The next two questions pertain to the following situation:

Consider the following two systems:

    A: three interacting harmonic oscillators with total energy 6ε.

    B: two interacting harmonic oscillators, with total energy 4ε.

1. What is the ratio of entropies for the two systems?

a. $\sigma_A/\sigma_B = 0$
b. $\sigma_A/\sigma_B = 0.86$
c. $\sigma_A/\sigma_B = 1$
d. $\sigma_A/\sigma_B = 2.07$
e. $\sigma_A/\sigma_B = 3.5$

(handwritten annotations)

$$\sigma = \ln \Omega$$

$$\Omega_\# = \frac{(q + N - 1)!}{(N-1)! \, q!}$$

$$\Omega_B = \frac{(4 + 2 - 1)!}{4! \, 1!} = 5$$

$$\sigma_B = 1.61$$

$$\Omega_A = \frac{(6 + 3 - 1)!}{2! \, 6!} = 28$$

$$\sigma_A = 3.33$$

2. Now the systems are brought into contact and allowed to reach equilibrium. The increase in entropy due to the process is

a. 0
b. 1.97
c. 6.41

(handwritten annotations)

$$\sigma_i = \sigma_A + \sigma_B = 3.33 + 1.61 = 4.94$$

$$\Omega_f = \frac{(10 + 5 - 1)!}{10! \, 4!} = 1001 \qquad \sigma_f = 6.91$$

$$\Delta\sigma = 1.97$$

3. Two types of atoms have energy levels as shown in the figure. All of the excited states are at the same energy, E_2, but atoms of type 1 have three states at that energy, while atoms of type 2 have only one. A box of gas contains both types of atoms in equilibrium at temperature, T. Compare the probability that an atom of type 1 is in its ground state (*i.e.*, has energy E_1) with the probability that an atom of type 2 is in its ground state.

Type 1   Type 2
E_2

E_1
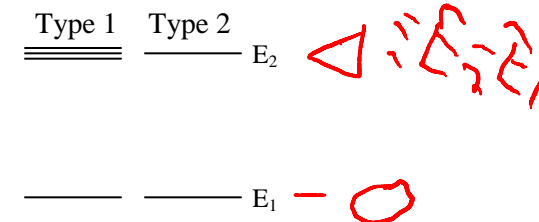
a. Atoms of type 1 are more likely to be in the ground state.
b. Atoms of type 2 are more likely to be in the ground state.
c. Atoms of the two types are equally likely to be in the ground state.

Type 1 : more ways to be excited : less likely in ground.

$$P_1(\text{ground}) < P_2(\text{ground})$$

$$P_1 = \frac{e^{-0}}{e^{-0} + d_1 e^{-\Delta/kT}} = \frac{1}{1 + d_1 e^{-\Delta/kT}}$$

$$d_1 = 3$$

$$T = \frac{\Delta}{k \ln 3}$$

At what T $P(1) = P(2)$?

Type 2: $T = \infty$

Type 1

$$\frac{P(2)}{P(1)} = \frac{3 e^{-\Delta/kT}}{1.1/T} = 1$$

$$U = -\vec{\mu} \cdot \vec{B}$$

The next three questions are related:

Consider a large collection of spins with dipole moment $\mu = 1 \times 10^{-23}$ J/Tesla.

4. If B = 1 Tesla, for what temperature will twice as many spins be pointing up (along the field) as down?

a. T = 2.1 K
b. T = 5.2 K
c. T = 14.5 K
d. T = 23.7 K
e. T = 88.4 K

$$\downarrow \quad \sim +\mu B$$

$$\uparrow \quad -\mu B$$

$$\frac{P(\uparrow)}{P(\downarrow)} = 2 = \frac{e^{+(\mu B/kT)}/Z}{e^{-\mu B/kT}/Z} = e^{2\mu B/kT}$$

$$\frac{2\mu B}{kT} = \ln 2$$

$$\frac{2\mu B}{k \ln 2} = T$$

5. Assuming there are N spins, what is the heat capacity in the limit of high temperature?
Hint: What happens to the energy at high temperature?

a. $C_V = \dfrac{\mu^2 B^2}{kT^2}$

b. $C_V = \dfrac{Nk^2T^2}{\mu B}$

c. $C_V = \dfrac{N\mu\, B}{kT^2}$

d. $C_V = \dfrac{N\mu^2 B^2}{kT^2}$

e. $C_V = \dfrac{Nk^2T}{\mu\, B}$

$$C = \frac{Q}{dT} = \frac{dU + dW}{dT}$$

$$[C] = \frac{[U]}{[T]} = \frac{[kT]}{[T]} = [k]$$

C (better increase in N) : Not a

$C(T \to 0) \to 0$

T very low: Lowest energy $U(\text{small } T) = N(-\mu B)$

T high: Equally likely ground or excited

$$U = N\langle E \rangle = N\left[\left(\tfrac{1}{2}\right)(-\mu B) + \tfrac{1}{2}(+\mu B)\right] = 0$$

C must $\to 0$ as $T \to \infty$

U

$-N\mu B$

6. Now let there be 10^6 spins and let B go to zero. After the spins have completely randomized, approximately what is the probability of observing $N_{up} - N_{down} = 1000$?

a. $P = 5 \times 10^{-2}$
b. $P = 5 \times 10^{-3}$
c. $P = 5 \times 10^{-4}$
d. $P = 5 \times 10^{-5}$
e. $P = 5 \times 10^{-6}$

$$P = \frac{\Omega}{\Omega_{tot}}$$

$$2^N$$

$$N_{up} - N_{down} = 1000$$

$$\Omega(m) = 2^N \sqrt{\frac{2}{\pi N}} \, e^{-m^2/2N}$$

$$N_{up} + N_{down} = 10^6$$

$$N_{up} = \frac{10^6}{2} + 500$$

$$N_{down} = \frac{10^6}{2} - 500$$

$$\Omega = \frac{N!}{N_{up}! \, N_{down}!}$$

$$\Rightarrow P = \sqrt{\frac{2}{\pi \, 10^6}} \times e^{-1000^2/2 \cdot 10^6}$$

(handwritten top margin)
$\Delta S = ?$ $\Delta S_{Ar} + \Delta S_{N_2}$ $\Delta S_{Ar} = \Delta S_{Ar}$ (T changes)
$+ \Delta S_{Ar}$ (V changes) $\sim C_{Ar} \ln \frac{T_f}{T_{Ar}}$
$\sim N_{Ar} k \ln \frac{V_f}{V_{Ar}}$

The next two questions pertain to the following situation:

7. A box of total volume V initially has an insulating partition, which separates N_{Ar} Argon atoms (monatomic, each with mass m_{Ar}) at initial temperature T_{Ar} from N_{N2} nitrogen molecules (diatomic, each with mass m_{N2}) at initial temperature T_{N2}. The partition is suddenly removed, and the gases allowed to equilibrate. The final temperature T_f is

a. $T_f = \dfrac{3N_{Ar}T_{Ar} - 5N_{N2}T_{N2}}{3N_{Ar} + 5N_{N2}}$

b. $T_f = \dfrac{N_{Ar}T_{Ar} - N_{N2}T_{N2}}{N_{Ar} + N_{N2}}$

c. $T_f = \dfrac{5N_{Ar}T_{Ar} + 3N_{N2}T_{N2}}{5N_{Ar} + 3N_{N2}}$

d. $T_f = \dfrac{3N_{Ar}T_{Ar} + 5N_{N2}T_{N2}}{3N_{Ar} + 5N_{N2}}$

e. $T_f = \dfrac{N_{Ar}T_{Ar} + N_{N2}T_{N2}}{N_{Ar} + N_{N2}}$

(handwritten work)

Heat flow from hot to cold

Assume $T_{Ar} > T_f > T_{N_2}$

$|Q_{Ar}| = |Q_{N_2}|$

$C_{Ar}(T_{Ar} - T_f) = C_{N_2}(T_f - T_{N_2})$

$T_f = \dfrac{C_{Ar}T_{Ar} + C_{N_2}T_{N_2}}{C_{Ar} + C_{N_2}} \Rightarrow \dfrac{3N_{Ar}T_{Ar} + 5N_{N_2}T_{N_2}}{3N_{Ar} + 5N_{N_2}}$

$U = \alpha N k T$

$C = \alpha N k$

$C_{Ar}(\alpha = \tfrac{3}{2}) = \tfrac{3}{2}N_{Ar}k$

$C_{N_2}(\alpha = \tfrac{5}{2}) = \tfrac{5}{2}N_{N_2}k$

$T_{Ar}' = T_{N_2}'$

8. If U'_{Ar} and U'_{N2} are the final energies of each component, which of the following is true in equilibrium:

a. $\dfrac{U'_{Ar}}{N_{Ar}} = \dfrac{U'_{N2}}{N_{N2}}$

b. $U'_{Ar} = U'_{N2}$

c. $m_{Ar}\left\langle v^2_{Ar}\right\rangle = m_{N2}\left\langle v^2_{N2}\right\rangle$

$N_{Ar} <<< N_{N_2}$

$\dfrac{\frac{3}{2}N_{Ar}kT}{N_A} \overset{?}{=} \dfrac{\frac{5}{2}N_N kT}{N_N}$

9. The vibrational mode of the N_2 molecule acts like a harmonic oscillator with energy spacing, ε = 0.292 eV. Estimate the probability that a molecule of N_2 in equilibrium at room temperature (T = 300 K) is in the **first excited vibrational state** (not the ground state).

a. Probability = 1.54×10^{-10}
b. Probability = 1.24×10^{-5}
c. Probability = 0.0885
d. Probability = 0.292
e. Probability = 0.99999

$$P(\varepsilon) = \frac{e^{-\varepsilon/kT}}{e^{-0} + e^{-\varepsilon/kT} + e^{-2\varepsilon/kT} + \cdots}$$

$$\sim e^{-\varepsilon/kT} = e^{-11.3} = 1.2\times10^{-5}$$

$$\frac{\varepsilon}{kT} = \frac{0.292\,eV}{8.6\times10^{-5}\,eV/K \cdot 300} = 11.3$$

10. As absolute temperature goes to zero, the heat capacity of N one-dimensional harmonic oscillators approaches

a.   0

b. $\frac{1}{2} Nk$

c.   Nk

$$\langle E \rangle = \frac{\varepsilon}{e^{\varepsilon/k_BT} - 1} \approx \frac{\varepsilon}{e^{\varepsilon/k_BT}}$$

$$\langle E \rangle = \varepsilon\, e^{-\varepsilon/k_BT}$$

$$U = N\varepsilon\, e^{-\varepsilon/k_BT}$$

$$C = \frac{dU}{dT}$$

$$= N\varepsilon\, e^{-\varepsilon/k_BT}\left(\frac{-\varepsilon}{k}\right)\frac{-1}{T^2}$$

$$= \frac{N\varepsilon^2}{kT^2}\, e^{-\varepsilon/k_BT} \;\Rightarrow 0 \text{ as } T \to 0$$

The next two questions pertain to the following situation:

11. The semiconductor silicon (Si) has a band gap of 1.14 eV, and a density of $5 \times 10^{28}/m^3$. The quantum density associated with its valence and conduction bands is $1.72 \times 10^{25}/m^3$. How many free holes are there in the valence band at 300K if one in 500 million Si atoms is replaced with a phosphorous (an electron donor)?

   a) $n_h = 2.1 \times 10^7/m^3$
   b) $n_h = 2.1 \times 10^9/m^3$
   c) $n_h = 2.1 \times 10^{11}/m^3$

$$n_e n_h = n_i^2 = n_G^2 e^{-\Delta/kT}$$

$$n_i = n_G e^{-\Delta/2kT}$$
$$= 1.7 \times 10^{25} e^{-1.14eV/2k300}$$
$$= 4.5 \times 10^{15} m^{-3}$$

$$n_e = n_{e,\,ho\,donor} + n_d$$
$$= n_i + \frac{1}{500 \times 10^6} 5 \times 10^{28}/m^3$$
$$= 4.5 \times 10^{15} + 10^{20}/m^3 = 10^{20}/m^3$$

$$n_h = \frac{n_i^2}{n_e} = \frac{(4.5 \times 10^{15})^2}{10^{20}}$$

12. What will be the carrier density in the doped silicon as the temperature goes to zero?

$$n_i (T \to 0) \Rightarrow 0$$

   a) 0
   b) $5 \times 10^{15}/m^3$
   c) $10^{20}/m^3$

As $T \to 0$, even donor electrons freeze $\to 0$

not free.

electron in Phosphorus = 0.045eV

$$Z = 1 + 4e^{-\Delta/kT}$$

The next two problems are related:

13. A molecule has one electronic state with energy E_0, and four with energy $E_1 = E_0 + 0.03$ eV. At T = 273K, what is the relative number of molecules at each energy level?

$E_1 =$

$E_0 \longrightarrow \circ$

E_0

$$\frac{P_1}{P_0} = \frac{4e^{-\Delta/kT}/Z}{1 \cdot e^{0}/Z}$$

a. $N_1 < N_0$
b. $N_1 > N_0$
c. The answer cannot be determined from the information given.

$$kT = \frac{0.03 eV}{k \cdot 273} = 1.275$$

$$\frac{P_1}{P_0} = 4e^{-\Delta/kT}$$

$$= 4e^{-1.275} = 1.12$$

14. What is the average energy of a molecule in the previous case (assume $E_0 = 0$)?

$$\langle E \rangle = E_0 P(E_0) + E_1 P(E_1)$$

a. 0.0065 eV
b. 0.016 eV
c. 0.021 eV

$$0 \cdot P(E_0) + 0.03eV \frac{4e^{-\Delta/kT}}{1 + 4e^{-\Delta/kT}} \quad 1.12$$

$$= 0.06eV$$

The next two problems are related:

The next two problems are related:

A sphere of radius 1m is released into deep space (away from the sun), with an initial temperature of 72 °C.

15. What is the wavelength of the peak in the 'blackbody radiation' from the sphere?

   a. 830 nm
   b. 8.4 micrometers
   c. 40 micrometers

$$\lambda_{max} \cdot T = 0.0029 \text{ m-k}$$

$$\lambda_{max} = \frac{0.0029}{72}$$
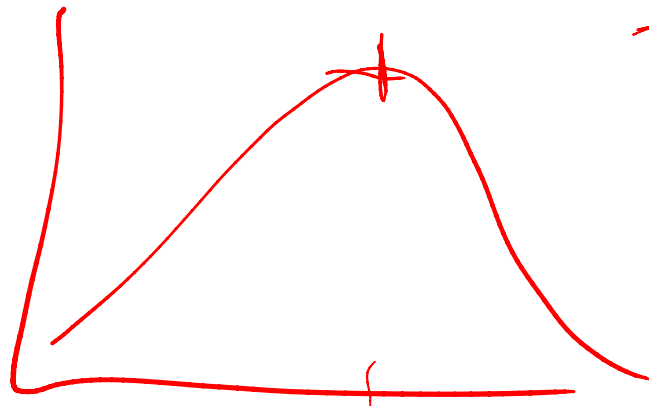
$$\lambda \quad \frac{0.0029}{72+273} = 8.4 \mu m \qquad = \frac{40 \mu m}{} \text{ NO!}$$

$$345 k$$

$r = 1m$

16. Approximating the sphere as a perfect "black body", and assuming its heat capacity is 10^4 J/K, about how long does the sphere take to cool to 335 K?

   a. 0.01 s
   b. 0.1 s
   c. 1 s
   d. 10 s
   e. 100 s

$$\frac{Power}{Area} = flux = J = \sigma_{SB} T^4$$

$$\frac{1}{5.67 \cdot 10^{-8}}$$

$$P = \sigma T^4 (4\pi r^2) = \frac{dQ}{dt} = C\frac{dT}{dt}$$

$$dQ = CdT$$

$$dT = 345 - 335 = 10K$$

$$dt = \frac{CdT}{\sigma T^4 4\pi r^2} = \frac{10^4 \cdot 10K}{\sigma (340)^4 4\pi r^2}$$

$$= 10.5 s$$

17.   Brick A has mass $m_A = 1$ kg, with specific heat $c_A = 1000$J/kg-K, initially at temperature $T_A = 100$ K. Brick B has mass $m_B = 2$ kg, with specific heat $c_B = 2000$J/kg-K, initially at temperature $T_B = 200$ K. The bricks are put in thermal contact with each other (but are isolated from the rest of the world). After the two-brick system reaches thermal equilibrium, by how much, ΔS_{tot}, has their total entropy changed?

a. $\Delta S_{tot} = -81$ J/K

b. $\Delta S_{tot} = 0.0$ J/K

c. $\Delta S_{tot} = 81$ J/K

d. $\Delta S_{tot} = 166$ J/K

e. $\Delta S_{tot} = 1009$ J/K

$$dS = \frac{dQ}{T} = \frac{c\, dT}{T}$$

$$\Delta S = c \int \frac{dT}{T} = c \ln \frac{T_A}{T_i}$$

$$|Q_A| = |Q_B|$$

$$\Rightarrow T_f = \frac{c_A T_A + c_B T_B}{c_A + c_B} = \frac{1 \cdot 1000 \cdot 100 + 2 \cdot 2000 \cdot 200}{1 \cdot 1000 + 2 \cdot 2000} = 180$$

$$\Delta S_{tot} = 1 \cdot 1000 \ln \frac{180}{100} + 2 \cdot 2000 \ln \frac{180}{200}$$
$$= 587.8 - 421.4 = 166.3$$

13 of 34 pages
(36 problems)

The next two problems are related:

A substance has the following chemical potential vs T diagram at a particular pressure.
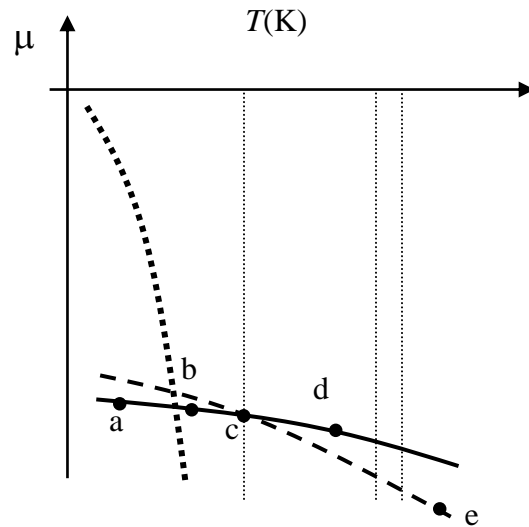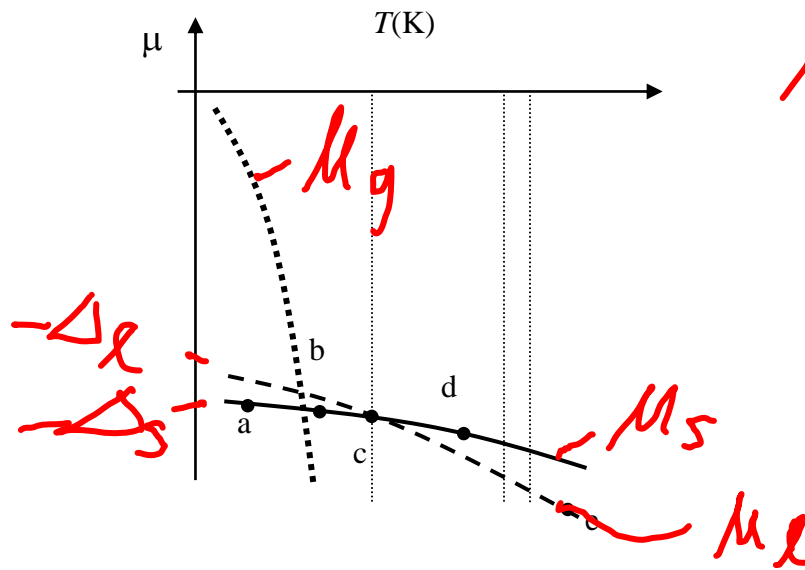

18. Which of the points corresponds to a substance that is in thermal equilibrium?

a.
b.
c.
d.
e.

lowest μ for its T.

A substance has the following chemical potential vs T diagram at a particular pressure.

μ $T(K)$

$\mu = \dfrac{\partial f}{\partial N}$

$F = U - TS$

$-\mu_g$

$-\Delta \ell = -\dfrac{\partial \mu}{\partial S}$

Minimize

b

d

μ_s

a

c

μ_ℓ

e

$G = N_g \mu_g + N_\ell \mu_\ell + N_s \mu_s$

19. Which of the points corresponds to a substance that is about to sublime?
   a.
   b.
   c.
   d.
   e.

$S \to g$

20. Calculate the chemical potential, μ, of Argon gas at room temperature, $T = 300$ K, and atmospheric pressure, $p = 1.01 \times 10^5$ Pa. Ignore the effect of external forces, such as gravity.

a. $\mu = -2.49 \times 10^{-9}$ eV
b. $\mu = -0.42$ eV
c. $\mu = 0$ eV
d. $\mu = +0.42$ eV
e. $\mu = +2.49 \times 10^{-9}$ eV

$$\mu = kT \ln \frac{n}{n_Q} \qquad n = \frac{N}{V}$$

$$n_Q = 10^{30} m^{-3} \left(\frac{u}{m_p}\right)^{3/2} \left(\frac{T}{300 K}\right)^{3/2}$$

$$n_Q = \left(\frac{2\pi m \frac{1}{2}T}{h^2}\right)^{3/2}$$

$$\frac{(40)^{3/2}}{1} = 2.5 \times 10^{32} m^{-3}$$

$$pV = N kT$$

$$\frac{N}{V} = \frac{P}{kT} = \frac{10^5}{k \cdot 300}$$

$$= 2.4 \times 10^{25} m^{-3}$$

$$\lambda_{dB} \sim \frac{h}{p}$$

$$\frac{p^2}{2m} = \frac{3}{2} kT$$

$$\mu = -0.42 eV$$

$$He = 4 g/mol$$
$$= 2p + 2n$$
$$p \sim 1 g/mol$$
$$Ar = 40 g/m$$

21. A container holds N_{total} gas molecules in equilibrium at two different heights as pictured. What is the relationship between the chemical   potentials of the molecules at height h_1 and $h_2 = 2h_1$?

$h_2 = 2h_1$

h_1

a) $\mu_1 = \mu_2/4$
b) $\mu_1 = \mu_2/2$
c) $\mu_1 = \mu_2$
d) $\mu_1^2 = \mu_2$
e) $\mu_1 = \mu_2^2$

$$F = F(h_1) + F(h_2)$$
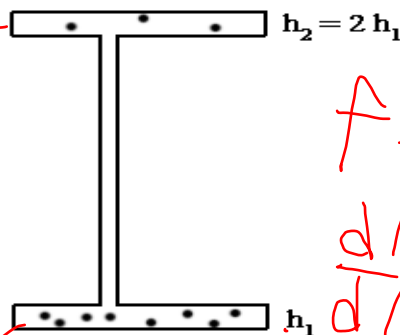
$$\frac{dF}{dN_1} = \frac{dF(h_1)}{dN_1} + \frac{dF(h_2)}{dN_1}$$
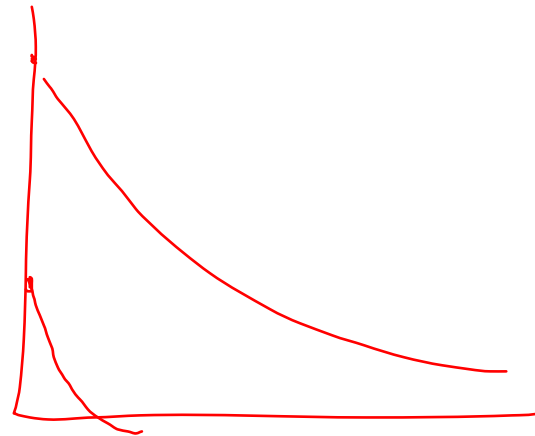
$$dN_1 = -dN_2$$

$$= \frac{dF(h_1)}{dN_1} - \frac{dF(h_2)}{dN_2}$$

$$\mu_1 - \mu_2 = 0$$

$$kT \ln \frac{n(h_1)}{n_Q(h_1)} + mgh_1$$

$$kT \ln \frac{n(h_2)}{n_Q(h_2)} + mgh_2$$

$$\frac{n(h_2)}{n(h_1)} = e^{-mg(h_2-h_1)n_Q(h_2)/kT}$$

22. The gas molecules are 80% N_2 and 20% O_2 at sea level, i.e., $n_{N2}/n_{O2} = 4$. Neglecting the effects of thermal mixing, at what height is the ratio $n_{N2}/n_{O2} = 1$? (Assume a constant temperature $T = 260$ K.)

   a. 33 km
   b. 76 km
   c. There is no altitude above sea level where the ratio is 1.

The next two problems are related:

23. A window is made of two layers of material that have different thermal conductivities, $\kappa_2 > \kappa_1$. The temperature on one side of the window is higher than the other: $T_2 > T_1$. Which orientation of the window (see the figure) gives more heat flow through the window?
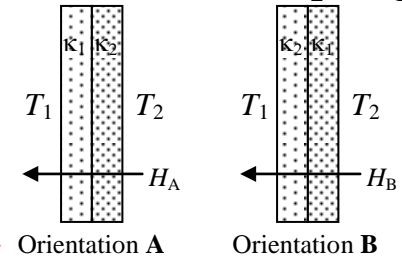
a. $H_A > H_B$ (**A** has more heat flow.)
b. $H_A = H_B$ (The heat flows are equal.)
c. $H_A < H_B$ (**B** has more heat flow.)

$$H = \frac{\Delta T}{R_{th}} = \frac{\Delta T \, A \, K}{d}$$

$$O \quad R_1 \quad R_2 \quad V$$

$$R = R_1 + R_2$$

Same current flow

24.   Now consider that the layers have equal thickness, and $\kappa_2 = 3\kappa_1$. The temperatures $T_1 = 10°C$
       and $T_2 = 20°C$. What is the relative temperature T_i at the interface between the two
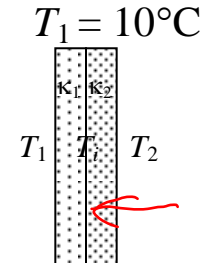   layers?

   a. 12.5°C
   b. 15°C
   c. 17.5°C
   d. 18°C
   e. 20° C

Heat flow into interface
     =   "   flow out of   "

$$\frac{\Delta T_{2i}}{R_2} = \frac{\Delta T_{i1}}{R_1}$$

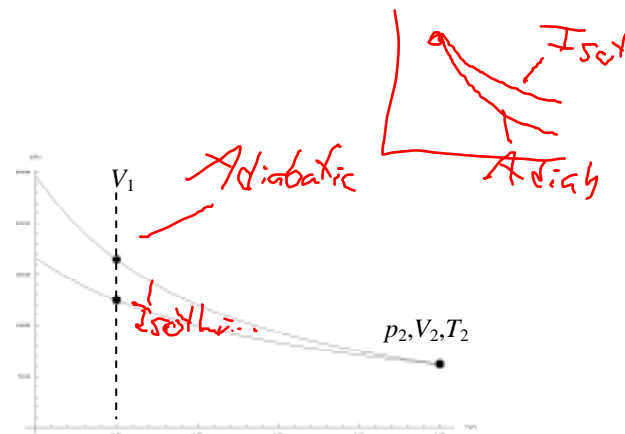$$\frac{(T_2 - T_i) A 3\kappa}{d} = \frac{(T_i - T_1) A \kappa}{d}$$

$$3T_2 - 3T_i = T_i - T_1$$

$$4T_i = 3T_2 + T_1$$

$$T_i = \frac{3 \cdot 20 + 10}{4} = 17.5$$

25. Compare the isothermal and adiabatic expansions from
$V_1 = 0.04$ m³ to $V_2 = 0.08$ m³ of two moles of an ideal diatomic gas,
as shown in the figure. Both processes end up at the same state,
$T_2 = 300$ K, and $p_2 = 62.3$ kPa. Calculate the ratio, W_I/W_A, of the
work done by the two processes (isothermal divided by adiabatic).

a. $W_I/W_A = 0.40$
b. $W_I/W_A = 0.87$
c. $W_I/W_A = 1.40$
d. $W_I/W_A = 2.00$
e. $W_I/W_A = 3.46$

21 of 34 pages
(36 problems)

26. A Carnot heat engine achieves 33.3% efficiency when operating between temperatures T_h and T_c. If it is operated as a refrigerator operating between the same two reservoirs, how much work, W, must we supply in order to remove 1 kJ of heat from the cold reservoir?

a. $W = 500$ J
b. $W = 666$ J
c. $W = 1$ kJ
d. $W = 2$ kJ
e. $W = 3$ kJ

$$\varepsilon = 1 - \frac{T_c}{T_H}$$

Carnot

Heat pump

$$T_H$$

$$T_c$$

$$\frac{Q_c}{T_c} = \frac{Q_H}{T_H}$$

$$W_{on} = Q_H - Q_c$$

$$= Q_c\left(\frac{Q_H}{Q_c} - 1\right)$$

$$= Q_c\left(\frac{Q_H}{Q_c} - 1\right)$$

$$= 1000\left(\frac{3}{2} - 1\right) = 500\,J$$

$$W_{by} = Q_H - Q_c$$

$$\varepsilon = \frac{W_{by}}{Q_H} = \frac{Q_H - Q_c}{Q_H} = 1 - \frac{Q_c}{Q_H}$$

$$\frac{Q_c}{Q_H} = \frac{w}{w}$$

27. The following chemical reaction occurs: $2N_2O_5 \rightarrow 4NO_2 + O_2$. Which of the following correctly expresses the relationship between the chemical potentials of all three species?

a) $2\mu_{N_2O_5} = 4\mu_{NO_2} + \mu_{O_2}$

b) $2\mu_{N_2O_5} = \mu_{NO_2} + 4\mu_{O_2}$

c) $\mu^2_{N_2O_5} = \mu^4_{NO_2} + \mu_{O_2}$

d) $\mu_{N_2O_5} = \mu_{NO_2} + \mu_{O_2}$

e) $2\mu_{N_2O_5} = 4\mu_{NO_2} = \mu_{O_2}$

$$\mu = kT \ln \frac{n}{n_Q}$$

$$\frac{n_{N_2O_5}^2}{n_{NO_2}^4 \, n_{O_2}} = K(T)$$

28. Two identical blocks each have heat capacity 100 J/K. One block is at temperature 500K and the other is at temperature 100K. Which block has the higher free energy relative to the environment at 300 K?

a) the cold block
b) the hot block
c) they have the same free energy

$$\Delta F = \Delta U - T_{env}\Delta S$$

500K: $C(500-300) - 300\, C\, ln\left(\dfrac{500}{300}\right)$

$= C\,(46.7)$

100K $C(100-300) - 300\, C\, ln\left(\dfrac{100}{300}\right)$

$= C\, 129.6$

29. A block of material has a temperature-dependent heat capacity given by $C_V(T) = 5$ J/K $+ T \times (2$ J/K^2). How much does the entropy of this object change as its temperature is increased from 10°C to 40°C at constant volume?

313K

   a) −66.9 J/K
   b) −60.5 J/K
   c) 0.50 J/K
   d) 60.5 J/K
   e) 66.9 J/K

$$dS = \frac{dQ}{T} = \frac{C_V \, dT}{T}$$

$$\Delta S = \int \frac{C \, dT}{T} = \int 5 \frac{dT}{T} + \int \frac{2T}{T} dT$$

$$= 5 \ln \frac{313}{283} + 2(313 - 283)$$

$$= 60.5$$

30. The star Rigel (in the constellation Orion) is a blue supergiant. It is very large ($R_{Rigel} = 70 R_{Sun}$) and very bright, i.e., has very high total radiated power $P_{Rigel}/P_{Sun} = 80,000$). How much hotter is it than the sun? That is, what is the ratio, T_{Rigel}/T_{Sun}?

a. $T_{Rigel}/T_{Sun} = 2$
b. $T_{Rigel}/T_{Sun} = 16$
c. $T_{Rigel}/T_{Sun} = 70$
d. $T_{Rigel}/T_{Sun} = 4,900$
e. $T_{Rigel}/T_{Sun} = 78,000$

$$P = J \cdot A = \sigma T^4 \, 4\pi R^2$$

$$\frac{P_R}{P_S} = 80,000 = \frac{\sigma T_R^4 \, 4\pi R_R^2}{\sigma T_S^4 \, 4\pi R_S^2}$$

$$\frac{T_R}{T_S} = \left(\left(\frac{R_S}{R_R}\right)^2 80,000 \right)^{1/4} = 2$$

$$\lambda_R T_R = \text{const}$$

$$\lambda_S T_S \qquad \frac{\lambda_R}{\lambda_S} = \frac{T_S}{T_R} = \frac{1}{2} \qquad \lambda_R = \frac{\lambda_S}{2} \Rightarrow \underline{blue!}$$

$$\frac{1}{70}$$

31. Consider a 2-dimensional gas, in which particles are allowed to move only in a plane. What is the root-mean-square velocity of the particles in this 2-d gas?

a. $v_{rms} = 0$

b. $v_{rms} = \sqrt{\dfrac{2kT}{m}}$

c. $v_{rms} = \sqrt{\dfrac{3kT}{m}}$

$$\frac{1}{2}m\langle v^2\rangle = \frac{1}{2}m\, v_{rms}^2$$

$$= \frac{1}{2}m\langle v_x^2\rangle + \frac{1}{2}m\langle v_y^2\rangle$$

$$= \frac{1}{2}kT + \frac{1}{2}kT = kT$$

$$v_{rms}^2 = \frac{2kT}{m}$$

32. A 1-liter (non-elastic) balloon is filled with pure helium gas. The balloon is in a room 10x10x10m at 300 K and 1 atm. (Assume the air in the room is all nitrogen and oxygen molecules.) Now we pop the balloon, allowing the He to diffuse throughout the room. By what amount does the Free energy of the He gas change?

a) -1382 J
b) -691 J
c) 0
d) + 691 J
e) +1382 J

$$\Delta F = \Delta \mu N = \Delta U - T \Delta S$$

$$\Delta \mu = RT \ln \frac{n_2}{n_Q} - RT \ln \frac{n_1}{n_Q}$$

$$= kT \ln \frac{n_2}{n_1} = kT \ln \left(\frac{N/N_2}{N/V_1} \right)$$

$$N = \frac{pV}{kT} = \frac{10^5 \cdot 10^{-3}}{k \cdot 300}$$

$$= kT \ln \frac{V_1}{V_2} = k \cdot 300 \ln \frac{10^{-3} m^3}{1000 m^3}$$

$$= -5.7 \times 10^{-20} J / molecule$$

The next two problems are related:

33. Argon (molecular weight 40 g/mole) is a monatomic compound. If liquid argon is confined to a container and held at a constant temperature of 80.5 K, what is the approximate vapor pressure of gaseous argon, assuming the liquid has no entropy and a binding energy of 0.1 eV? [Note: At 1 atm, the boiling point is 87.3 K.]

   a) 2 atm
   b) 0.2 atm
   c) 0.02 atm
   d) 0.002 atm
   e) 0.0002 atm

$$\mu_g = \mu_\ell$$

$$kT \ln \frac{n}{n_Q} = kT \ln \frac{P}{P_Q} = -\Delta$$

$$P_Q = kT n_Q$$

$$P = P_Q e^{-\Delta/kT}$$

$$= n_Q kT e^{-\Delta/kT}$$
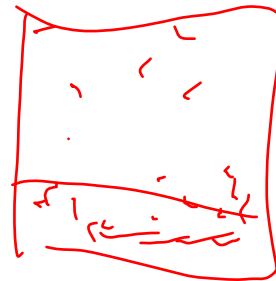
$$= 10^{30} m^{-3} \left(\frac{40}{1}\right)^{3/2} \left(\frac{80.5K}{300}\right)^{3/2} k (80.5) e^{-0.1/k \cdot 80.5}$$

$$= 0.21 \, atm \quad (0.21 \times 10^5 \, Pa)$$

34. The measured value of the latent heat of vaporization of argon (at 1 atm) is 6.43 kJ/mol.

Use this to estimate the binding energy.

a) 0.027 eV
b) 0.059 eV
c) 0.067 eV
d) 0.11 eV
e) 0.4 eV

Handwritten work:

How much heat to go from $\ell \to g$

$L = Q_{\ell-g} = dU + pdV$

$dU = L - pdV$

$pdV = p(V_f - V_i) \approx p(V_f) = nRT$

$p = 1 \text{ atm}$
$p = 10^5 \text{ Pa}$

1 mole: $L = 6430 J$

$= 1 \cdot 8.314 \cdot (87.3 K)$

$= 725 J$ $T_{boiling}$ @ 1 atm

$\dfrac{dU}{mole} = 6430J - 725J = N \cdot \Delta$

$\Delta = \dfrac{5705J}{6 \times 10^{23}} \cdot \dfrac{eV}{1.6 \times 10^{-19} J} = 0.059 eV$

The next two questions pertain to the following situation:

In a hydrogen atom, the electron (e) is electrostatically bound to the proton (p) with an energy −13.6 eV. Inside a star, the density is $\sim 10^{24}/m^3$, and the temperature is 7000 K.

35. ***Note: This problem may be longer/more difficult; therefore, you may want to do it last.***
What is the density of free electrons (and protons)? You may assume that $n_H = 1 \times 10^{24}$ m^{-3} and that $m_H = m_p = 1836 \, m_e$.

a. 6.0×10^{13} m^{-3}
b. 2.7×10^{15} m^{-3}
c. 5.1×10^{17} m^{-3}
d. 2.1×10^{18} m^{-3}
e. 4.7×10^{20} m^{-3}

$$n$$

$$e + p \leftrightarrow H$$

$$\frac{n_e n_p}{n_H} = \frac{n_{Qe} n_{Qp}}{n_{QH}} e^{-\Delta/kT} \qquad n_{Qp} = n_{QH}$$

$$n_e = n_p$$

$$n_e^2 = n_H n_{Qe} e^{-\Delta/kT}$$

$$n_e = \sqrt{n_H n_{Qe}} \; e^{-\Delta/2kT}$$

$$\Delta = +13.6\,eV$$

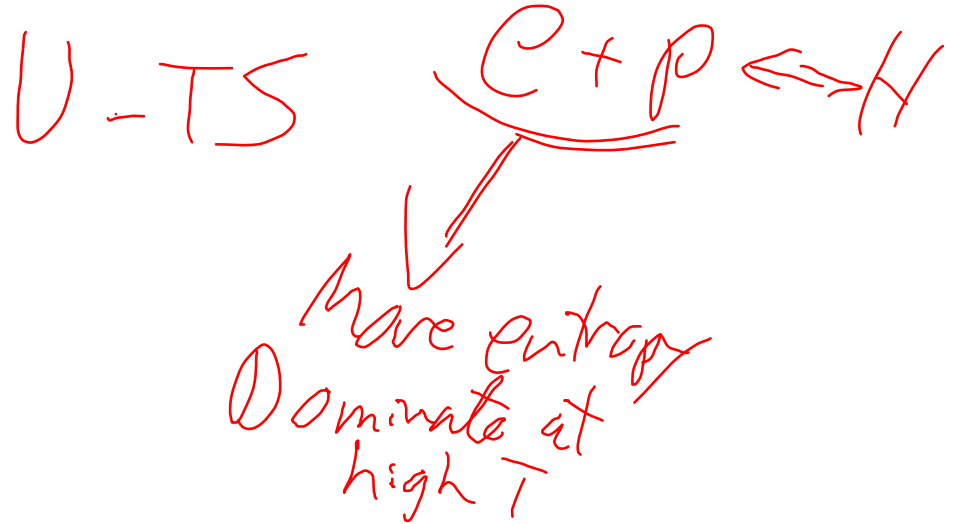$$\Rightarrow 4.7 \times 10^{20} \, m^{-3}$$

$$n_{Qe} = 10^{30} m^{-3} \left(\frac{m_e}{m_p}\right)^{3/2} \left(\frac{T = 7000K}{300K}\right)^{3/2} = 1.4 \times 10^{27} m^{-3}$$

$$\frac{1}{1836}$$

36. Which of the following would *increase* the *fraction* of unbound protons?

a. increase the temperature.
b. increase the pressure.
c. increasing the binding energy Δ.

$$U - TS \qquad C + P \Longleftrightarrow H$$

more entropy
Dominate at
high T

Answers:

1	d
2	b
3	b
4	a
5	d
6	c
7	d
8	c
9	b
10	a
11	c
12	a
13	b
14	b
15	b
16	d
17	d
18	a
19	b
20	b

21	c
22	c
23	b
24	c
25	b
26	a
27	a
28	a
29	d
30	a
31	b
32	a
33	b
34	b
35	e
36	a