Table 1: Resistivity of a few materials at different temperatures

Material	Temperature (K)	resistivity
Copper	60	9.7×10^{-10} Ωm
Copper	273 K	1.5×10^{-8} Ωm
Copper	400 K	2.4×10^{-8} Ωm
Silicon	298 K	0.5 Ω m
Silicon	1375 K	9.1×10^{-5} Ωm

13 Semiconductors

13.1 Phenomenology of semiconductors

We can classify most materials as insulating, which means that they do not conduct electricity very well[1], or conducting, in which the electrons are free to move around easily and electricity can be conducted easily.

In Table 1 are some values of electrical resistivity versus temperature for a semiconductor (silicon) and a conductor (copper). There are both quantitative and qualitative differences betweent these resistivities. First, note that near room temperature, the silicon is a very poor conductor, with a resistivity about 10^8 larger than copper. However, raising the temperature of silicon lowers the resistivity, while raising the temperature of copper only slightly **increases** the resistivity. In this section we will understand why the semiconductor behaves in the way that it does.

13.2 Spherical cow model of a semiconductor.

This is not a very realistic model of semiconductors, but it captures the basic behavior of them. If you take classes that specialize on semiconductors then you will use quantum mechanics to make more sophisticated models of them. In this chapter, we will model of semiconductors as a bunch of atoms, each of which has two possible states for electrons. The lower energy state is "filled," which means that in the lowest energy state it has an electron in it. The rule is that electrons can only move to unoccupied states, and there is only one electron allowed per state.

If an electron moves to an unoccupied energy state (called the conduction states, or conduction band) then there's room for it to move around. On the other hand, if the electron is removed from the filled level (called the valence states, or valence band) then the 'hole' can move around as well through collective motion of the electrons in the filled level.

The conductivity is proportional to the number of electrons and holes in the material. The number of electrons n_e is:

$$n_e = NP(e) \tag{1}$$

and the number of holes n_h is:

$$n_h = NP(h), \tag{2}$$

Where N is the number of atoms, and $P(h)$ is the probability that an electron migrates from the semiconductor to the metal. Similarly, $P(e)$ is the probability that an electron migrates from the metal to the insulator. This migration of charge is the conduction!

13.3 Boltzmann factors

For a metal nearby where the chemical potential is μ, the probability of atom i taking on an electron is:

$$P(e,i) = \frac{1}{Z}e^{-(E_c-\mu)/kT} \tag{3}$$

[1]although with enough voltage, everything can be a conductor!

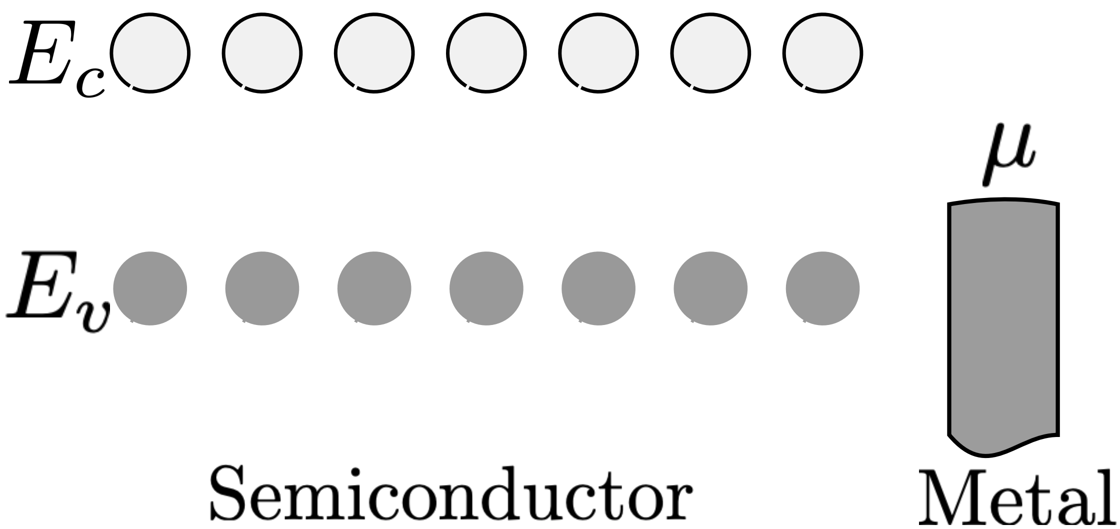

Figure 1: A sketch of the model we are using to understand semiconductors. Each atom in the semiconductor has a low energy state with energy E_v that is filled with an electron (with probability close to 1, similar to our two-state systems!) at low temperature, and a high energy state with energy E_c that is not filled at low temperature. Nearby there is a metal which can either absorb electrons creating a hole, or donate electrons, creating an electron in the conduction states.

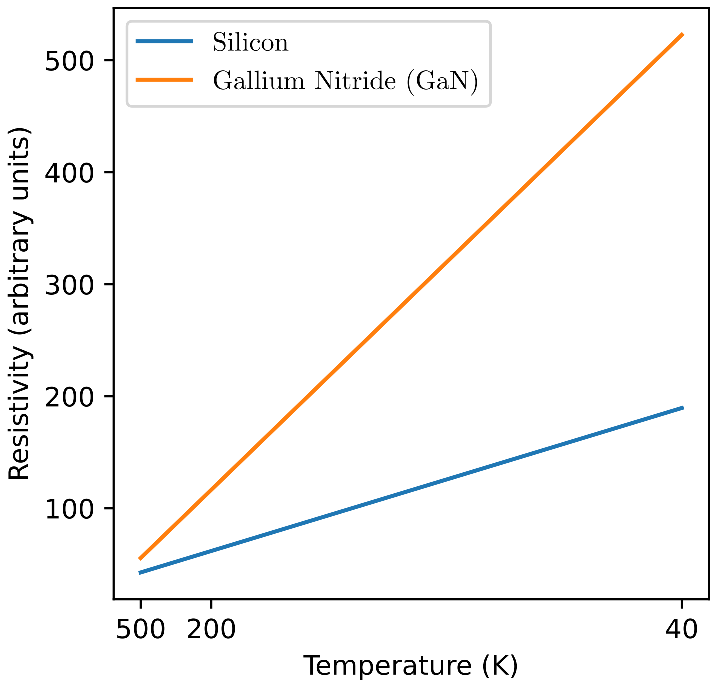

Figure 2: Arrhenius plots for materials with different gaps. The larger gap material has a larger slope on this plot. Note that we are actually plotting $1/2kT$ on the x-axis.

and similarly for atom i losing an electron:

$$P(h,i) = \frac{1}{Z} e^{-(\mu - E_v)/kT} \tag{4}$$

13.4 Neutral limit

Our previous equations are perfectly fine to use; however, they are not intrinsic to the semiconductor, since they depend on the type of metal attached to it through μ. In most applications, the amount of current we send through the semiconductor is very small compared to the number of atoms in the material itself. In that situation, we can very accurately say that the number of holes and electrons are equal because the system is neutral; that is, $n_h = n_e$.

Using neutrality,

$$n_e = n_h = \sqrt{n_e n_h} = \frac{1}{Z} e^{-(E_c - \mu)/2kT} e^{-(\mu - E_v)/2kT}. \tag{5}$$

Combining the exponentials,

$$n_e = n_h = \frac{1}{Z} e^{-(E_c - E_v)/2kT}, \tag{6}$$

which only depends on the properties of the semiconductor; in particular the so-called **gap** $\Delta = E_c - E_v$, the energy difference between the conduction and valence bands.

13.5 Measuring the gap: Arrhenius plots of semiconductors

The gap is one of the primary ways that we characterize semiconductors. To build a transistor or other device out of a semiconductor, it is important that in its intrinsic state, it mostly does not conduct electricity.

Depending on how exactly you would like the material to behave, you may select a different gap. For example, silicon which has a gap of 1.1 eV works quite well at room temperature; however, at very high temperatures it is essentially a conductor of electricity and therefore not typically useful. So at high temperatures or high voltages, you would probably want to use a larger gap material like GaN with a gap of 3.4 eV.

Since the conductivity is proportional to the density of electrons n_e, the resistivity ρ is given as follows:

$$\rho \propto 1/n_e \propto e^{\Delta/2kT}. \tag{7}$$

Therefore the resistivity is

$$\rho = Ce^{\Delta/2kT}, \tag{8}$$

where C is a constant that is determined by the specifics of the material. This constant, with some work, is computable by more advanced methods.

Taking the logarithm of Eqn 8,

$$\ln \rho = \ln C + \frac{\Delta}{2kT} \tag{9}$$

Therefore, when plotting the logarithm of the resistivity versus $1/(2kT)$, the slope is the gap. This is a quick and easy way to estimate the gap of semiconductors.