Lecture 4:

Classical Illustrations of Macroscopic Thermal Effects

- Heat capacity of solids & liquids
- Thermal conductivity

References for this Lecture:
Elements Ch 3,4A-C

Reference for Lecture 5:
Elements Ch 5

Last time: Heat capacity

Remember the 1st Law of Thermodynamics:

$Q = \Delta U + W_{by}$ (conservation of energy)

If we add heat to a system, it can do two things:

- Raise the temperature (internal energy increases)
- Do mechanical work (*e.g.*, expanding gas)

How much does the temperature rise?

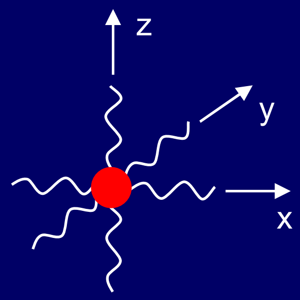
Define heat capacity to be the amount of heat required to raise the temperature by 1 K.	$C \equiv \dfrac{Q}{\Delta T}$

The heat capacity is proportional to the amount of material.
It can be measured either at constant volume (C_V) or constant pressure (C_P).

It depends on the material, and may also be a function of temperature.

Heat Capacity of a Solid

The atoms in a solid behave like little balls connected by springs. Here's one atom:
It has x, y, and z springs as well as x, y, and z motion.



If T is not too low, the equipartition theorem applies, and each kinetic and potential term contributes ½ kT to the internal energy:

$$U = 3(\tfrac{1}{2}kT) + 3(\tfrac{1}{2}kT) = 3kT$$

Equipartition often works near room temperature and above.

Therefore, a solid with N atoms has this heat capacity:

$$\boxed{C = 3Nk = 3nR}$$

Note: For solids (and most liquids), the volume doesn't change much, so $C_P \sim C_V$ (no work is done).

The temperature dependence of C is usually much larger in solids and liquids than in gases (because the forces between atoms are more important).

Heat Capacity and Specific Heat

Heat capacity: The heat energy required to raise the temperature of an object by 1K (=1° C).

It depends on the amount of material. Units: J / K

$$\boxed{C = \frac{Q}{\Delta T}}$$

Upper case "C"

Specific heat: The heat capacity normalized to a standard amount of material (mass or moles).

It only depends on the kind of material.

Normalize to mass: Units: J/kg·K → $\boxed{c = \frac{C}{m}}$

Lower case "c"

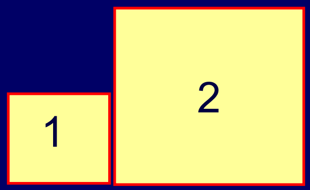
Normalize to moles: Units: J/mole·K → $\boxed{c_{mol} = \frac{C}{n}}$
"molar specific heat"

Normalize to volume: Units: J/m^3·K → $\boxed{c_{vol} = \frac{C}{V}}$
"volume specific heat"

Question: Which has the higher c, aluminum or lead?

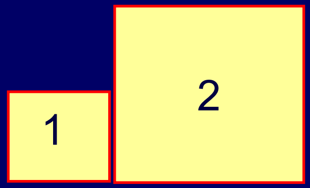
Exercise: Heat Capacity

Two blocks of the same material are put in contact. Block 1 has m_1 = 1 kg, and its initial temperature is T_1 = 75° C. Block 2 has m_2 = 2 kg, and T_2 = 25° C. What is the temperature after the blocks reach thermal equilibrium?



Solution

Two blocks of the same material are put in contact. Block 1 has m_1 = 1 kg, and its initial temperature is T_1 = 75° C. Block 2 has m_2 = 2 kg, and T_2 = 25° C. What is the temperature after the blocks reach thermal equilibrium?



The two blocks have the same (unknown) specific heat. However, the heat capacity of block 2, $C_2 = cm_2$, is twice as large as that of block 1, $C_1 = cm_1$.

We can use the 1st law (conservation of energy) to determine the final temperature:

$$U_f = c(m_1+m_2)T_f$$
$$= U_i = cm_1T_1 + cm_2T_2 .$$

Solve for T_f: $$\boxed{T_f = \frac{m_1T_1 + m_2T_2}{m_1 + m_2} = \frac{1\times 75 + 2\times 25}{3} = 41.7^\circ \text{ C}}$$

Question:
Suppose we measure temperature in Kelvin. Will we get a different answer?

Act 1

An m_1 = 485-gram brass block sits in boiling water (T_1 = 100° C). It is taken out of the boiling water and placed in a cup containing m_2 = 485 grams of ice water (T_2 = 0° C). What is the final temperature, T_F, of the system (*i.e.*, when the two objects have the same T)? (c_{brass} = 380 J/kg·K; c_{water} = 4184 J/kg·K)

a. T_F < 50° C

b. T_F = 50° C

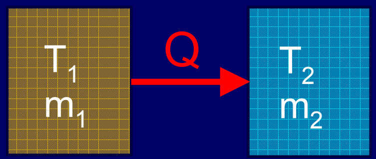
c. T_F > 50° C

Solution

An m_1 = 485-gram brass block sits in boiling water (T_1 = 100° C). It is taken out of the boiling water and placed in a cup containing m_2 = 485 grams of ice water (T_2 = 0° C). What is the final temperature, T_F, of the system (*i.e.*, when the two objects have the same T)? (c_{brass} = 380 J/kg·K; c_{water} = 4184 J/kg·K)

$\boxed{\text{a. } T_F < 50° \text{ C}}$ b. T_F = 50° C   c. T_F > 50° C

Solution:
Heat flows from the brass to the water. No work is done, and we assume that no energy is lost to the environment.



Remember: $Q = C\Delta T = mc\Delta T$

Brass (heat flows out): $Q_1 = \Delta U_1 = m_1c_1(T_F-T_1)$

Water (heat flows in): $Q_2 = \Delta U_2 = m_2c_2(T_F-T_2)$

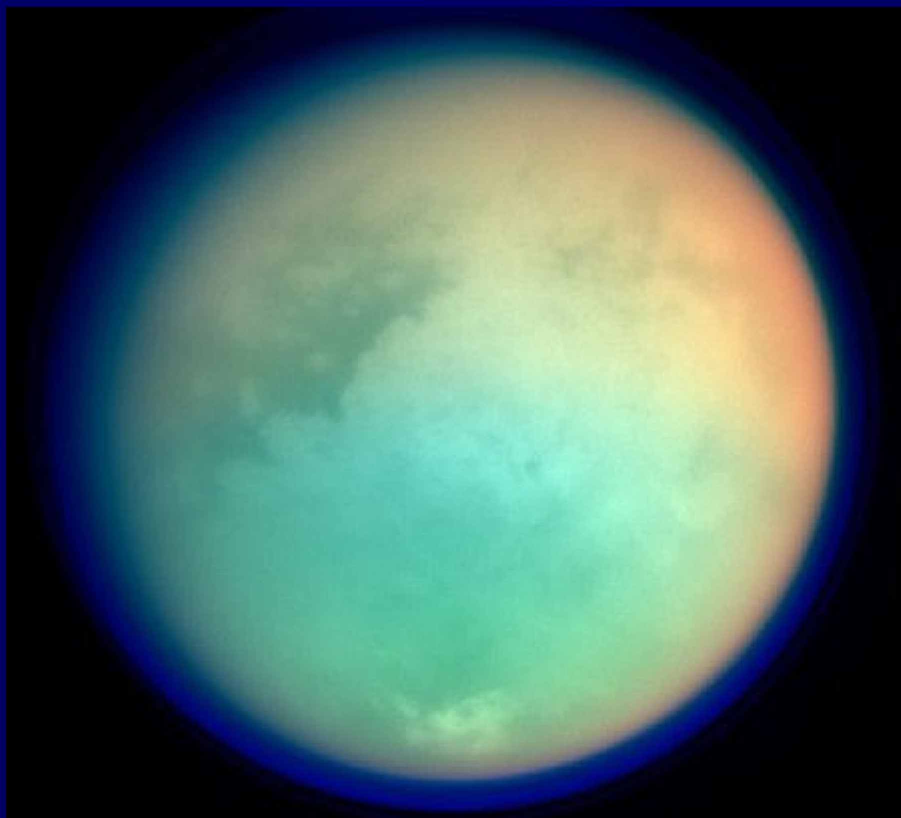
Energy is conserved: $Q_1 + Q_2 = 0$

Solve for T_F: $T_F = (m_1c_1T_1+m_2c_2T_2) / (m_1c_1+m_2c_2)$

$= (c_1T_1+ c_2T_2) / (c_1+ c_2) = 8.3°$ C

We measured T_F = _____ ° C.

Exercise: Spacecraft Heat Shields



This false-color view of Titan (moon of Saturn) is a composite of images captured by Cassini's infrared camera, which can penetrate some of Titan's clouds. Light and dark regions in the upper left quadrant are unknown types of terrain on Titan's surface.

The Huygens spacecraft entered the atmosphere on Jan. 14, 2005, initially traveling at ~6 km/s. After decelerating from friction, the heat shield was jettisoned, and three parachutes were deployed to allow a soft landing.

What is the temperature rise on entry, assuming that half of the thermal energy goes into the ship (and half to the atmosphere)?
Assume c_{steel} = 500 J/kg-K.

Solution

v = 6 km/s
c = 500 J/kg-K

Use conservation of energy (1st law of thermodynamics).
Half of the initial kinetic energy becomes internal thermal energy.

$$\frac{1}{2}\left(\frac{1}{2}mv^2\right) = C\,\Delta T = cm\,\Delta T$$

Note that m cancels.

$$\Rightarrow \quad \Delta T = \frac{v^2}{4c} = \frac{(6\times 10^3\,\text{m/s})^2}{4\times 500\ \text{J/kg-K}} = 18{,}000\,\text{K!}$$

The problem is that steel melts at ~1700 K! For this reason, the heat shield is *not* made of steel, but rather a ceramic that burns off ("ablates"). Also, the ceramic has a very low thermal conductivity!

Act 2

1 calorie* is defined to be the energy needed to raise the temperature of 1 gram of water 1° C (= 1 K). Therefore, given that c_{H2O} = 4184 J/kg·K, 1 calorie = 4.18 Joule.

Upper case "C" lower case "c"

1 food Calorie = 1000 calorie = 4180 J

If you weigh 80 kg, consume 2000 Cal/day, and could actually convert this entirely into work, how high could you climb?

a. 1 km b. 10 km c. 100 km

*The amount of heat actually depends somewhat on the temperature of the water, so there are actually several slightly different "calorie" definitions.

Solution

1 calorie is defined to be the energy needed to raise the temperature of 1 gram of water 1° C (= 1 K). Therefore, given that c_{H2O} = 4184 J/kg·K, 1 calorie = 4.18 Joule.

Upper case "C" lower case "c"

1 food Calorie = 1000 calorie = 4180 J

If you weigh 80 kg, consume 2000 Cal/day, and could actually convert this entirely into work, how high could you climb?

a. 1 km $\boxed{\text{b. 10 km}}$ c. 100 km

2000 Cal = 8.36×10^6 J = mgh

So, h = 8.36×10^6 J / (80 kg $\times$ 9.8 m/s^2) = 10.7 km

Note: Tour de France riders consume 6000-9000 Cal/day.

Home Exercises: Cooking

While cooking a turkey in a microwave oven that puts out 500 W of power, you notice that the temperature probe in the turkey shows a 1°C temperature increase every 30 seconds. If you assume that the turkey has roughly the same specific heat as water (c= 4184 J/kg-K), what is your estimate for the mass of the turkey?

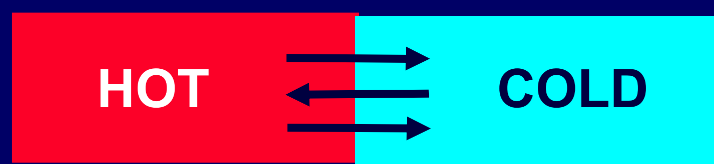
3.6 kg

You place a copper ladle of mass m_L=0.15 kg (c_L = 386 J/kg-K) - initially at room temperature, T_{room}= 20° C - into a pot containing 0.6 kg of hot cider (c_c = 4184 J/kg-K), initially at 90° C. If you forget about the ladle while watching a football game on TV, roughly what is its temperature when you try to pick it up after a few minutes?

88° C = 190° F

Heat Conduction

Thermal energy randomly diffuses equally in all directions, like gas particles (next lecture). More energy diffuses out of a high T region than out of a low T region, implying net energy flow from HOT to COLD.



The heat current, H, depends on the gradient of temperature,

For a continuous change of T along x: $H \propto dT / dx$

For a sharp interface between hot and cold: $H \propto \Delta T$

Heat Conduction (2)

Heat current density J is the heat flow per unit area through a material.
Units: Watts/m^2

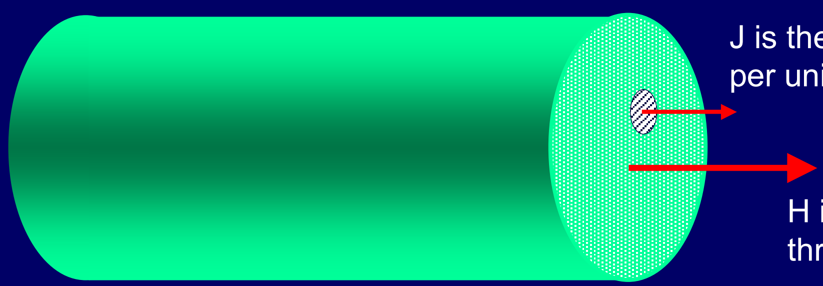
$$\boxed{J = -\kappa\, dT / dx}$$

(- sign because heat flows toward cold)

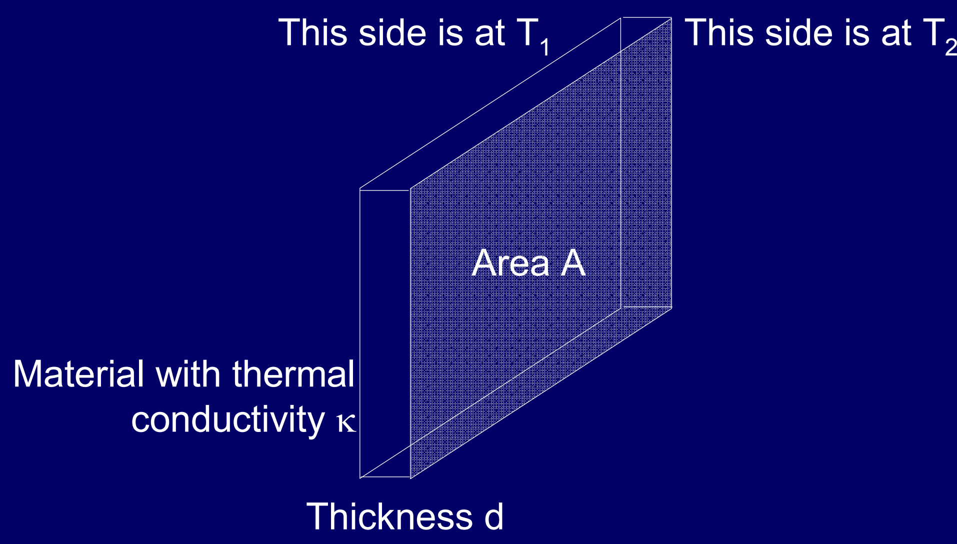
Thermal conductivity κ is the proportionality constant, a property of the material.
Units: Watts/m·K

Total heat current H is the total heat flow through the material.
Units: Watts

$$\boxed{H = J \cdot A}$$



Typical Problem in Heat Conduction

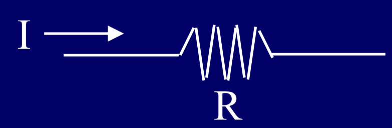


$$J = -\kappa \frac{dT}{dx} = -\kappa \frac{T_1 - T_2}{d} = \kappa \frac{\Delta T}{d}$$

$$H = J \cdot A = \left(\frac{\kappa A}{d}\right)\Delta T = \frac{\Delta T}{R_{thermal}},$$

$$\text{where } R_{thermal} \equiv \frac{d}{\kappa A}$$

Thermal resistance is defined to make the similarity to electrical current flow (Ohm's Law) clear:

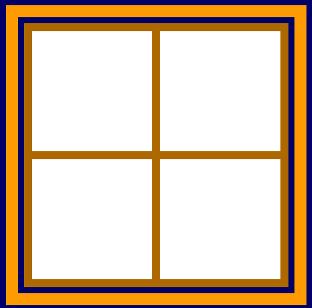


$$I = \frac{\Delta V}{R}$$

$\Delta V = V_1 - V_2$
= voltage drop

Exercise: Heat Loss Through Window

If it's 22°C inside, and 0°C outside, what is the heat flow through a glass window of area 0.3 m^2 and thickness 0.5 cm ?



The thermal conductivity of glass is about 1 W/m·K.

Solution

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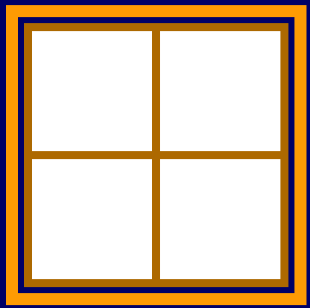


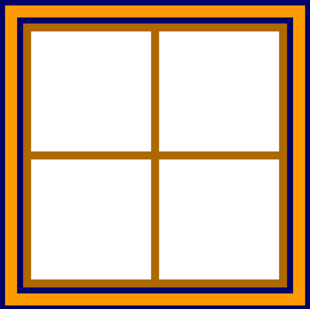
$$H = J \cdot A = \left(\frac{\kappa A}{d}\right)\Delta T$$

$$H = \left(1\frac{W}{mK}\right)\left(\frac{0.3m^2}{5\times 10^{-3}m}\right)(22K) = 1320\ W$$

That's a lot! Windows are a major cause of high heating bills.

ACT 3

How much heat is lost through a double-pane version of that window, with an 0.5-cm air gap?
The thermal conductivity of air is about 0.03 W/(m K).



Hint: Ignore the glass, which has a much higher conductivity than air.
H is limited by the high resistance air gap.

A) 20 W B) 40 W C) 1320 W d) 44,000 W

Solution

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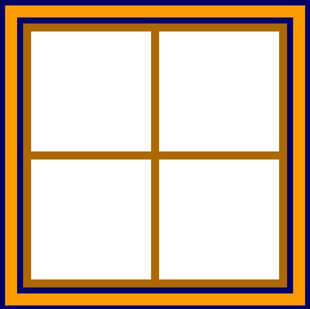


Hint: Ignore the glass, which has a much higher conductivity than air.
H is limited by the high resistance air gap.

A) 20 W $\boxed{\text{B) 40 W}}$ C) 1320 W   d) 44,000 W

$$H = \left(\underbrace{0.03\frac{W}{mK}}_{\text{air}}\right)\left(\frac{0.3m^2}{5\times10^{-3}m}\right)(22K) = 39.6\ W \longleftarrow \ll 1320\ W$$

Note: Large air gaps don't always work, due to convection currents.

Thermal conductivities (κ at 300 K):

air	0.03 W/m-K
wood	0.1 W/mK
glass	1 W/m-K
aluminum	240 W/m-K
copper	400 W/m-K

How small can κ be ???

Aerogel 8×10^{-5} W/m-K

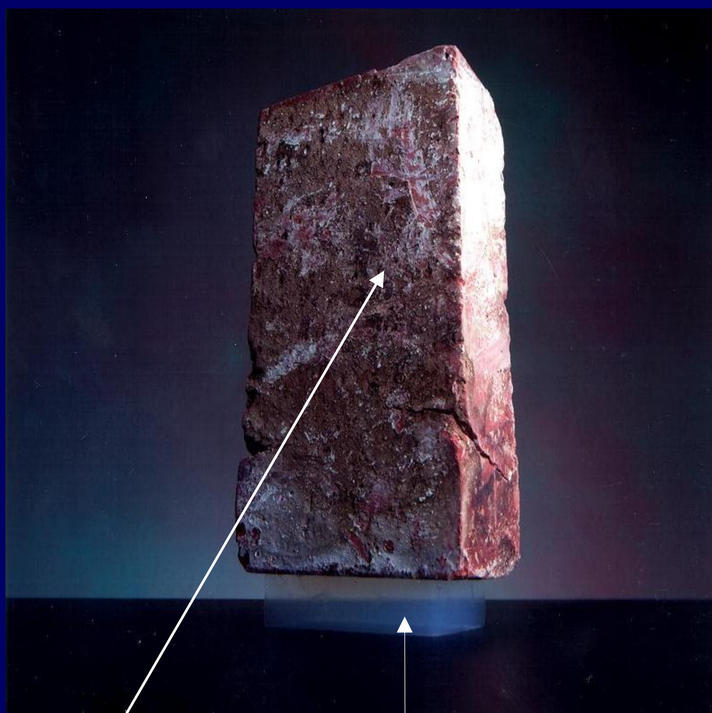
What's aerogel?

Aerogel

An artificial substance formed by specially drying a wet silica gel, resulting in a solid mesh of microscopic strands.

Used on space missions to catch comet dust

The least dense solid material known (ρ = 1.9 mg/cm^3. ρ_{air} = 1.2 mg/cm^3). 98% porous, but nevertheless, quite rigid:



2.5 kg brick   2 g aerogel

κ = 8 x 10^{-5} W/m-K

Next Time

- Thermal Diffusion – how long does it take?
- Random Walk and Particle Diffusion