Table 1: Resistivity of a few materials at different temperatures

Material	Temperature (K)	resistivity
Copper	60	9.7×10^{-10} Ωm
Copper	273 K	1.5×10^{-8} Ωm
Copper	400 K	2.4×10^{-8} Ωm
Silicon	298 K	0.5 Ω m
Silicon	1375 K	9.1×10^{-5} Ωm

13 Semiconductors

13.1 Phenomenology of semiconductors

We can classify most materials as insulating, which means that they do not conduct electricity very well[1], or conducting, in which the electrons are free to move around easily and electricity can be conducted easily. The main difference between them is that a conductor has lower electrical resistivity at lower temperatures, while the insulator has higher electrical resistivity at lower temperatures.

In Table 1 are some values of electrical resistivity versus temperature for a semiconductor (silicon) and a conductor (copper). There are both quantitative and qualitative differences between these values of resistivity. First, note that near room temperature, the silicon is a very poor conductor, with a resistivity about 10^8 larger than copper. However, raising the temperature of silicon lowers the resistivity, while raising the temperature of copper only slightly **increases** the resistivity. Despite the name, a semiconductor is a type of insulator that happens to conduct electricity a little bit at room temperature. In this section we will understand why the semiconductor behaves in the way that it does.

13.2 Simple model of a semiconductor.

This is not a very realistic model of semiconductors, but it captures the basic behavior of them. If you take classes that specialize on semiconductors then you will use quantum mechanics to make more sophisticated models of them. In this chapter, we will model of semiconductors as a bunch of atoms, each of which has two possible states for electrons. The lower energy state is "filled," which means that in the lowest energy state it has an electron in it. The rule is that electrons can only move to unoccupied states, and there is only one electron allowed per state.

If an electron moves to an unoccupied energy state (called the conduction states, or conduction band) then there's room for it to move around. On the other hand, if the electron is removed from the filled level (called the valence states, or valence band) then the 'hole' can move around as well through collective motion of the electrons in the filled level.

The conductivity is proportional to the number of electrons in the conduction band and holes in the valence band of the material. These come from the metal which is The number of electrons n_e that migrate from the metal is:

$$n_e = NP(e), \tag{1}$$

where $P(e)$ is the probability for an electron to migrate from the metal to a single atom. We will define the chemical potential of electrons in the metal as μ, so the probability of atom i taking on an electron is:

$$P(e) = \frac{1}{Z} e^{-(E_c-\mu)/kT}, \tag{2}$$

where N is the number of atoms.

We also have to consider another process: an electron migrating from the atom to the metal to create a hole. The number of holes n_h is:

$$n_h = NP(h), \tag{3}$$

[1] With enough voltage, anything can be a conductor!

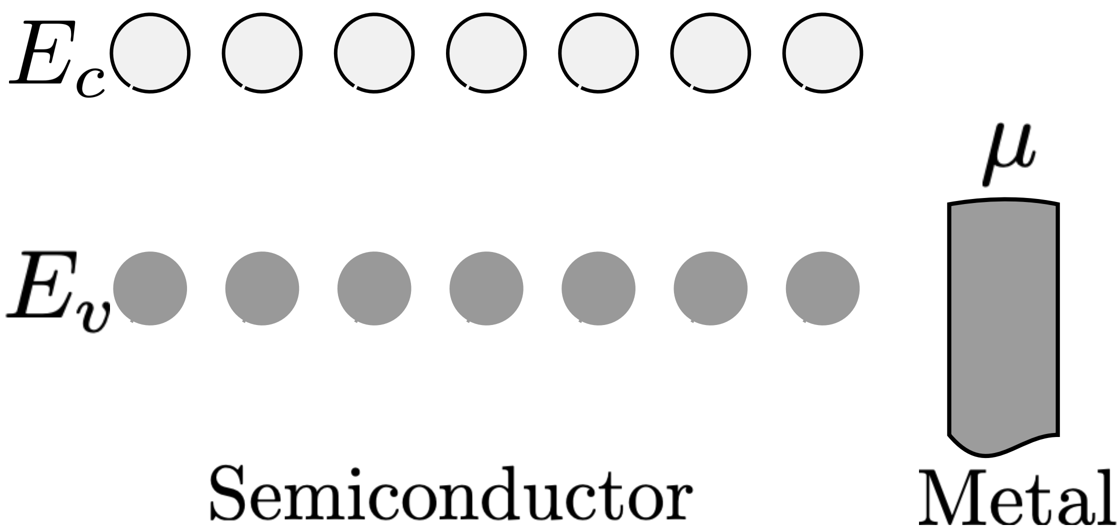

Figure 1: A sketch of the model we are using to understand semiconductors. Each atom in the semiconductor has a low energy state with energy E_v that is filled with an electron (with probability close to 1, similar to our two-state systems!) at low temperature, and a high energy state with energy E_c that is not filled at low temperature. Nearby there is a metal which can either absorb electrons creating a hole, or donate electrons, creating an electron in the conduction states. The gap is the energy difference $E_c - E_v$.

and similarly for atom i losing an electron:

$$P(h) = \frac{1}{Z} e^{-(\mu - E_v)/kT} \tag{4}$$

13.3 Neutral limit

Our previous equations are perfectly fine to use; however, they are not intrinsic to the semiconductor, since they depend on the type of metal attached to it through μ. In most applications, the amount of current we send through the semiconductor is very small compared to the number of atoms in the material itself. In that situation, we can very accurately say that the number of holes and electrons are equal because the system is neutral; that is, $n_h = n_e$.

Using neutrality,

$$n_e = n_h = \sqrt{n_e n_h} = \frac{1}{Z} e^{-(E_c - \mu)/2kT} e^{-(\mu - E_v)/2kT}. \tag{5}$$

Combining the exponentials,

$$n_e = n_h = \frac{1}{Z} e^{-(E_c - E_v)/2kT}, \tag{6}$$

which only depends on the properties of the semiconductor; in particular the so-called **gap** $\Delta = E_c - E_v$, the energy difference between the conduction and valence bands.

Since the conductivity is proportional to the density of electrons n_e, the resistivity ρ is given as follows:

$$\rho \propto 1/n_e \propto e^{\Delta/2kT}. \tag{7}$$

Therefore the resistivity is

$$\rho = C e^{\Delta/2kT}, \tag{8}$$

where C is a constant that is determined by the specifics of the material. This constant, with some work, is computable by more advanced methods. Note the factor of 2 here, as opposed to a standard Boltzmann-like relationship.

This explains the observation we made at the beginning of the chapter; that semiconductors have the opposite behavior of resistivity with respect to temperature as compared to conductors. Conductors (or metals) have zero gap, so the model we made does not apply to them. For a conductor, temperature inhibits electrical conductivity because the vibrations in the atoms get in the way of the electrons' motion. On the other hand, for a semiconductor, increased temperature is helpful to conductivity, because the electrons are helped into the conduction band by the temperature fluctuations.

The gap is one of the primary ways that we characterize semiconductors. Depending on how exactly you would like the material to behave, you may select a different gap. For example, silicon which has a gap of 1.1 eV works quite well at room temperature; however, at very high temperatures it is essentially a conductor of electricity and therefore not typically useful. So at high temperatures or high voltages, you would probably want to use a larger gap material like GaN with a gap of 3.4 eV.

13.4 Measuring the gap: Arrhenius plots of semiconductors

Taking the logarithm of Eqn 8,

$$\ln \rho = \ln C + \frac{\Delta}{2kT} \tag{9}$$

Therefore, when plotting the logarithm of the resistivity versus $1/(2kT)$, the slope is the gap. This is a quick and easy way to estimate the gap of semiconductors.

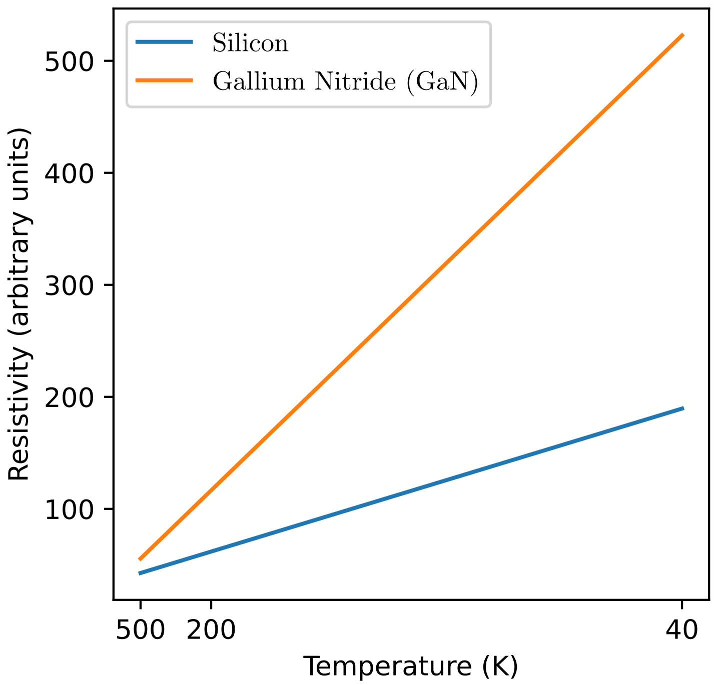

Figure 2: Arrhenius plots for materials with different gaps. The larger gap material has a larger slope on this plot. Note that we are actually plotting $1/2kT$ on the x-axis.