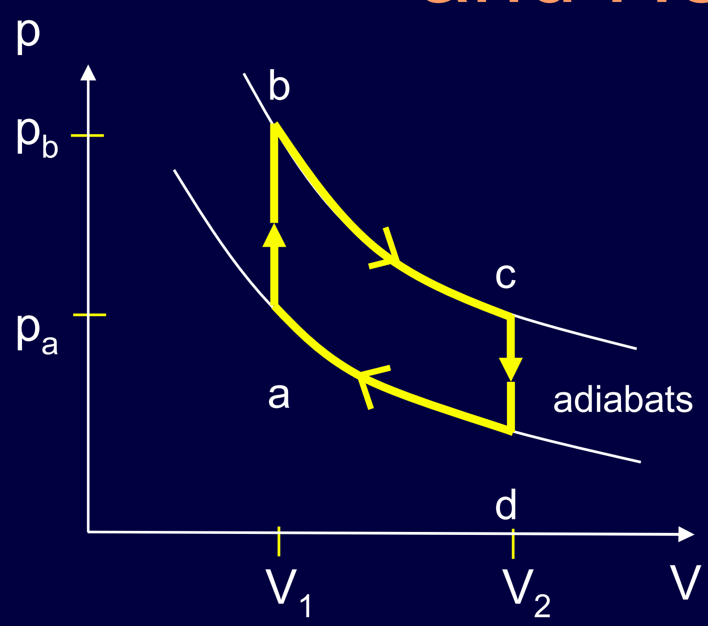
Act 0

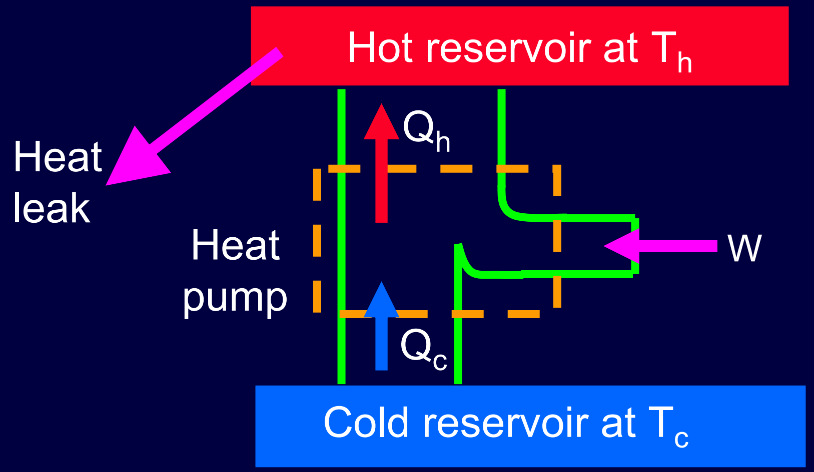
The midterm exam was:

a. Just right
b. Too easy
c. Too hard
d. Too long
e. Too long and too hard

Lecture 14

Carnot engines, Refrigerators and Heat Pumps





Reading for this Lecture:
Elements Ch 4E-F, 10A-B

Heat Engine Efficiency

We pay for the heat input, Q_H, so:

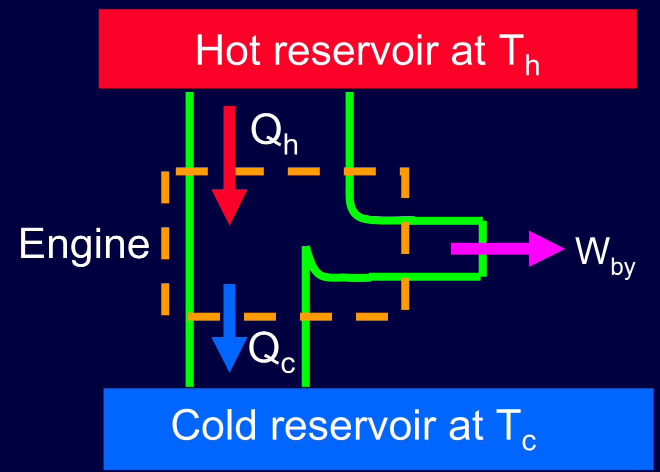
Define the efficiency

$$\varepsilon \equiv \frac{\text{work done by the engine}}{\text{heat extracted from reservoir}} = \frac{\text{results}}{\text{cost}}$$

$$\boxed{\varepsilon \equiv \frac{W_{by}}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}}$$

Valid for all heat engines.
(Conservation of energy)

Cartoon picture of a heat engine:



Remember:

We define Q_h and Q_c as *positive*.
The arrows define direction of flow.

What's the best we can do?

The Second Law will tell us.

Heat Engine Summary

For all cycles: $\varepsilon = 1 - \frac{Q_c}{Q_h}$ Some energy is dumped into the cold reservoir.

For the Carnot cycle: $\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$ Q_c cannot be reduced to zero.

Carnot (best) efficiency: $\varepsilon_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$ Only for reversible cycles.

Carnot engines are an idealization - impossible to realize.
They require very slow processes, and perfect insulation.
When there's a net entropy increase, the efficiency is reduced:

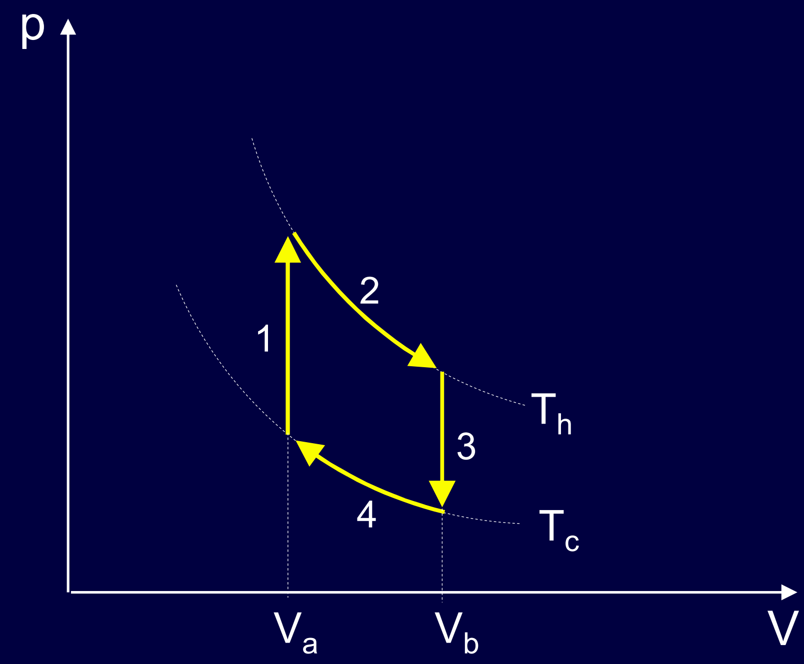
$$\varepsilon = \varepsilon_{\text{Carnot}} - \frac{T_C \Delta S_{tot}}{Q_H}$$

Act 1: Stirling Efficiency

Will our Stirling engine achieve Carnot efficiency?

a) Yes

b) No

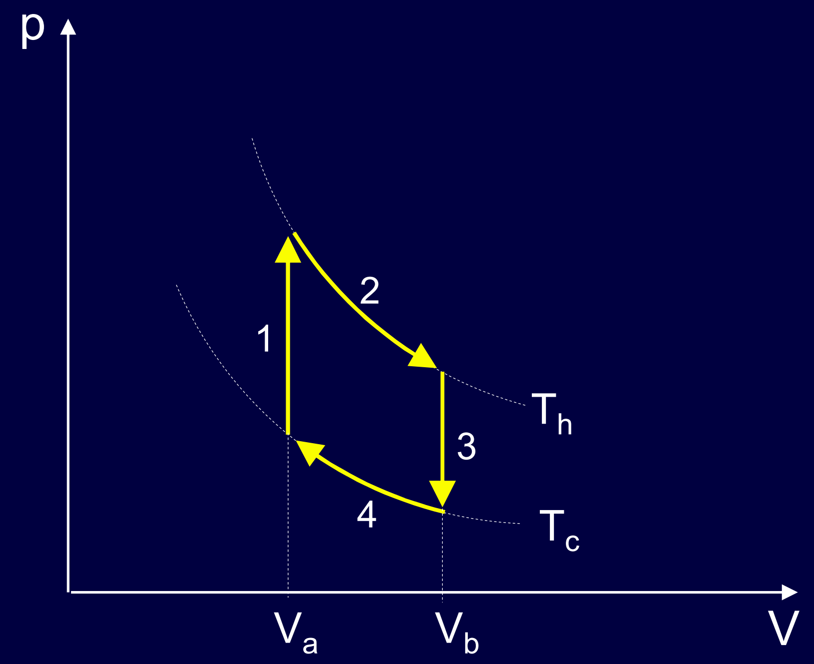


Solution

Will our Stirling engine achieve Carnot efficiency?

a) Yes

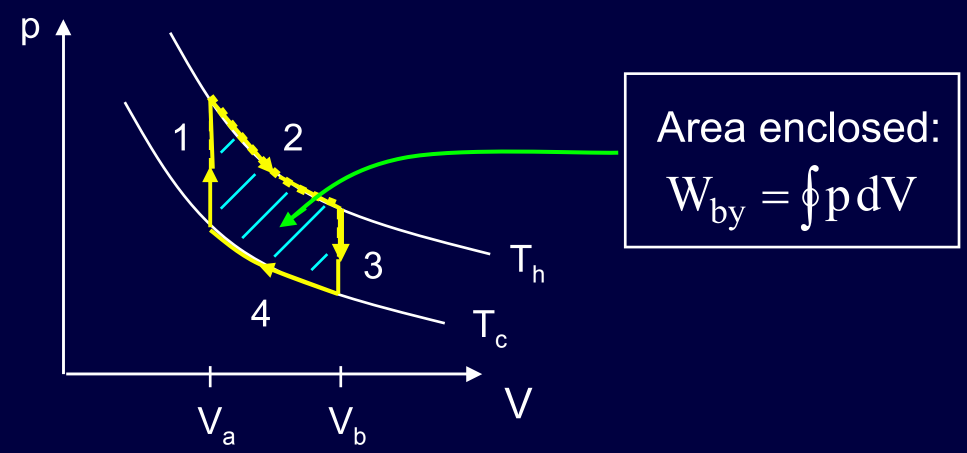
b) No



Processes 1 and 3 are irreversible.
(isochoric heating and cooling)

1: Cold gas touches hot reservoir.
3: Hot gas touches cold reservoir.

Example: Efficiency of Stirling Cycle



Total work done by the gas is the sum of steps 2 and 4:

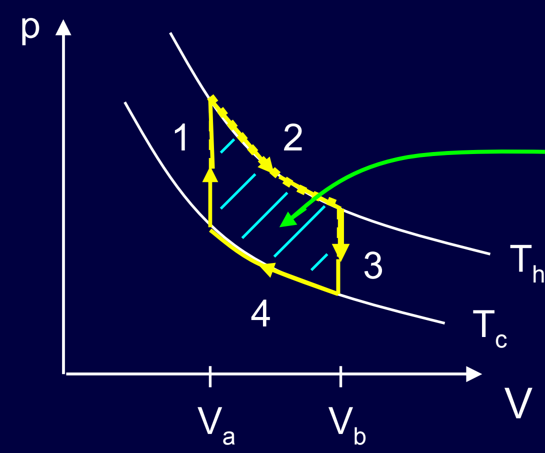
$$W_2 = \int_{V_a}^{V_b} p dV = NkT_h \int_{V_a}^{V_b} \frac{dV}{V} = NkT_h \ln\left(\frac{V_b}{V_a}\right)$$

$$W_4 = \int_{V_b}^{V_a} p dV = NkT_c \int_{V_b}^{V_a} \frac{dV}{V} = -NkT_c \ln\left(\frac{V_b}{V_a}\right)$$

$$W_{by} = W_2 + W_4 = Nk\left(T_h - T_c\right)\ln\left(\frac{V_b}{V_a}\right)$$

We need a temperature difference if we want to get work out of the engine.

Solution



Area enclosed:
$W_{by} = \oint p\,dV$

Heat extracted from the hot reservoir, exhausted to cold reservoir:

$$Q_h = Q_1 + Q_2 = \alpha Nk(T_h - T_c) + NkT_h \ln\left(\frac{V_b}{V_a}\right)$$

$$-Q_c = Q_3 + Q_4 = -\alpha Nk(T_h - T_c) - NkT_c \ln\left(\frac{V_b}{V_a}\right)$$

Solution

Let's put in some numbers:

$V_b = 2V_a$
$\alpha = 3/2$ (monatomic gas)
$T_h = 373K$ (boiling water)
$T_c = 273K$ (ice water)

$$\varepsilon \equiv \frac{W_{by}}{Q_h} = \frac{(T_h - T_c)\ln\left(\frac{V_b}{V_a}\right)}{\frac{3}{2}(T_h - T_c) + T_h \ln\left(\frac{V_b}{V_a}\right)}$$

$(\ell n\ 2 = 0.69)$

$$= \frac{100(0.69)}{150 + 373(0.69)} = 16.9\%$$

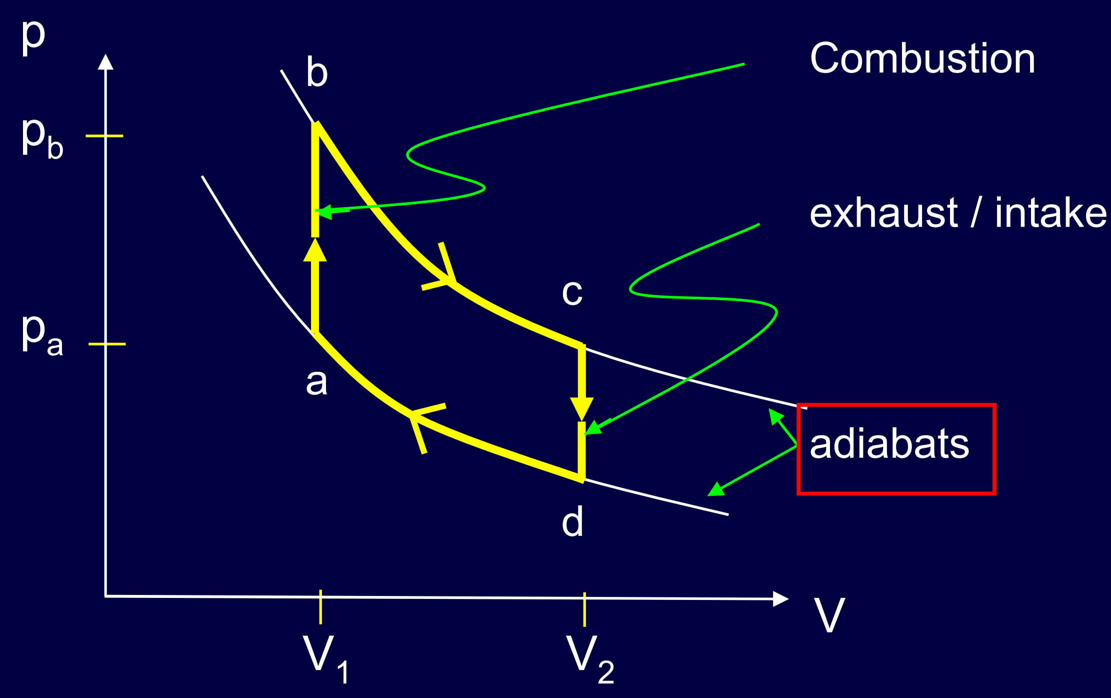
For comparison:

$\varepsilon_{carnot} = 1 - 273/373 = 26.8\%$

Example: Gasoline Engine

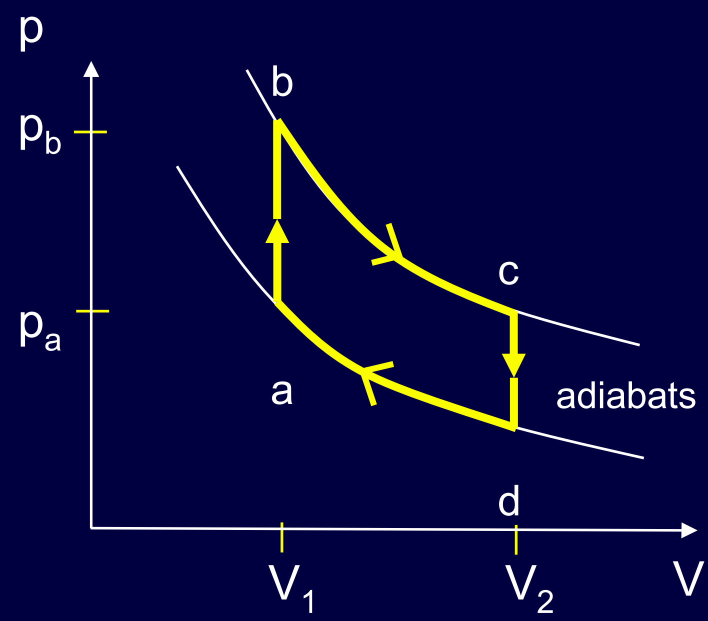
It's not really a heat engine because the input energy is via fuel injected directly into the engine, not via heat flow. There is no obvious hot reservoir. However, one can still calculate work and energy input for particular gas types.

We can treat the gasoline engine as an Otto cycle:



b→c and d→a are nearly adiabatic processes,
because the pistons move too quickly for much heat to flow.

Solution



Calculate the efficiency:

$$Q_{in} = C_v(T_b - T_a)$$

$$W_{by} = W_{b\to c} - W_{d\to a}$$

$$W_{b\to c} = U_b - U_c = C_v(T_b - T_c) \quad \text{because } Q_{b\to c} = 0$$

$$W_{d\to a} = U_d - U_a = C_v(T_d - T_a) \quad \text{because } Q_{d\to a} = 0$$

$$W_{by} = C_v(T_b - T_c) - C_v(T_a - T_d)$$

$$\varepsilon = \frac{W_{by}}{Q_{in}} = \frac{C_v(T_b - T_c) - C_v(T_a - T_d)}{C_v(T_b - T_a)} = 1 - \frac{(T_c - T_d)}{(T_b - T_a)}$$

Solution

Write it in terms of volume instead of temperature.
We know the volume of the cylinders.

$$\varepsilon = 1 - \frac{(T_c - T_d)}{(T_b - T_a)}$$

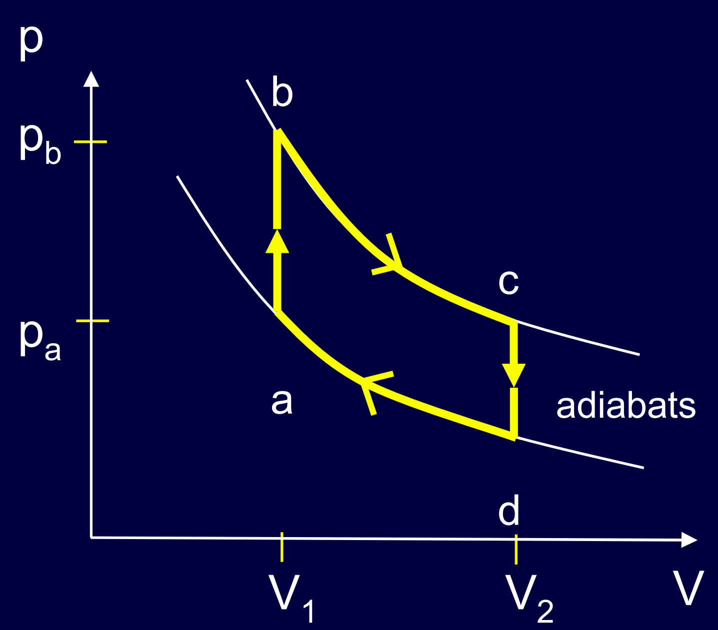
$$T_c{}^{\alpha} V_2 = T_b{}^{\alpha} V_1 \Rightarrow T_c = T_b (V_1 / V_2)^{1/\alpha}$$

$$T_d{}^{\alpha} V_2 = T_a{}^{\alpha} V_1 \Rightarrow T_d = T_a (V_1 / V_2)^{1/\alpha}$$

$$\therefore \frac{(T_c - T_d)}{(T_b - T_a)} = \frac{(T_b - T_a)(V_1 / V_2)^{1/\alpha}}{(T_b - T_a)}$$

$$= \left(\frac{V_1}{V_2}\right)^{1/\alpha} = \left(\frac{V_2}{V_1}\right)^{-1/\alpha} = \left(\frac{V_2}{V_1}\right)^{1-\gamma}$$

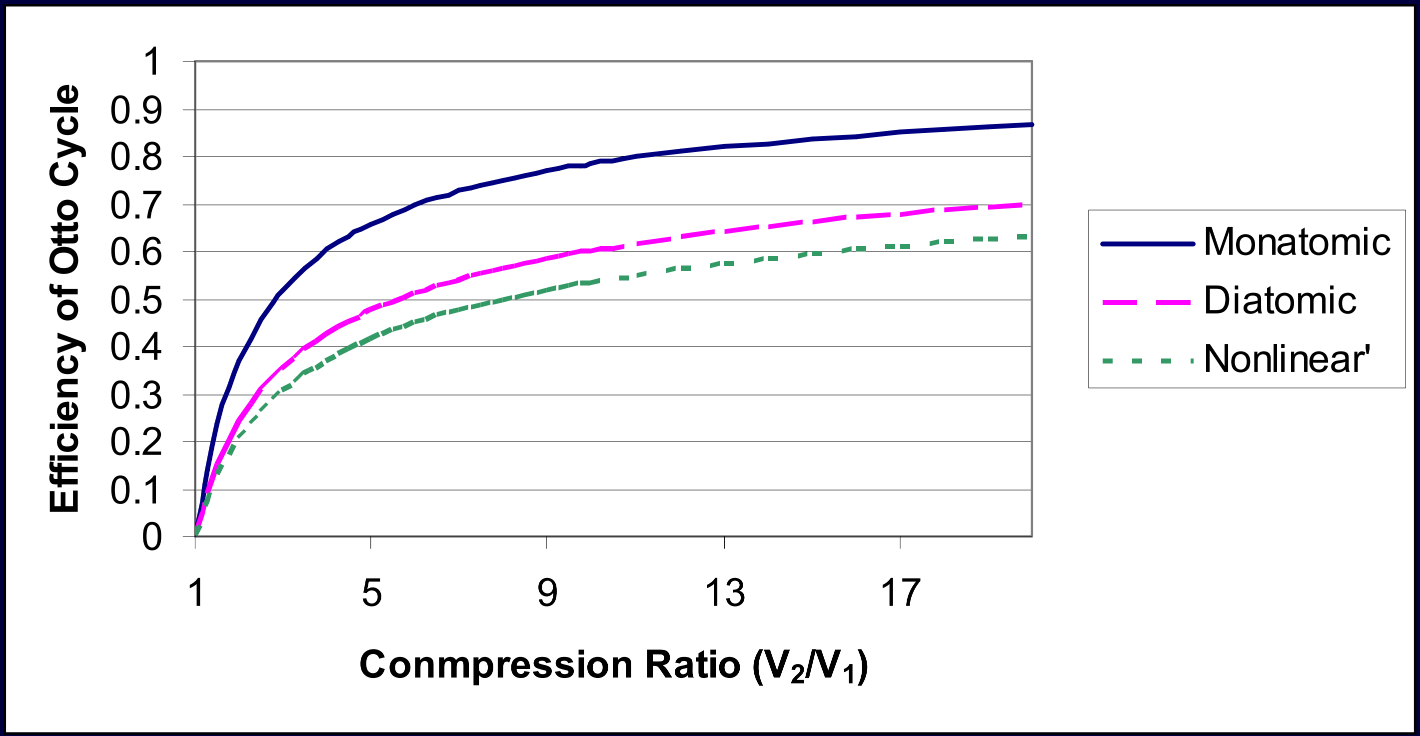
$$\varepsilon = 1 - \left(\frac{V_2}{V_1}\right)^{1-\gamma}$$



Compression ratio $\equiv V_2/V_1 \approx 10$
γ = 1.4 (diatomic gas) $\rightarrow \varepsilon$ = 60%

(in reality about 30%, due to friction etc.)

Solution



Why not simply use a higher compression ratio?

- If V_2 big, we need a huge, heavy engine (OK for fixed installations).
- If V_1 small, the temperature gets too high, causing premature ignition. High compression engines need high octane gas, which has a higher combustion temperature.

How to Achieve Carnot Efficiency

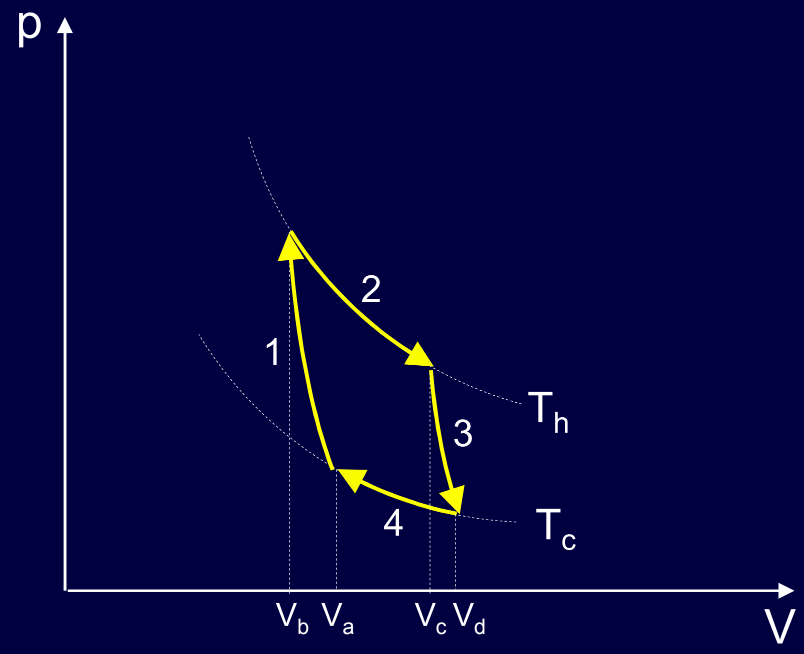
To achieve Carnot efficiency, we must replace the isochors (irreversible) with reversible processes. Let's use adiabatic processes, as shown:

Processes 1 and 3 are now adiabatic.
Processes 2 and 4 are still isothermal.

This cycle is reversible, which means:

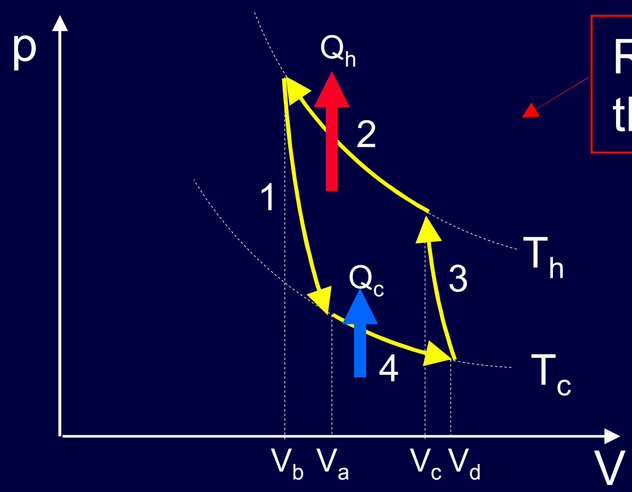
S_{tot} remains constant: $\Rightarrow \varepsilon = \varepsilon_{Carnot}$.

This thermal cycle is called the Carnot cycle, and an engine that implements it is called a Carnot heat engine.

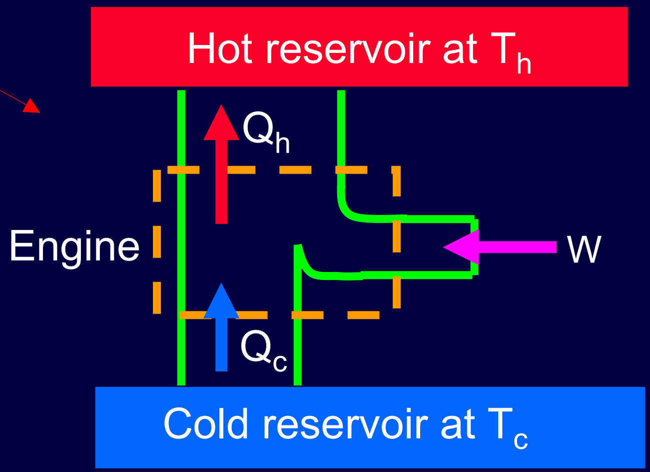


Run the Engine in Reverse

The Carnot cycle is reversible (each step is reversible):



Reverse all the arrows



When the engine runs in reverse:

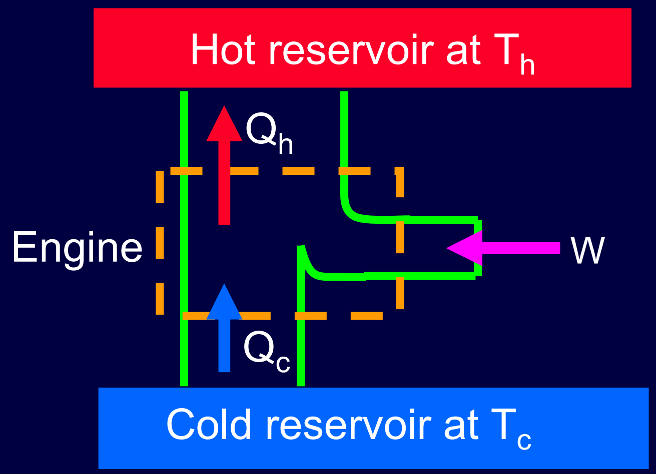
Heat is transferred from cold to hot by action of work on the engine.
Note that heat never flows spontaneously from cold to hot;
the cold gas is being heated by adiabatic compression (process 3).

$Q_c / Q_h = T_c / T_h$ is still true. (Note: Q_h, Q_c, and W are still positive!)

Refrigerators and Heat Pumps

Refrigerators and heat pumps are heat engines running in reverse.

How do we measure their performance?
It depends on what you want to accomplish.



Refrigerator:
We want to keep the food cold (Q_c).
We pay for W (the electric motor in the fridge).

So, the coefficient of performance, K is:

It's not called "efficiency".

$$K \equiv \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{T_c}{T_h - T_c}$$

Heat pump:
We want to keep the house warm (Q_h).
We pay for W (the electric motor in the garden).

The coefficient of performance, K is:

$$K \equiv \frac{Q_h}{W} = \frac{Q_h}{Q_h - Q_c} = \frac{T_h}{T_h - T_c}$$

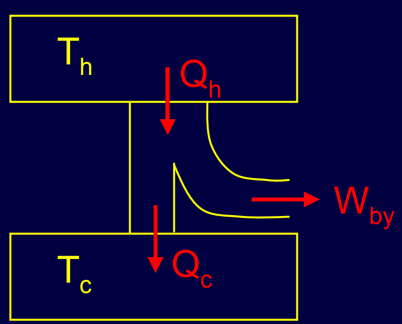
Helpful Hints in Dealing with Engines and Fridges

Sketch the process (see figures below).
Define Q_h and Q_c and W_{by} (or W_{on}) as positive and show directions of flow.

Determine which Q is given.

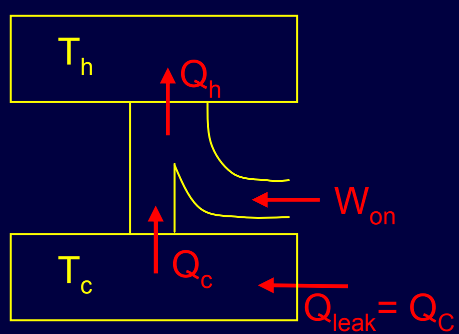
Write the First Law of Thermodynamics (FLT).

We considered three configurations of Carnot cycles:

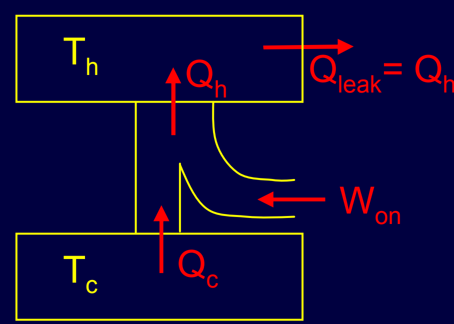


Engine:
We pay for Q_h,
we want W_{by}.
$W_{by} = Q_h - Q_c = \varepsilon Q_h$
Carnot: $\varepsilon = 1 - T_c/T_h$

This has large ε when $T_h - T_c$ is large.



Refrigerator:
We pay for W_{on},
we want Q_c.
$Q_c = Q_h - W_{on} = K W_{on}$
Carnot: $K = T_c/(T_h - T_c)$



Heat pump:
We pay for W_{on},
we want Q_h.
$Q_h = Q_c + W_{on} = K W_{on}$
Carnot: $K = T_h/(T_h - T_c)$

These both have large K when $T_h - T_c$ is small.

Act 2: Refrigerator

There is a 70 W heat leak (the insulation is not perfect) from a room at temperature 22° C into an ideal refrigerator. How much electrical power is needed to keep the refrigerator at -10° C? Assume Carnot performance.

a) < 70 W

b) = 70 W

c) > 70 W

Act 2: Refrigerator

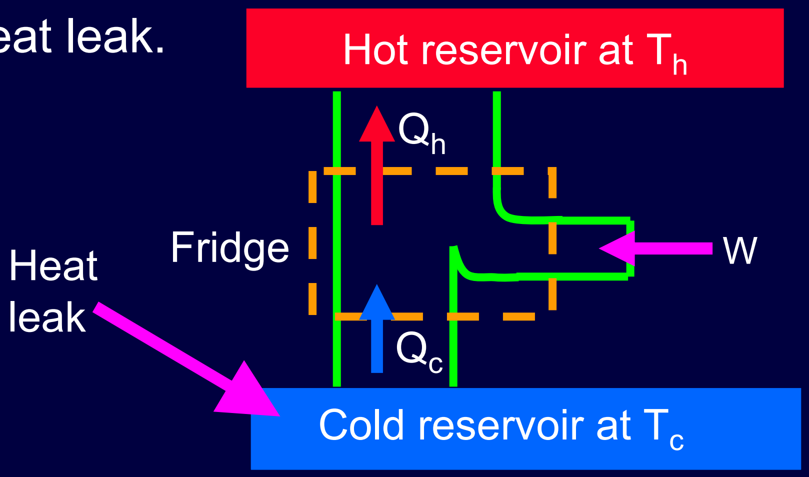
There is a 70 W heat leak (the insulation is not perfect) from a room at temperature 22° C into an ideal refrigerator. How much electrical power is needed to keep the refrigerator at -10° C?
Assume Carnot performance.

Hint: Q_c must exactly compensate for the heat leak.

a) < 70 W

b) = 70 W

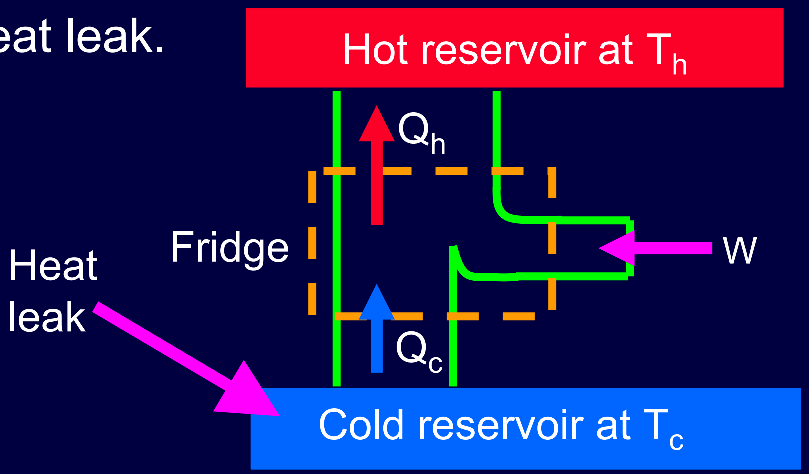
c) > 70 W



Solution

There is a 70 W heat leak (the insulation is not perfect) from a room at temperature 22° C into an ideal refrigerator. How much electrical power is needed to keep the refrigerator at -10° C? Assume Carnot performance.

Hint: Q_c must exactly compensate for the heat leak.



$$W = Q_h - Q_c = Q_c\left(\frac{Q_h}{Q_c} - 1\right) = Q_c\left(\frac{T_h}{T_c} - 1\right)$$

We need Q_c = 70 J each second.

Therefore we need

$$W \equiv 70\left(\frac{295}{263} - 1\right) J/s$$

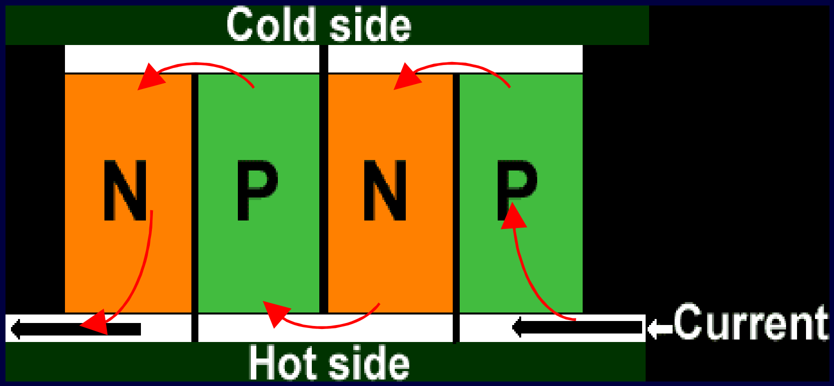
The motor power is 8.5 Watts.

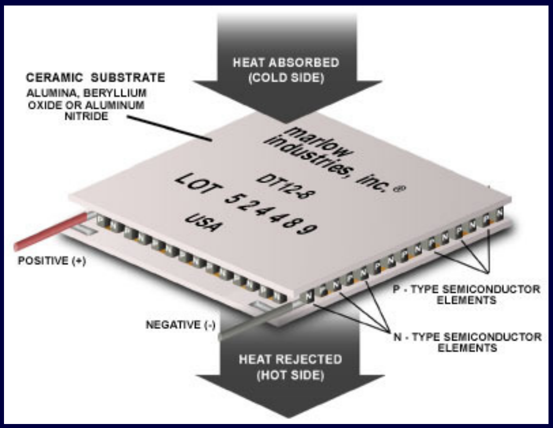
Watt = Joule/second.

This result illustrates an unintuitive property of refrigerators and heat pumps: When T_h - T_c is small, they pump more heat than the work you pay for.

Supplement: Peltier cooler

- Driving a current (~amps) through generates a temperature difference. 20-50°C typical
- Not so common – they're more costly, take a lot of power, and you still have to get rid of the heat! But....no moving parts to break.
- How's it work…





Electrons pushed from electron-deficit material (p-type) to electron-rich material (n-type); they slow down, cooling the top connector. Similarly, they heat up in going from n to p-type (bottom connector).

Despite the radically different construction, this heat pump must obey exactly the same limits on efficiency as the gas-based pumps, because these limits are based on the 1st and 2nd laws, not any details.

The Limits of Cooling

The maximum efficiency is $\varepsilon = \frac{Q_C}{W} \leq \frac{T_C}{T_H - T_C}$

Refrigerators work less well as T_h - T_c becomes large.

The colder you try to go, the less efficient the refrigerator gets. The limit as T_C goes to zero is zero efficiency !

Since heat leaks will not disappear as the object is cooled, you need to supply more work the colder it gets. The integral of the power required diverges as $T_c \rightarrow 0$.

Therefore you cannot cool a system to absolute zero.

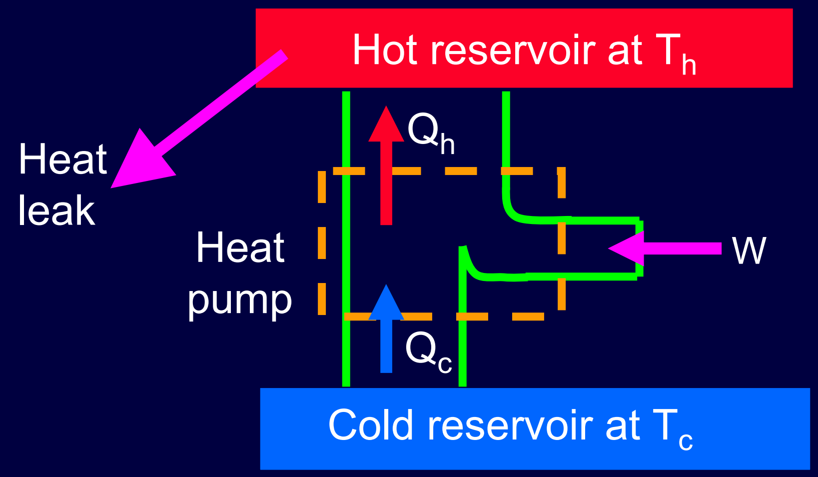
Act 3: Heat Pump

Suppose that the heat flow out of your 20° C home in the winter is 7 kW. If the temperature outside is -15° C, how much power would an ideal heat pump require to maintain a constant inside temperature?

a) W < 7 kW

b) W = 7 kW

c) W > 7 kW



Solution

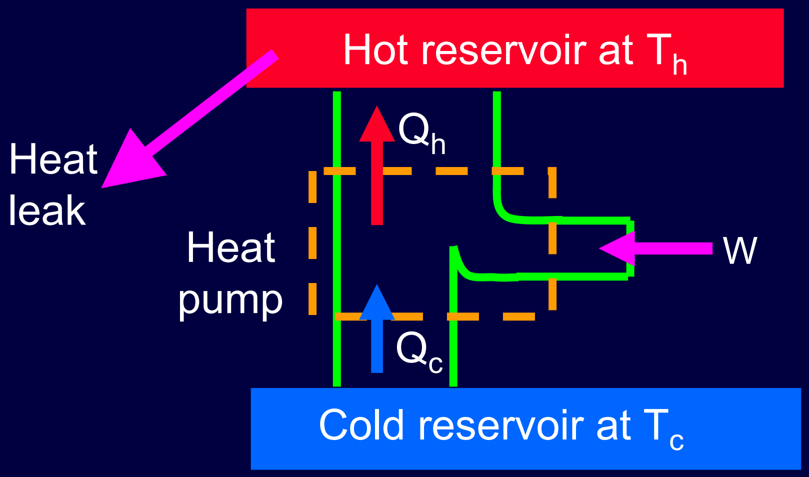
Suppose that the heat flow out of your 20° C home in the winter is 7 kW. If the temperature outside is -15° C, how much power would an ideal heat pump require to maintain a constant inside temperature?

a) W < 7 kW

b) W = 7 kW

c) W > 7 kW

This is what you'd need with an electric blanket or furnace.



$$W = Q_h - Q_c = Q_h\left(1 - \frac{Q_c}{Q_h}\right) = Q_h\left(1 - \frac{T_c}{T_h}\right)$$

We need Q_h = 7000 J each second.

Therefore we need

$$W \equiv 7000\left(1 - \frac{258}{293}\right) = 836 J/s$$

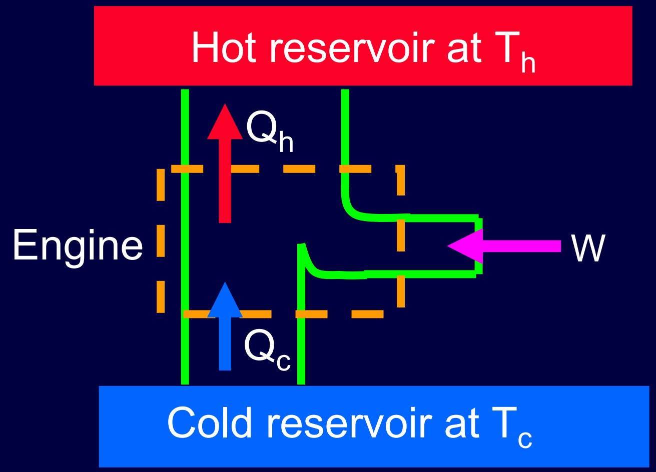
The electric company must supply 836 Watts, much less than the 7 kW that a furnace would require!

Beware:

Real heat pumps are not nearly ideal, so the advantage is smaller.

ACT 4:
Entropy Change in Heat Pump

Consider a Carnot heat pump.



1) What is the sign of the entropy change of the hot reservoir during one cycle?

a) $\Delta S_h < 0$ b) $\Delta S_h = 0$ c) $\Delta S_h > 0$

2) What is the sign of the entropy change of the cold reservoir?

a) $\Delta S_c < 0$ b) $\Delta S_c = 0$ c) $\Delta S_c > 0$

3) Compare the magnitudes of the two changes.

a) $|\Delta S_c| < |\Delta S_h|$ b) $|\Delta S_c| = |\Delta S_h|$ c) $|\Delta S_c| > |\Delta S_h|$

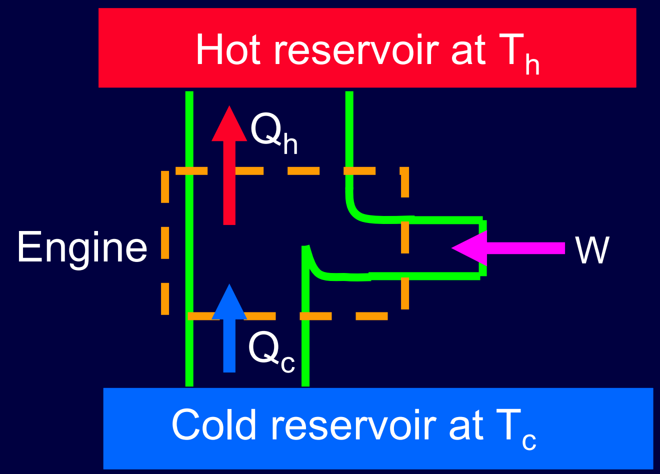
Solution

Consider a Carnot heat pump.

1) What is the sign of the entropy change of the hot reservoir during one cycle?

a) $\Delta S_h < 0$ b) $\Delta S_h = 0$ c) $\Delta S_h > 0$

Energy (heat) is entering the hot reservoir, so the number of available microstates is increasing.



2) What is the sign of the entropy change of the cold reservoir?

a) $\Delta S_c < 0$ b) $\Delta S_c = 0$ c) $\Delta S_c > 0$

3) Compare the magnitudes of the two changes.

a) $|\Delta S_c| < |\Delta S_h|$ b) $|\Delta S_c| = |\Delta S_h|$ c) $|\Delta S_c| > |\Delta S_h|$

Solution

Consider a Carnot heat pump.

1) What is the sign of the entropy change of the hot reservoir during one cycle?

a) $\Delta S_h < 0$ b) $\Delta S_h = 0$ c) $\Delta S_h > 0$

Energy (heat) is entering the hot reservoir, so the number of available microstates is increasing.

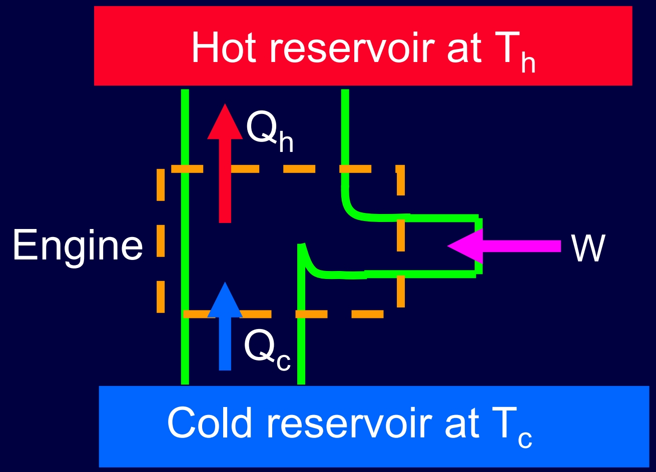
2) What is the sign of the entropy change of the cold reservoir?

a) $\Delta S_c < 0$ b) $\Delta S_c = 0$ c) $\Delta S_c > 0$

Energy (heat) is leaving the cold reservoir.

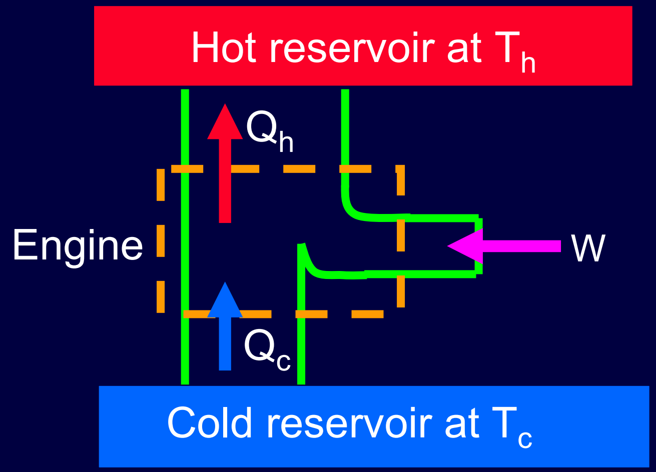
3) Compare the magnitudes of the two changes.

a) $|\Delta S_c| < |\Delta S_h|$ b) $|\Delta S_c| = |\Delta S_h|$ c) $|\Delta S_c| > |\Delta S_h|$



Solution

Consider a Carnot heat pump.



1) What is the sign of the entropy change of the hot reservoir during one cycle?

a) $\Delta S_h < 0$ b) $\Delta S_h = 0$ **c) $\Delta S_h > 0$**

Energy (heat) is entering the hot reservoir, so the number of available microstates is increasing.

2) What is the sign of the entropy change of the cold reservoir?

a) $\Delta S_c < 0$ b) $\Delta S_c = 0$ c) $\Delta S_c > 0$

Energy (heat) is leaving the cold reservoir.

3) Compare the magnitudes of the two changes.

a) $|\Delta S_c| < |\Delta S_h|$ **b) $|\Delta S_c| = |\Delta S_h|$** c) $|\Delta S_c| > |\Delta S_h|$

It's a reversible cycle, so $\Delta S_{tot} = 0$. Therefore, the two entropy changes must cancel. Remember that the entropy of the "engine" itself does not change.

Note: We've neglected the heat leak OUT of the hot reservoir. In fact, this must equal Q_h (why?). If that heat leaks directly into the cold reservior, this will be irreversible…

Next Time

Available Work and Free Energy