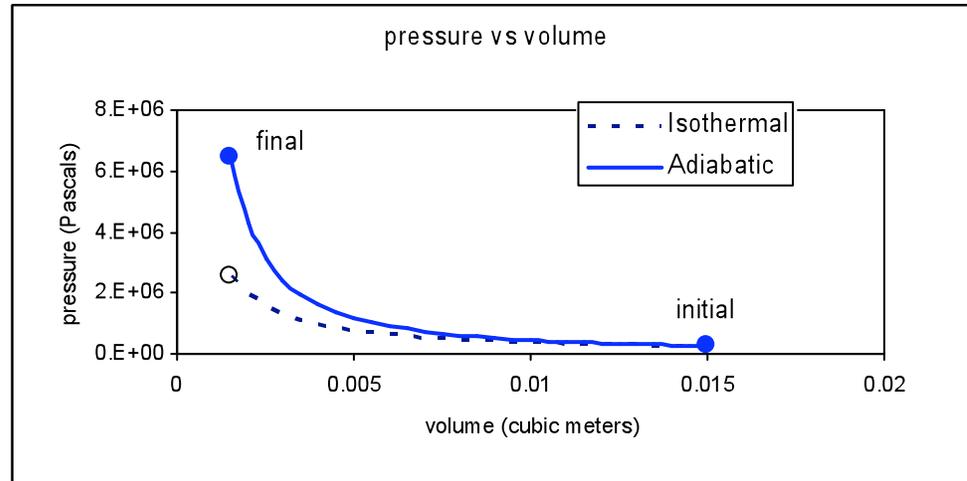


- a) Last week, we considered the problem of *isothermal* compression: 1.5 moles of an ideal diatomic gas at temperature  $35^\circ\text{C}$  were compressed **isothermally** from a volume of  $0.015\text{ m}^3$  to a volume of  $0.0015\text{ m}^3$ . The  $pV$ -diagram for the isothermal process is shown below. Now we consider an **adiabatic** process, with the same starting conditions and the same final volume. Is the final temperature higher, lower, or the same as the in isothermal case? Sketch the adiabatic processes on the  $p$ - $V$  diagram below and compute the final temperature. (Ignore vibrations of the molecules.)



For an adiabatic process  $V_i T_i^\alpha = V_f T_f^\alpha$ , where  $\alpha = 5/2$  for the diatomic gas. In this case  $T_i = 273\text{ K} + 35^\circ\text{C} = 308\text{ K}$ . We find  $T_f = T_i (V_i/V_f)^{1/\alpha} = (308\text{ K}) (10)^{2/5} = 774\text{ K}$ .

- b) According to your diagram, is the final pressure greater, lesser, or the same as in the isothermal case? Explain why (*i.e.*, what is the energy flow in each case?). Calculate the final pressure.

We argued above that the final temperature is greater for the adiabatic process. Recall that  $p = nRT/V$  for an ideal gas. We are given that the final volume is the same for the two processes. Since the final temperature is greater for the adiabatic process, the final pressure is also greater. We can do the problem numerically if we assume an idea gas with constant  $\alpha$ .

In an adiabatic processes,  $pV^\gamma = \text{constant}$ , where  $\gamma = (\alpha + 1) / \alpha$ . Here, the gas is diatomic and non-vibrational, so  $\alpha = 5/2$ , and  $\gamma = 7/5$ . Note that  $\gamma$  is always greater than one. Then for an adiabatic process,  $p_f V_f^\gamma = p_i V_i^\gamma$ , or  $p_f = p_i (V_i/V_f)^\gamma$ . For an isothermal process,  $p_f V_f = p_i V_i$ , or  $p_f = p_i (V_i/V_f)$ . Since  $\gamma > 1$ , the final pressure must be greater in the adiabatic process. Numerically,

$$p_f = p_i (V_i/V_f)^\gamma = 2.56 \times 10^5 \text{ Pa} \times (0.015 \text{ m}^3/0.0015 \text{ m}^3)^{7/5} = 6.43 \times 10^6 \text{ Pa}$$

Recall that in the isothermal process the final presume was  $2.56 \times 10^5 \text{ Pa}$ . Thus the final pressure is greater in the adiabatic process.

- c) Based on your diagram, would the amount of work done on the gas be larger, smaller, or the same as in the isothermal case (the work done in the isothermal case was 8844 J)? What about the heat flow? Explain your answers briefly. Compute the work done on the gas,  $W_{on}$ , for this process. [Hint: You could do the integral  $-\int p dV$  from  $V_i$  to  $V_f$ , because the pressure  $p$  can be written as a function of  $V$ . Alternatively, you could avoid the integral by noticing that work is simply related to  $\Delta U$  for an adiabatic process, and that  $\Delta U$  is simply related to  $\Delta T$  for an ideal gas.]

The short way:

We can answer the question using the First Law,  $\Delta U = Q_{in} + W_{on}$ . For the adiabatic process  $Q_{in} = 0$ , by definition, so  $\Delta U = W_{on}$  for the adiabatic process. But  $\Delta U = \alpha n R \Delta T$ , so,

$$W_{on} = \alpha n R \Delta T = (5/2)(1.5 \text{ mol})(8.314 \text{ J/mol K})(774 \text{ K} - 308 \text{ K}) = 1.45 \times 10^4 \text{ J}$$

The long way:

$pV^\gamma = \text{constant}$  (We'll use  $p_f V_f^\gamma$ .) So,  $p = p_f V_f^\gamma / V^\gamma$

$$\begin{aligned} W_{on} = -W_{by} &= -\int_{V_i}^{V_f} p dV = -p_f V_f^\gamma \int_{V_i}^{V_f} \frac{dV}{V^\gamma} \\ &= \frac{p_f V_f^\gamma}{\gamma-1} V^{-\gamma+1} \Big|_{V_i}^{V_f} = \frac{p_f V_f^\gamma}{\gamma-1} \left( \frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}} \right) = 1.45 \times 10^4 \text{ J, as before.} \end{aligned}$$

Recall that for the isothermal process the work done on the gas was 8850 J. The work done on the gas is greater in the adiabatic process.

Incidentally, the derivation of the adiabatic formulas is as follows: For small changes in  $T$  and  $V$ , the first law ( $\Delta U = -W_{by}$ ) tells us:

$$\alpha N k dT = -p dV = -(NkT/V) dV$$

which implies

$$\begin{aligned} \alpha dT/T &= -dV/V \\ \alpha \int (dT/T) &= -\int (dV/V) \\ \alpha \ln T &= -\ln V + \text{constants} \end{aligned}$$

yielding,

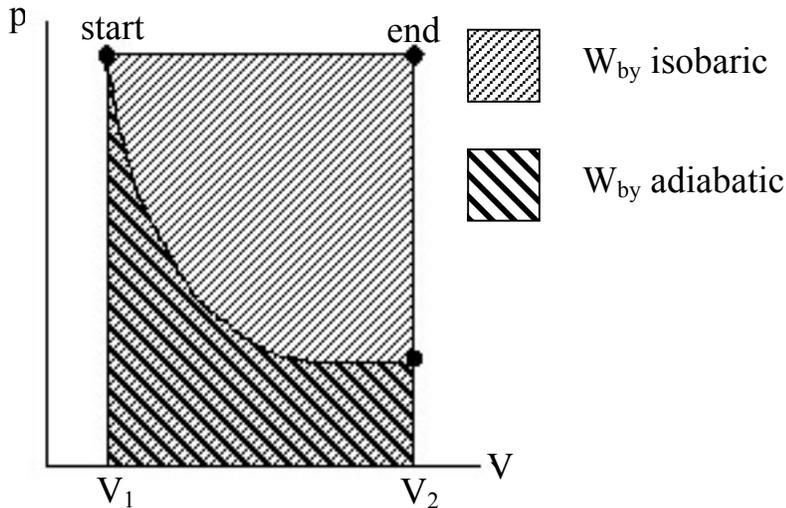
$\begin{aligned} VT^\alpha &= \text{constant, or} \\ pV^\gamma &= \text{constant} \end{aligned}$
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(adiabatic process)

with  $\gamma = (\alpha+1)/\alpha$ . These functional forms apply only in the temperature range where  $\alpha$  and  $\gamma$  are constant.

2. For the following parts, assume each process takes the same ideal gas between the same two volumes,  $V_1$  and  $V_2$ .

a) Which does more work on the surroundings, an adiabatic expansion or an isobaric expansion? (a p-V sketch may help) Which one has the smallest final temperature? As a group, talk together and come up with a physical reason why its temperature must be smaller. (Hint: think about each term in the First Law of Thermodynamics for each process)

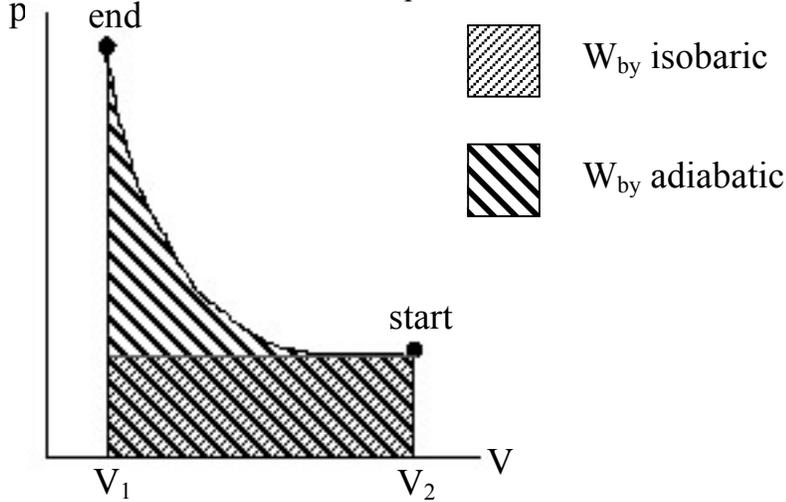


An expansion takes energy out of the gas as it does work on the environment. This tends to lower the temperature of the gas. Since the adiabatic process doesn't take in any heat, its pressure and temperature drop drastically.

The isobaric process must take in heat from the environment to offset this energy loss; otherwise, its internal energy would decrease and the speed of the gas molecules would drop. This would reduce the impulse and frequency of their impacts with the wall, decreasing the pressure. To keep this from happening, heat comes in from the environment, raising the temperature of the gas and keeping the pressure constant.

To tell for sure that the isobaric case must have a higher temperature, realize that it has the same volume as the adiabatic outcome, but a higher pressure. This means the gas molecules must be striking the walls harder and/or more frequently, implying faster speeds and therefore a higher temperature.

b) Which requires a larger  $W_{on}$ , an adiabatic compression or an isobaric compression? Which one has the smallest final temperature?



Here, the adiabatic compression cannot release any of the energy put into it as work. It therefore keeps all the work done on it as internal energy and ends with a very high temperature.

The isobaric process must lower its temperature during this process. If it didn't, the gas molecules would strike the container walls more often (from the reduced volume). This process must therefore lose heat to the surroundings to slow the molecules down.

Since the adiabatic process ended with the same volume as the isobaric, but with a higher pressure, the adiabatic process ends up with the larger temperature.