c)  $7.5 \times 10^2 \text{ J}$

Two rotational modes:  $\text{KE}_{\text{rot}} = kT$  per molecule, so  $U_{\text{rot}} = NkT = pV$ .

One atmosphere is  $10^5 \text{ Pascals}$ , and 5 liters is  $5 \times 10^{-3} \text{ m}^3$ , so

$$pV = 5 \times 10^2 \text{ J}.$$

6. How much heat,  $Q$ , must flow out of the  $\text{H}_2$  gas if the balloon is to contract to a volume of 1 liter with no change in pressure?

- (a)  $Q = 0$       (b)  $Q = 6 \text{ l-atm}$    (c)  $Q = 10 \text{ l-atm}$   
(d)  $Q = 14 \text{ l-atm}$    (e)  $Q = 19 \text{ l-atm}$

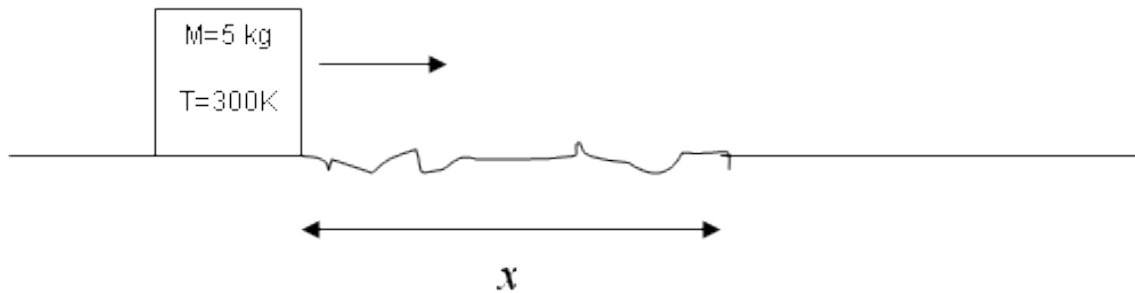
$Q = \Delta U + W_{\text{by}}$ . Positive  $Q$  means heat flows in.

$$\Delta U = \frac{5}{2} \Delta(pV) = -\frac{5}{2} p\Delta V = -10 \text{ l-atm}.$$

$$W_{\text{by}} = p\Delta V = -4 \text{ l-atm}$$

So,  $Q = -14 \text{ l-atm}$ . Heat flows out.

7. A 5-kg block with a specific heat of 250 J/kg-K is sliding across a frictionless surface. It is at a temperature of 300 K when it encounters a rough terrain of length  $x$ . It traverses the full length of the terrain and exits with a higher temperature of 301 K. A constant force of 80 N due to friction acts on the block while it is traversing the rough terrain. Assuming that the temperature change is due to friction and that all of the thermal energy generated is absorbed by the block, how long,  $x$ , is the rough terrain?



- (a)  $x = 11\text{ m}$       (b)  $x = 16\text{ m}$       (c)  $x = 22\text{ m}$   
(d)  $x = 28\text{ m}$       (e)  $x = 43\text{ m}$

The block has gained thermal energy at the expense of its CM kinetic energy.

$$\Delta U = C\Delta T = (250\text{ J/kg-K}) \times (5\text{ kg}) \times (1\text{ K}) = 1250\text{ J.}$$

This must be balanced by the negative  $Fd$  done by the friction:

$$d = 1250\text{ J} / 80\text{ N} = 15.6\text{ m.}$$

8. 5 liters of 90°C water from a kettle is mixed with 10 liters of 10°C water in a bucket to warm it up for washing a car. Find the temperature  $T$  of the mixed water, assuming no significant heat loss during the mixing. The specific heat of water is 4200 J/(kg K).

- (a)  $T = 10.5^\circ\text{C}$    (b)  $T = 20.6^\circ\text{C}$    (c)  $T = 36.7^\circ\text{C}$   
(d)  $T = 45.0^\circ\text{C}$    (e)  $T = 50.0^\circ\text{C}$

Energy is conserved – the cold water gains the heat that the hot water loses:

$$\Delta U_{hot} = (T_f - T_{hot})c_{H_2O}m_{hot} =$$
$$-\Delta U_{cold} = -(T_f - T_{cold})c_{H_2O}m_{cold}$$

Solve for  $T_f$ . The specific heat cancels out, because the two materials are the same:

$$T_f = (m_{hot}T_{hot} + m_{cold}T_{cold}) / (m_{hot} + m_{cold}) = 36.7^\circ\text{C}$$

9. You can put your hand in an oven at 200°C and even touch a baking cake, without serious harm. But you must avoid touching anything in the oven made from metal. Why is it not so harmful unless you touch metal?

- (a) When air, cake and metal are in thermal equilibrium, the metal is at a much higher temperature than the air and the cake.  
(b) Metals are good thermal conductors, so the energy can be rapidly conducted to your hand.  
(c) The metal has much lower thermal conductivity than the air.

a and c are both false, so they can't be the reason.

10. In the middle of the nineteenth century, James Joule performed a great series of experiments, which was part of the work leading to the law of conservation of energy. One of them was on his honeymoon, when he measured the temperature difference between water at the top and bottom of a waterfall. If the waterfall was 100 m high, what maximum temperature difference  $\Delta T_{max}$  could Joule expect due to the conversion of gravitational potential energy? The specific heat of water is 4200 J/(kg K).

(a)  $\Delta T_{max} = 0$  K   (b)  $\Delta T_{max} = 0.23$  K   (c)  $\Delta T_{max} = 1.0$  K

Set  $mgh = \Delta U = mc\Delta T$ . The mass cancels out:  $\Delta T = gh/c = 0.23$  K.

***This and the next question pertain to the following situation.***

Two holes are made in a 1-kg block of aluminum at a distance  $d$  from each other. A 48 W electric immersion heater is placed in one hole, and a thermometer in the other. Both objects make good thermal contact with the block. The heater is switched on for exactly 3 seconds and then switched off.

11. The time  $t$  it takes for heat to diffuse from the heater to the thermometer scales with the distance  $d$  as:

(a)  $t \sim d^{1/2}$  (b)  $t \sim d$  (c)  $t \sim d^2$

Heat flow is a diffusion process:  $\langle x^2 \rangle = 2Dt$  .

12. After the heater is switched off, the system is allowed to equilibrate. One finds that the temperature in the metal has risen from 20°C to 20.15°C. Calculate the specific heat of aluminum.

(a) 70 J/(kg K) (b) 590 J/(kg K) (c) 960 J/(kg K)

The heat has raised the internal energy (no work was done).

$$Q = (48 \text{ W})(3 \text{ s}) = 144 \text{ J} = \Delta U = mc\Delta T$$
$$\Rightarrow c = 144 \text{ J} / (1 \text{ kg} \times 0.15 \text{ K}) = 960 \text{ J}/(\text{kg k})$$



***This and the next question are related.***

**13.** Water molecules will diffuse in still air. If a bottle of water is opened, what will be the RMS distance  $r_{RMS} = \sqrt{\langle r^2 \rangle}$  a water molecule has travelled away from the bottle after 20 s? Assume the room temperature is 300 K, and that the mean free path of the water molecules in dry air at 300 K is  $10^{-7}$  m. The molar mass of water is 0.018 kg.

- (a)  $r_{RMS} = 0.2$  mm      (b)  $r_{RMS} = 0.5$  mm      (c)  $r_{RMS} = 1$  mm  
(d)  $r_{RMS} = 2$  cm      (e)  $r_{RMS} = 5$  cm

This is diffusion in 3D:  $\langle x^2 \rangle = 6Dt = 6(l^2 / 3\tau)t = 6(lv / 3)t$ .

We need to calculate the rms speed,  $v$ , of the molecules.

Use equipartition:

$$\frac{1}{2}mv^2 = \frac{3}{2}kT \Rightarrow v_{rms} = \sqrt{3kT / m} = \sqrt{3kT / (m_{mol} / N_A)} = 644 \text{ m/s}$$

$$\text{So, } r_{rms} = \sqrt{6(10^{-7} \text{ m})(644 \text{ m/s})(20 \text{ s}) / 3} = 0.051 \text{ m}$$

If the temperature of the air is increased from 300 K to 310 K, what will happen to the diffusion radius  $r_{RMS}$  calculated above (assume the mean-free path is not affected)?

- (a)  $r_{RMS}$  will increase. (b)  $r_{RMS}$  will decrease. (c)  $r_{RMS}$  will not change.

Higher temperature means larger  $KE_{CM}$ , or larger  $v_{rms}$ .

This will increase the diffusion rate.

15. Calculate the dimensionless entropy  $\sigma$  of 20 distinguishable spins for the configuration with  $N_{up} = N_{down}$ .

- (a)  $\sigma = 0$       (b)  $\sigma = 12.1$       (c)  $\sigma = 27.2$   
(d)  $\sigma = 30.2$       (e)  $\sigma = 42.3$

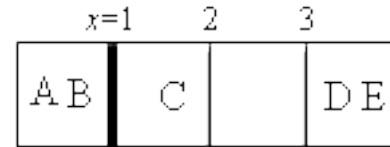
How many ways to choose (make the spin be up) 10 objects from a collection of 20 objects?  $\Omega = 20!/10!10! = 1.85 \times 10^5$ .

So,  $\sigma = \ln(\Omega) = 12.1$  .

*This and the next question are related.*

16. Considering volume exchange but not energy exchange, calculate the equilibrium position,  $x$ , of the partition. There are four **multiple-occupancy** cells, with two **distinguishable** particles on the left and three on the right. The figure shows one microstate, with the partition at  $x = 1$ .

- (a)  $x = 1$     (b)  $x = 2$     (c)  $x = 3$



**Guess:** Position 2 makes the densities most equal (1 and 1.5).

**Calculate:**  $\Omega = M^N$ , so for  $x = \{1, 2, 3\}$ :

$$\Omega_L = \{1, 4, 9\}, \text{ and } \Omega_R = \{27, 8, 1\}$$

Therefore,  $\Omega_{\text{tot}} = \{27, 32, 9\}$ , and the answer is  $x = 2$ .

17. The previous question assumed distinguishable particles. Calculate the change in the entropy if all the particles were instead indistinguishable, assuming that the partition is held at  $x = 1$  (and that the first cell still has two particles).

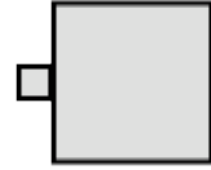
- (a)  $\sigma_{\text{indistinguishable}} - \sigma_{\text{distinguishable}} = -1.79$   
 (b)  $\sigma_{\text{indistinguishable}} - \sigma_{\text{distinguishable}} = -0.99$   
 (c)  $\sigma_{\text{indistinguishable}} - \sigma_{\text{distinguishable}} = 0.27$   
 (d)  $\sigma_{\text{indistinguishable}} - \sigma_{\text{distinguishable}} = 0.99$   
 (e)  $\sigma_{\text{indistinguishable}} - \sigma_{\text{distinguishable}} = 1.79$

Other things being equal, indistinguishable particles have less entropy, so the answer must be a or b.

The original entropy was  $\ln(27) = 3.30$ .

The final entropy is  $\ln(1) + \ln(5!/3!2!) = 2.30$

18. Two systems are in thermal equilibrium at temperature,  $T$ . A tiny bit of energy  $\delta E$  flows from the small system to the big system. By how much,  $\delta\sigma$ , does the total entropy of the two systems change?

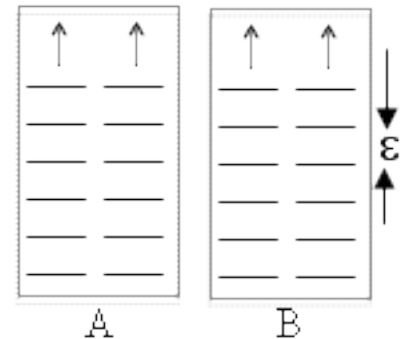


- (a)  $\delta\sigma = -T \delta E$    (b)  $\delta\sigma = 0$    (c)  $\delta\sigma = +T \delta E$

This was a hard question. In equilibrium the total entropy is maximum. Therefore,  $d\sigma/dU = 0$ . So, if a **tiny** amount of entropy flows, the entropy does not change. (The change is proportional to  $(\delta E)^2$ .)

Within a sufficiently small region around equilibrium, all macrostates are equally probable.

19. In this question, all the oscillators have the same energy level spacing,  $\varepsilon$ . Two systems, A and B, in thermal contact, each contain 2 oscillators and share a total of  $4\varepsilon$  between them. What is the probability,  $P$ , that system A has all the energy?



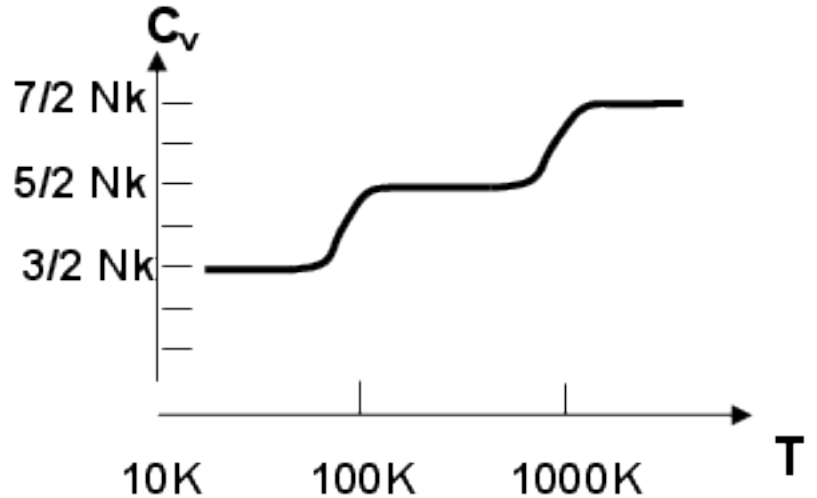
- (a)  $P = 0.042$    (b)  $P = 0.062$    (c)  $P = 0.14$   
 (d)  $P = 0.25$    (e)  $P = 0.50$

The total number of microstates is:  $\Omega_{tot} = \frac{(q+N-1)}{q!(N-1)!} = \frac{7!}{4!3!} = 35$ .

The number of microstates with all four energy packets on the left is:  $\Omega_{4L} = \frac{5!}{4!1!} = 5$ .

So, the probability is  $1/7$ .

20. Use the following diagram of the heat capacity of a particular diatomic molecule to estimate the *vibrational* frequency  $f$  of the molecule.



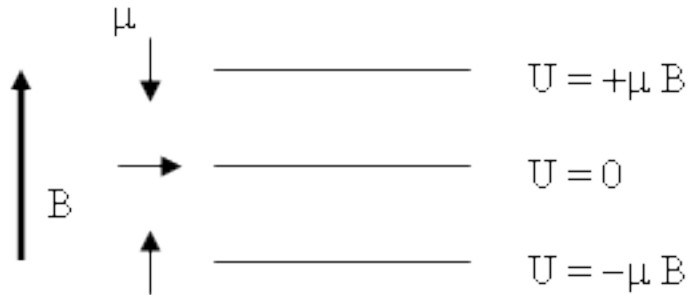
- (a)  $f = 2 \times 10^{10}$  Hz
- (b)  $f = 2 \times 10^{12}$  Hz
- (c)  $f = 2 \times 10^{13}$  Hz
- (d)  $f = 2 \times 10^{15}$  Hz
- (e) Cannot be determined from the information given.

$C_v$  rises from  $5/2 Nk$  to  $7/2 Nk$  when  $kT$  is large enough to excite the vibrational modes (oscillation of the chemical bond “springs”), that is when  $kT \sim hf$ . Because this is happening at  $T \sim 1000$  K,  
 $f \sim kT/h \sim 2 \times 10^{13}$  Hz.

NOTE: The  $3/2$  to  $5/2$  rise occurs when the molecule begins to rotate.

**The next three questions pertain to the following situation.**

In class we considered electron spins, which led to a magnetic moment that was either aligned with or opposed to a magnetic field. Here we consider an atom with a magnetic moment that can have one of three



orientations (three projections along the magnetic field -- up, down, and zero), leading to the energy level diagram shown below.

**21.** If the field magnitude is 5 Tesla, and the magnetic moment is  $\mu = 2 \times 10^{-23}$  J/T, what is the probability  $P_{ground}$  that a given atom will be in the lowest energy state at 10 K?

- (a)  $P_{ground} = 0$     (b)  $P_{ground} = 0.14$     (c)  $P_{ground} = 0.58$   
 (d)  $P_{ground} = 0.72$     (e)  $P_{ground} = 1$

The math is a bit simpler if we pick the lowest energy state to have  $E = 0$ . Then, the three Boltzmann factors are:

1,  $\exp(-\mu B/kT) = 0.485$ , and  $\exp(-2\mu B/kT) = 0.485^2 = 0.235$ .

So,  $P_{ground} = 1/(1+0.485+0.235) = 0.581$

**22.** At approximately what temperature,  $T$ , will the probability of being in the upper-most state be greater than the probability of being in the middle state?

- (a) This will occur when  $kT > \mu B$ .  
 (b) This will not occur at any temperature.  
 (c) Cannot be determined from the information given.

The Boltzmann factor for the upper state is always less than the one for the middle state, so it never happens in thermal equilibrium.

23. What is the limiting value  $S_0$  of the entropy of an ensemble of  $N$  such atoms (in a 1 Tesla field), as the temperature is lowered toward  $T = 0$ ?
- (a)  $S_0 = 0$       (b)  $S_0 = Nk$       (c)  $S_0 = \ln(3) Nk$

As  $T \rightarrow 0$ , all spins point the same direction. There is only one microstate.  $S_0 = 0$ .

*The next two questions are related.*

An advanced telescope spots the moon of a distant planet, and observes that the moon has an isothermal atmosphere of gaseous  $\text{CO}_2$  that extends  $\sim 1000$  m into the air (*i.e.*, this is the height at which the density falls to about 1/3 of its maximum value).

24. Assuming that the gravity on the moon is  $1 \text{ m/s}^2$ , estimate the temperature of the moon.
- (a)  $T = 5 \text{ K}$       (b)  $T = 25 \text{ K}$       (c)  $T = 155 \text{ K}$

$$\rho(h) = \rho_0 e^{-mgh/kT} \Rightarrow T = -mgh / k \ln(\rho / \rho_0)$$

The mass of a  $\text{CO}_2$  molecule is  $0.044 \text{ kg}/N_A = 7.30 \times 10^{-26} \text{ kg}$ .  
So,  $T = 4.8 \text{ K}$ .

25. Compare the gas density at a height of 2000 m to the density at 1000 meters.
- (a) The density at 2000 m is smaller by a factor of about 2.  
(b) The density at 2000 m is smaller by a factor of about 3.  
(c) The density at 2000 m is larger by a factor of about 2.

The density decreases exponentially. This means that the factor decrease from 1000 m to 2000 m is the same as from 0 to 1000 m.