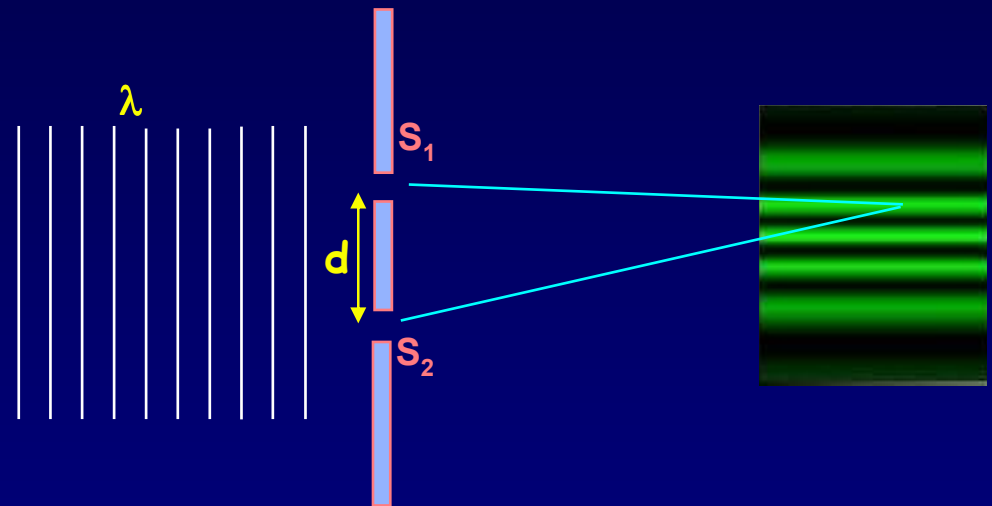
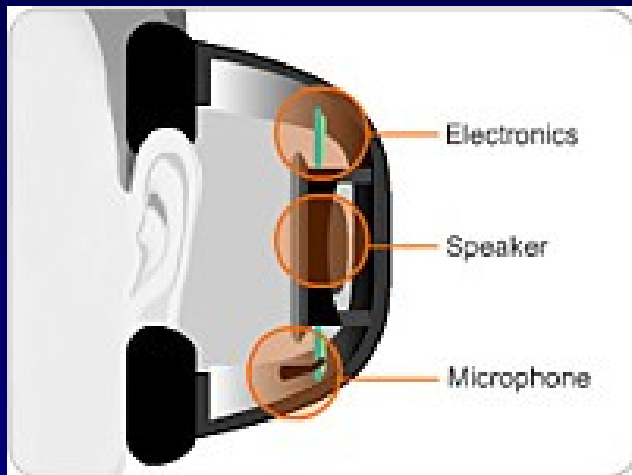


Lecture 2: Interference

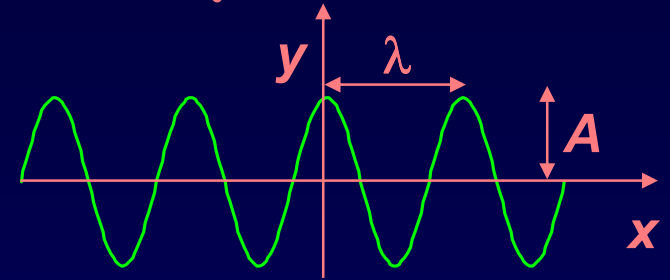


Today

- Interference of sound waves
- Two-slit interference

Review: Wave Summary

The formula $y(x,t) = A \cos(kx - \omega t)$ describes a harmonic plane wave of amplitude A moving in the $+x$ direction.



For a wave on a string, each point on the wave oscillates in the y direction with simple harmonic motion of angular frequency ω .

The wavelength is $\lambda = \frac{2\pi}{k}$; the speed is $v = \lambda f = \frac{\omega}{k}$

The intensity is proportional to the square of the amplitude: $I \propto A^2$

Superposition

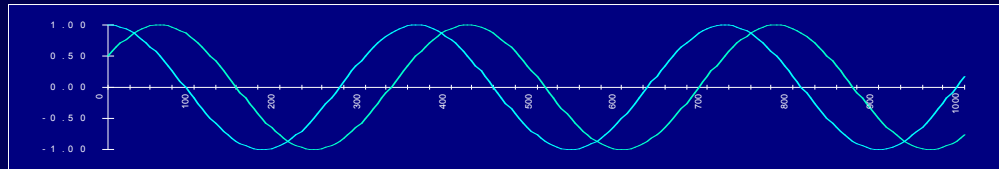
Because the wave equation is linear, arbitrary combinations of solutions will also be solutions. For unequal intensities, the maximum and minimum intensities are:

$$I_{\max} = |A_1 + A_2|^2$$

$$I_{\min} = |A_1 - A_2|^2$$

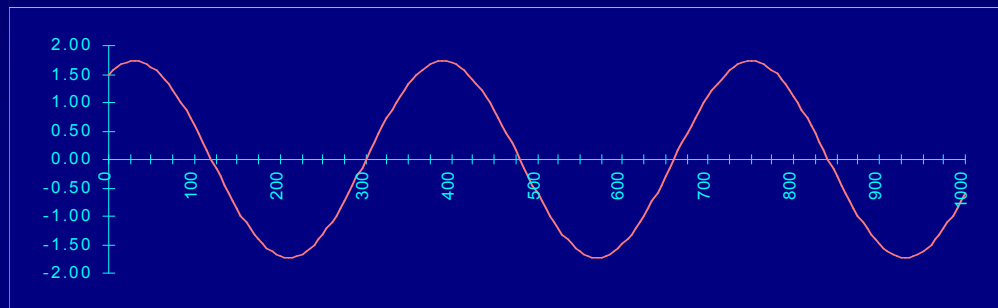
Superposing sine waves

If you added the two sinusoidal waves shown, what would the result look like?



The sum of two sines having the same frequency is another sine with the same frequency.

Its amplitude depends on their relative phases.

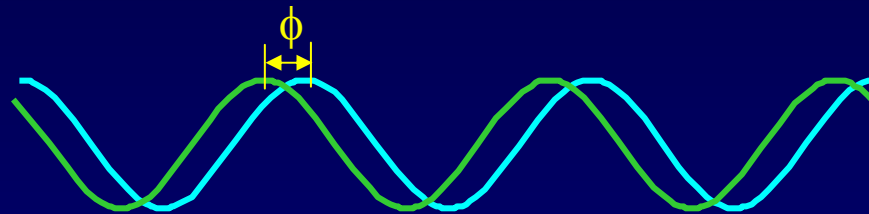


Let's see how this works.

Adding Sine Waves with Different Phases

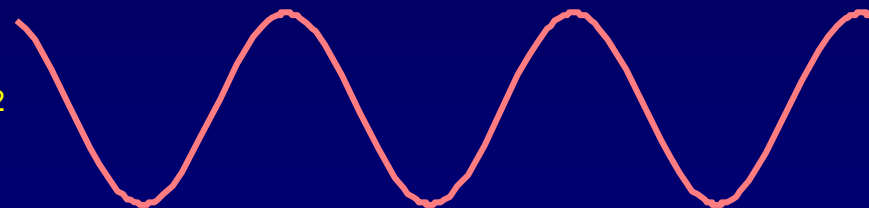
Suppose we have two sinusoidal waves with the same A_1 , ω , and k : $y_1 = A_1 \cos(kx - \omega t)$ and $y_2 = A_1 \cos(kx - \omega t + \phi)$
 One starts at phase ϕ after the other:

Spatial dependence
of 2 waves at $t = 0$:



Resultant wave:

$$y = y_1 + y_2$$



Use this trig identity:

$$A_1 (\cos \alpha + \cos \beta) = 2A_1 \cos\left(\frac{\beta - \alpha}{2}\right) \cos\left(\frac{\beta + \alpha}{2}\right)$$

$$y_1 + y_2$$

$$(\phi/2)$$

$$(kx - \omega t + \phi/2)$$

$$y = 2A_1 \cos(\phi/2) \cos(kx - \omega t + \phi/2)$$

Amplitude

Oscillation

Interference of Waves

What happens when two waves are present at the same place?

Always add amplitudes (pressures or electric fields).

However, we observe intensity (power).

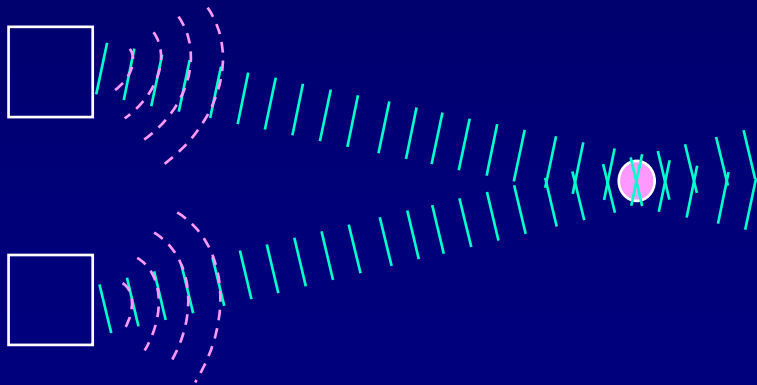
For equal A and ω :

$$A = 2A_1 \cos(\phi/2) \Rightarrow I = 4I_1 \cos^2(\phi/2)$$

Example:

Stereo speakers:

Listener:



Terminology:

Constructive interference:

waves are “in phase”
($\phi = 0, 2\pi, 4\pi, \dots$)

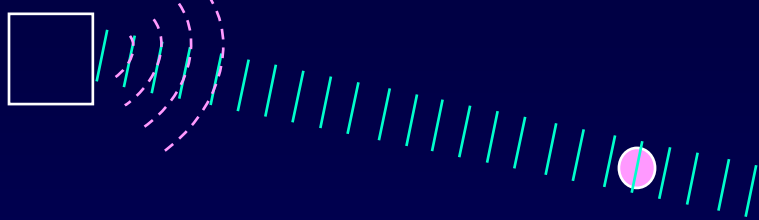
Destructive interference:

waves are “out of phase”
($\phi = \pi, 3\pi, 5\pi, \dots$)

Of course, ϕ can take on an infinite number of values. We won't use terms like “mostly constructive” or “slightly destructive”.

Example: Changing phase of the Source

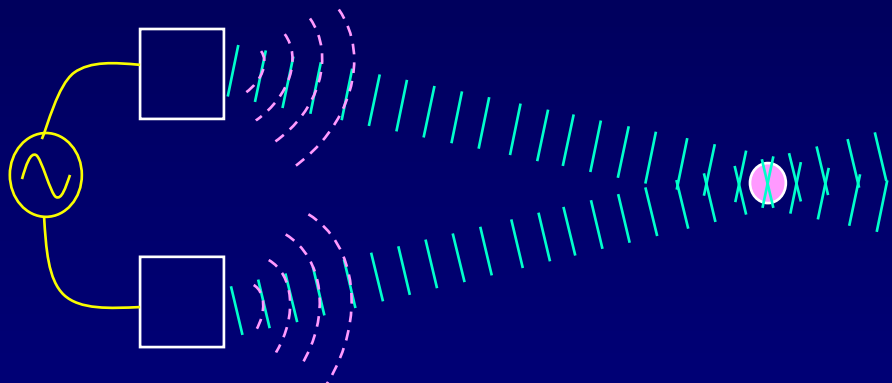
Each speaker alone produces an intensity of $I_1 = 1 \text{ W/m}^2$ at the listener:



$$I = I_1 = A_1^2 = 1 \text{ W/m}^2$$

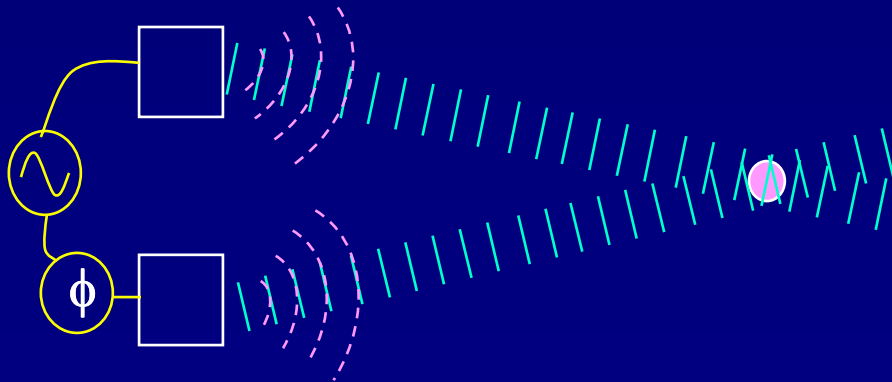


Drive the speakers in phase. What is the intensity I at the listener?



$$I =$$

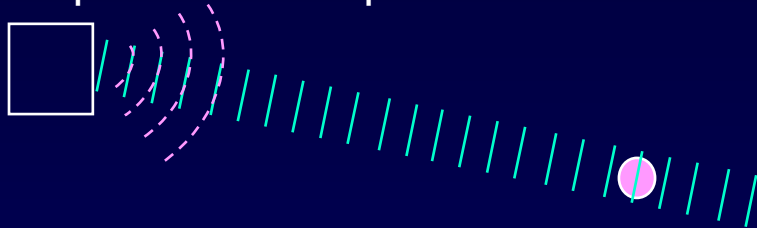
Now shift phase of one speaker by 90° . What is the intensity I at the listener?



$$I =$$

Example: Changing phase of the Source

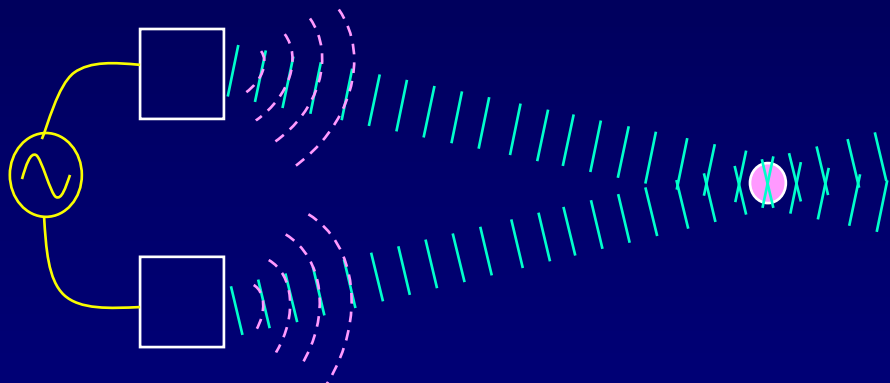
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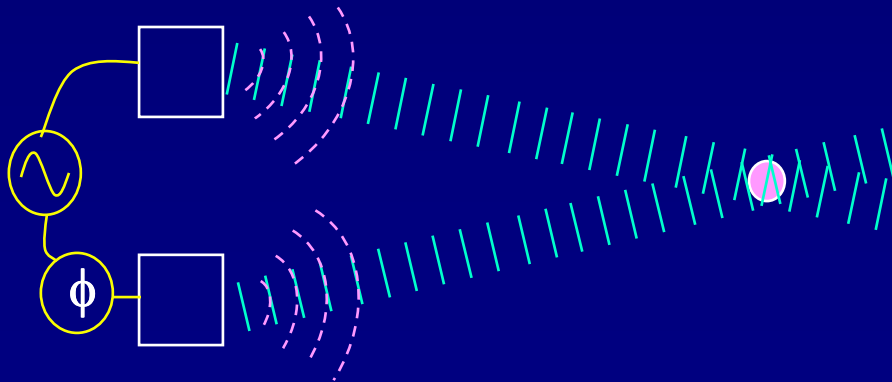


Drive the speakers in phase. What is the intensity I at the listener?



$$I = (2A_1)^2 = 4I_1 = 4 \text{ W/m}^2$$

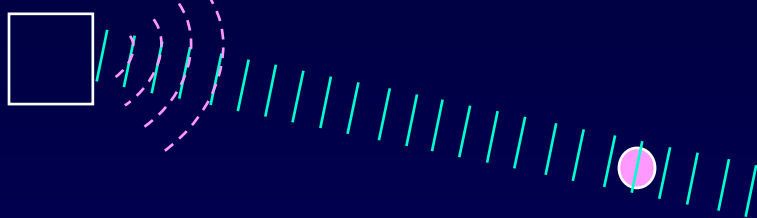
Now shift phase of one speaker by 90° . What is the intensity I at the listener?



$$I =$$

Example: Changing phase of the Source

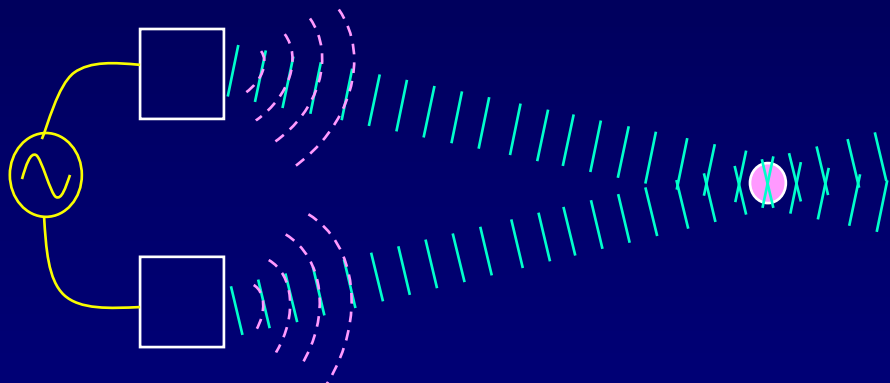
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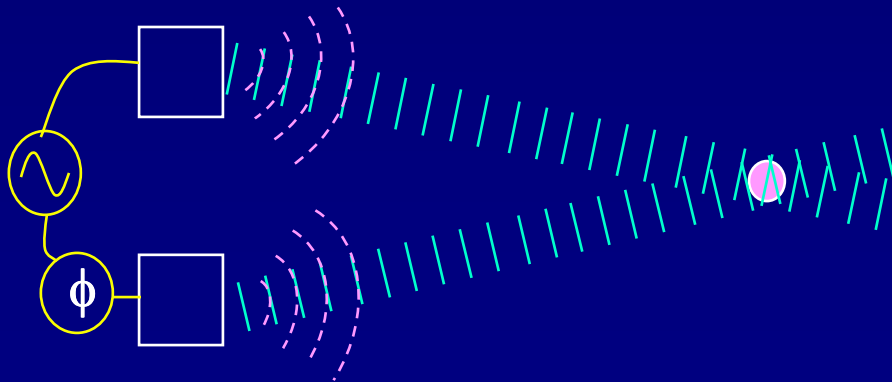


Drive the speakers in phase. What is the intensity I at the listener?



$$I = (2A_1)^2 = 4I_1 = 4 \text{ W/m}^2$$

Now shift phase of one speaker by 90° . What is the intensity I at the listener?

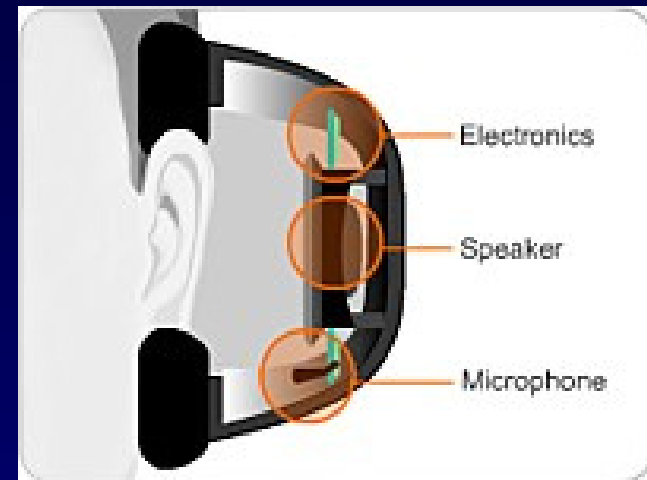


$$I = 4 I_1 \cos^2(45^\circ) = 2.0 I_1 = 2.0 \text{ W/m}^2$$

ACT 1:

Noise-cancelling Headphones

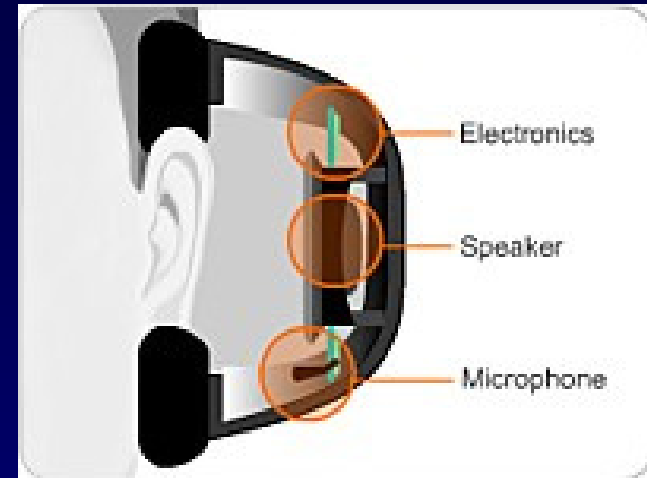
Noise-canceling headphones work using interference. A microphone on the earpiece monitors the instantaneous amplitude of the external sound wave, and a speaker on the inside of the earpiece produces a sound wave to cancel it.



1. What must be the phase of the signal from the speaker relative to the external noise?
 - a. 0
 - b. 90°
 - c. π
 - d. -180°
 - e. 2π
2. What must be the intensity I_s of the signal from the speaker relative to the external noise I_n ?
 - a. $I_s = I_n$
 - b. $I_s < I_n$
 - c. $I_s > I_n$

Solution

Noise-canceling headphones work using interference. A microphone on the earpiece monitors the instantaneous amplitude of the external sound wave, and a speaker on the inside of the earpiece produces a sound wave to cancel it.



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Destructive interference occurs when the waves are $\pm 180^\circ$ out of phase.

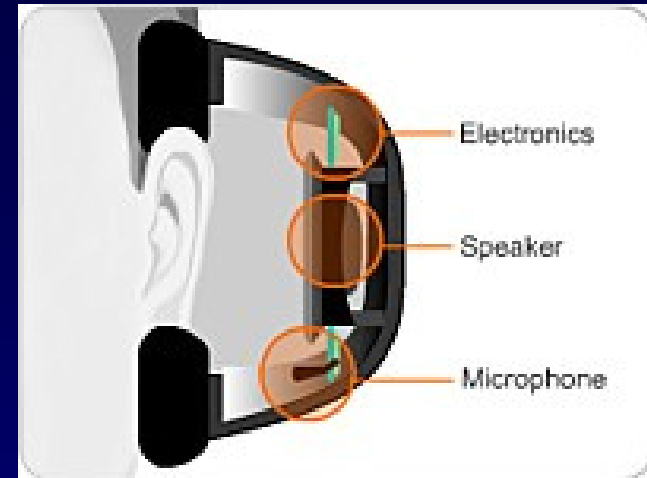
$180^\circ = \pi$ radians!

2. What must be the intensity I_s of the signal from the speaker relative to the external noise I_n ?

- a. $I_s = I_n$ b. $I_s < I_n$ c. $I_s > I_n$

Solution

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a. $I_s = I_n$

b. $I_s < I_n$

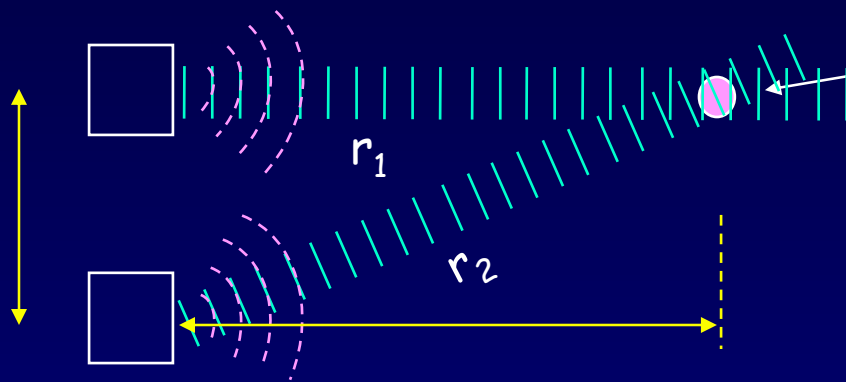
c. $I_s > I_n$

We want $A = A_s - A_n = 0$.

Note that I is never negative.

Interference Exercise

The relative phase of two waves also depends on the relative distances to the sources:



The two waves at this point are “out of phase”. Their phase difference ϕ depends on the path difference $\delta \equiv r_2 - r_1$.

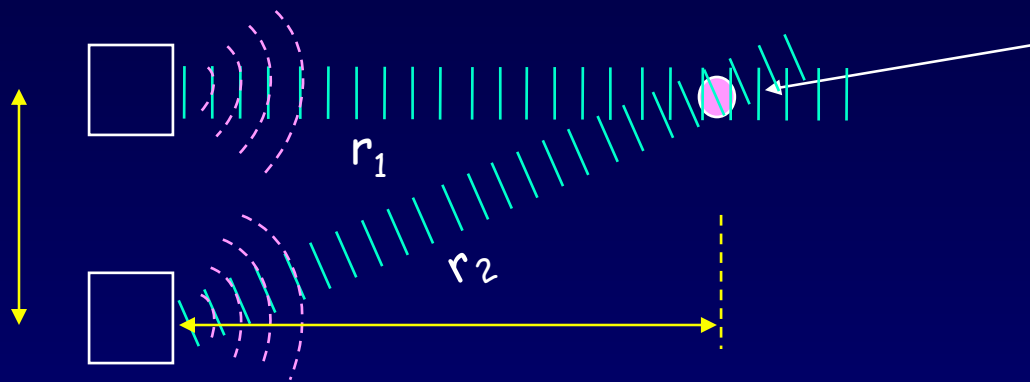
Each fraction of a wavelength of path difference gives that fraction of 360° (or 2π) of phase difference:

$$\frac{\phi}{2\pi} = \frac{\delta}{\lambda}$$

Path difference	Phase difference		I
δ	ϕ	$A = 2A_1 \cos(\phi/2)$	
0			
$\lambda/4$			
$\lambda/2$			
λ			

Solution

The relative phase of two waves also depends on the relative distances to the sources:



The two waves at this point are “out of phase”. Their phase difference ϕ depends on the path difference $\delta \equiv r_2 - r_1$.

Reminder: A can be negative.
“Amplitude” is the absolute value.

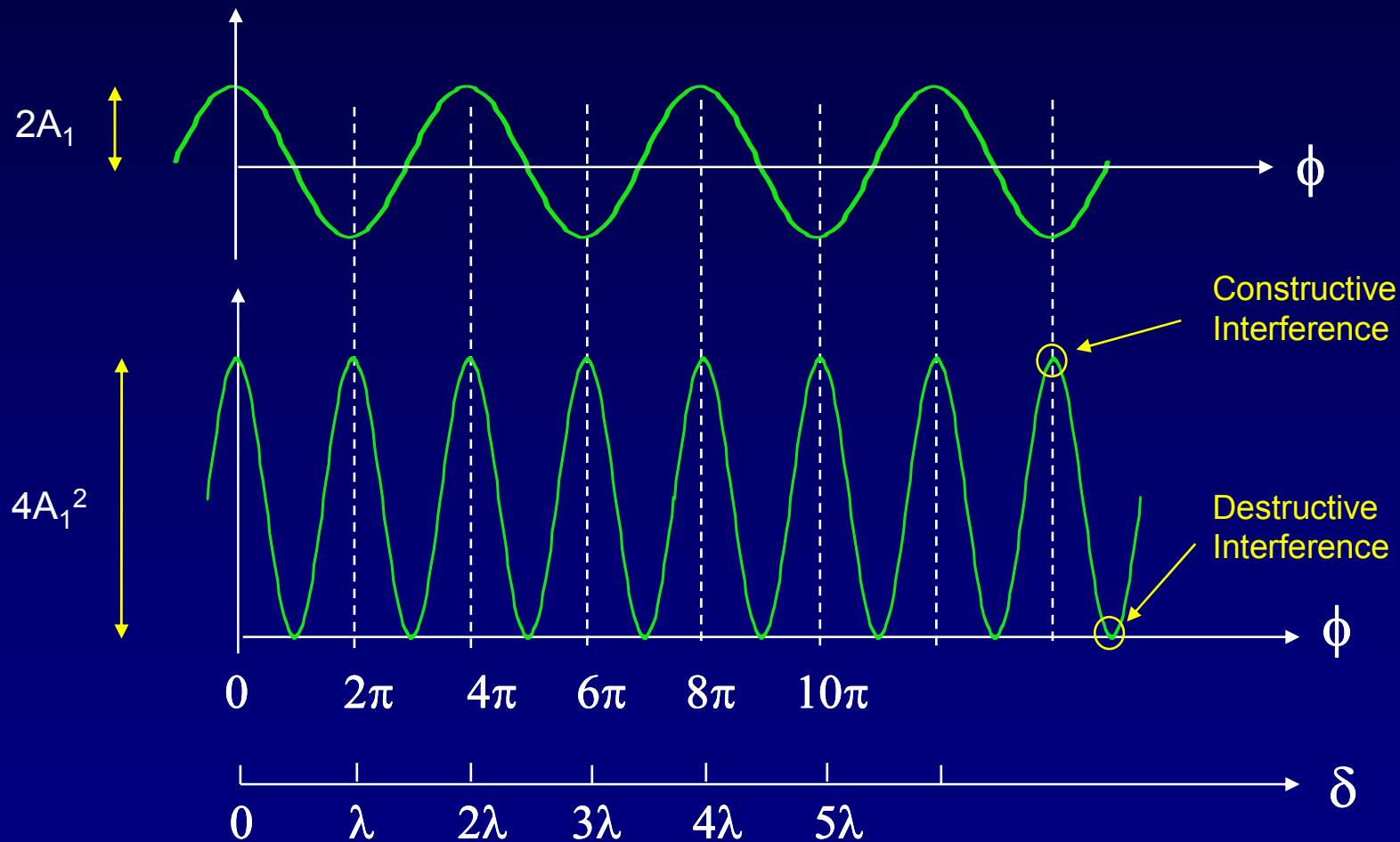
Path difference δ	Phase difference ϕ	$A = 2A_1 \cos(\phi/2)$	I
0	0	$2A_1$	$4I_1$
$\lambda/4$	$\pi/2$	$\sqrt{2}A_1$	$2I_1$
$\lambda/2$	π	0	0
λ	2π	$2A_1$	$4I_1$

Each fraction of a wavelength of path difference gives that fraction of 360° (or 2π) of phase difference:

$$\frac{\phi}{2\pi} = \frac{\delta}{\lambda}$$

Amplitude vs. Intensity for 2 Interfering Waves

Plot $2A_1\cos(\phi/2)$ and $4A_1^2\cos^2(\phi/2)$ as a function of ϕ .



Q: What is the spatial average intensity?

A: $I_{av} = 4I_1 \cdot 0.5 = 2I_1$ Does this make sense?

Summary

Interference of coherent waves

Resultant intensity of two equal-intensity waves of the same wavelength at the same point in space:

$$I = 4 I_1 \cos^2(\phi/2)$$

In order to calculate I ,
we need to know ϕ .

For unequal intensities, the maximum and minimum intensities are

$$I_{\max} = |A_1 + A_2|^2$$
$$I_{\min} = |A_1 - A_2|^2$$

The phase difference between the two waves may be due to a difference in their source phases or in the path difference to the observer, or both.

The difference due to path difference is:

$$\phi = 2\pi(\delta/\lambda)$$

$$\text{where } \delta = r_2 - r_1$$

Note: The phase difference can also be due to an index of refraction, because that will change the wavelength.

Light - Particle or Wave?

Diffraction of light played an important historical role.

- **1818**: French Academy held a science competition
- Fresnel proposed the diffraction of light.
- One judge, Poisson, knew light was made of particles, and thought Fresnel's ideas ridiculous; he argued that if Fresnel ideas were correct, one would see a bright spot in the middle of the shadow of a disk.
- Another judge, Arago, decided to actually do the experiment... (our lecture demo)

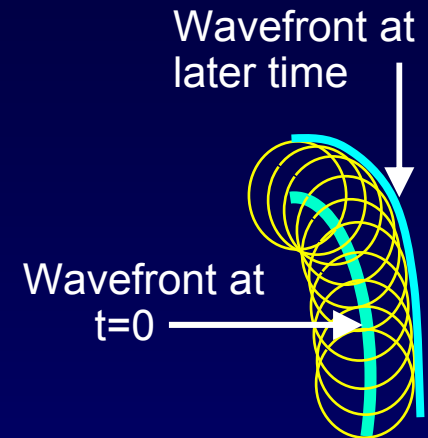
- **Conclusion:**
Light *must* be a wave, since particles don't diffract!

Huygens' principle

A Consequence of Superposition

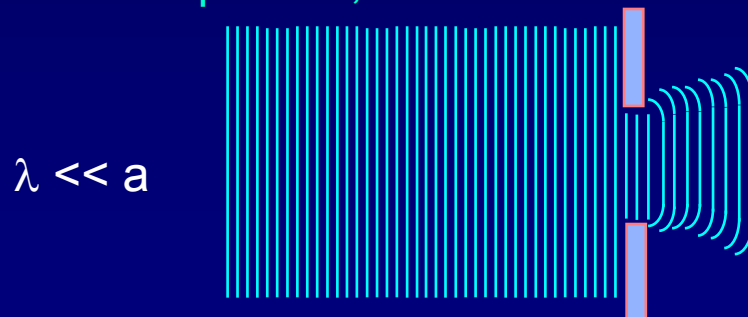
We will next study what happens when waves pass through one slit. We will use Huygens' principle (1678):

All points on a wave front (e.g., crest or trough) can be treated as point sources of secondary waves with speed, frequency, and phase equal to the initial wave.

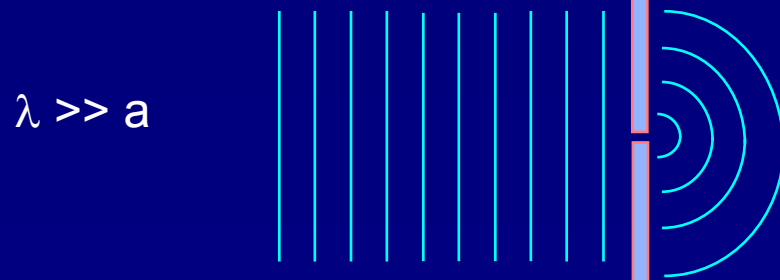


Q: What happens when a plane wave meets a small aperture?

A: The result depends on the ratio of the wavelength λ to the size of the aperture, a :

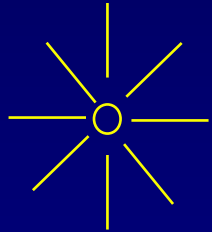


The transmitted wave is concentrated in the forward direction, and at near distances the wave fronts have the shape of the aperture. The wave eventually spreads out.

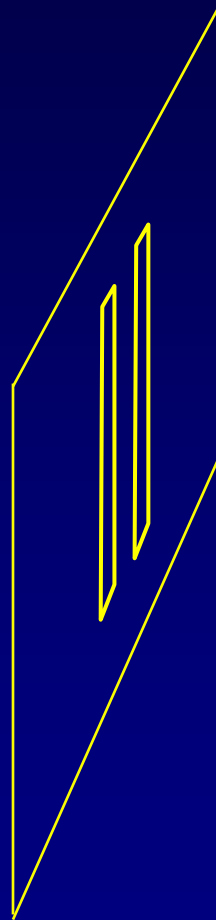


Similar to a wave from a point source. This effect is called *diffraction*.

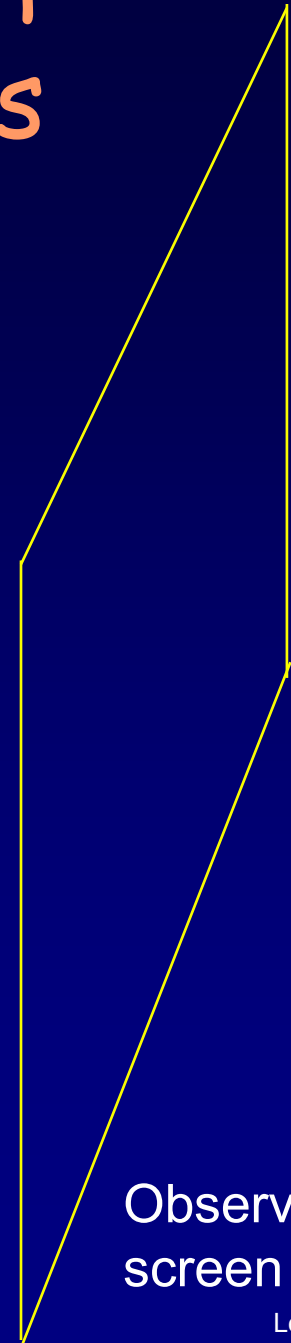
Transmission of Light through Narrow Slits



Monochromatic
light source
at a great distance,
or a laser.



Slit pattern



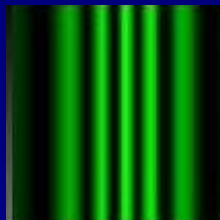
Observation
screen

Double-slit interference

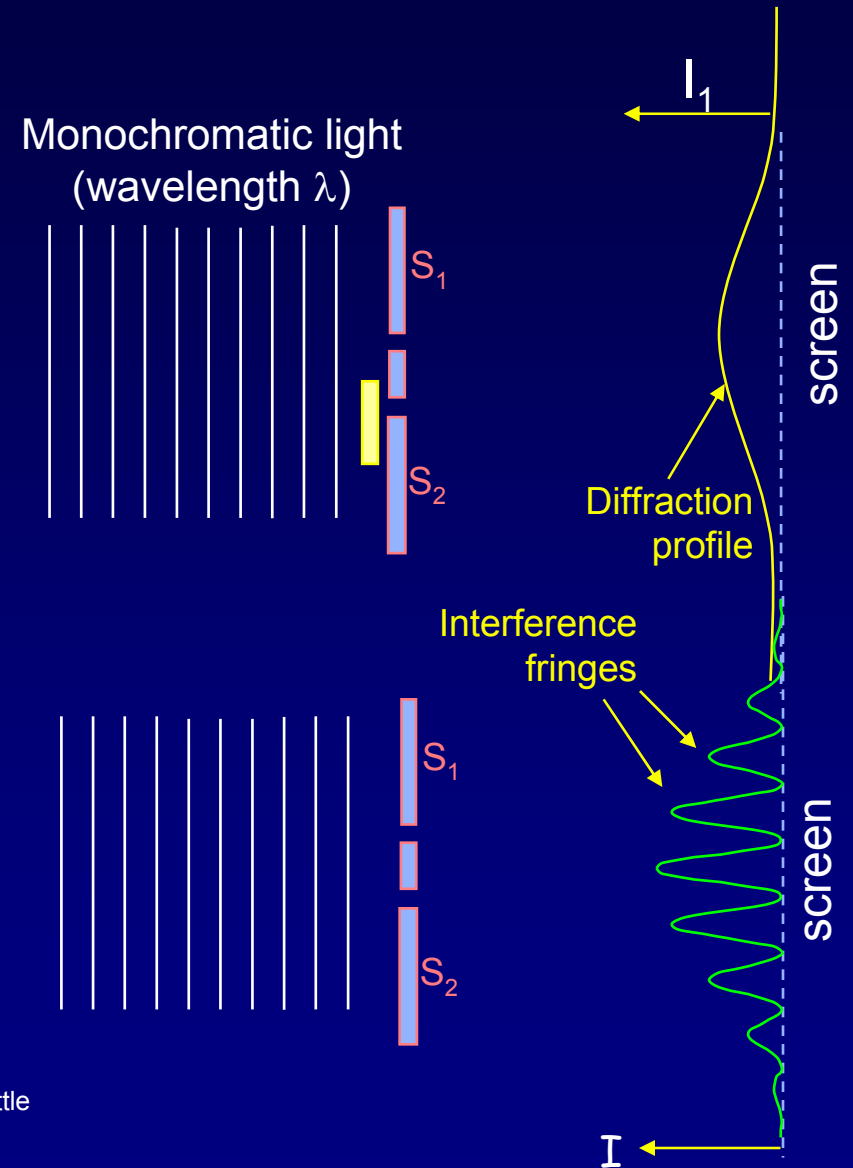
Light (wavelength λ) is incident on a two-slit (two narrow, rectangular openings) apparatus:

If either one of the slits is closed, a spread-out image of the open slit will appear on the screen. (The image is spread due to **diffraction**. We will discuss diffraction in more detail later.)

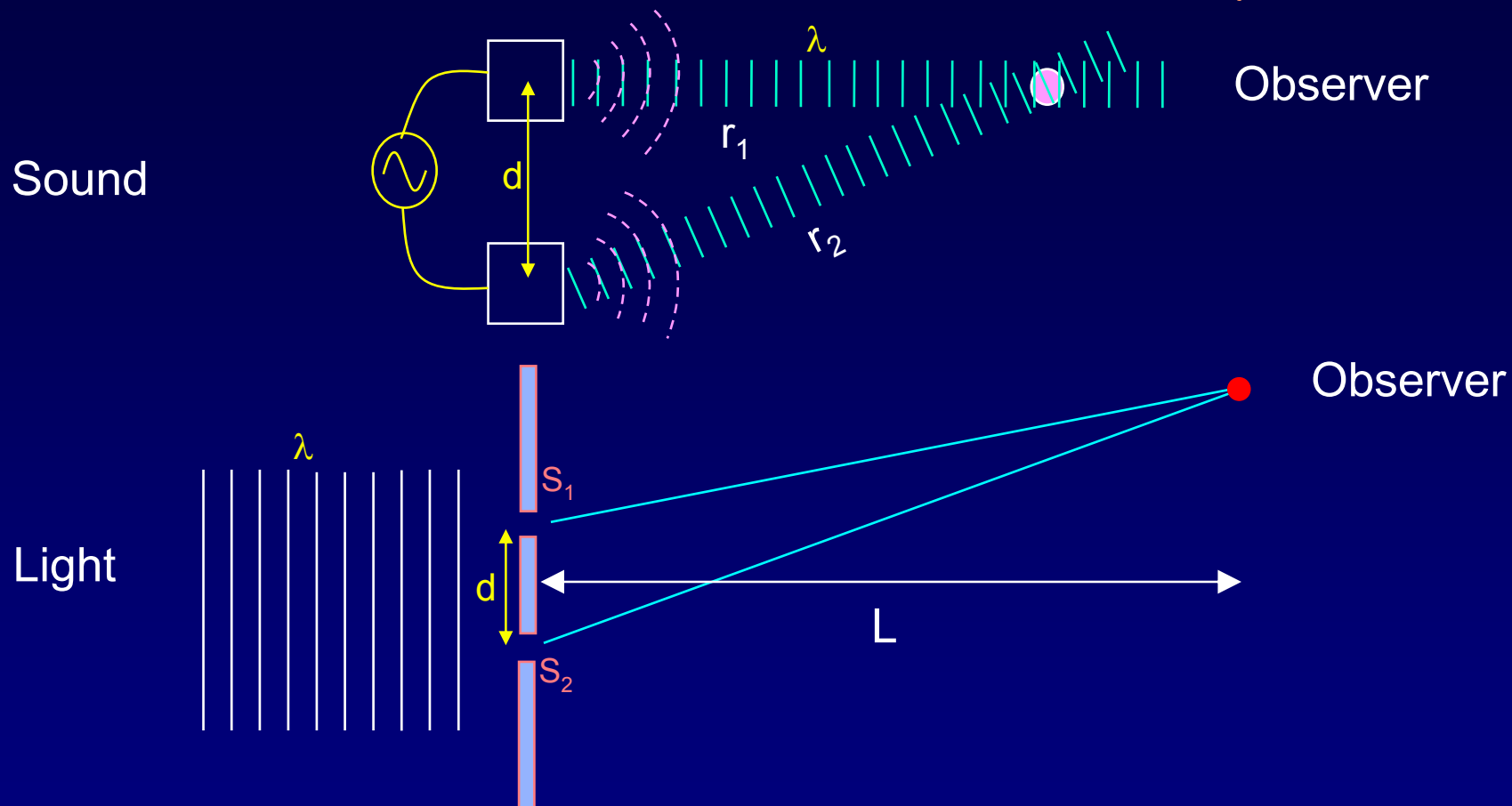
If both slits are open, we see interference “fringes” (light and dark bands), corresponding to constructive and destructive **interference** of the wave passing through the two slits.



Note: In the laser demo, there is little vertical spread, because the laser spot is small in that direction.



Sound and Light Waves Interfere the Same Way

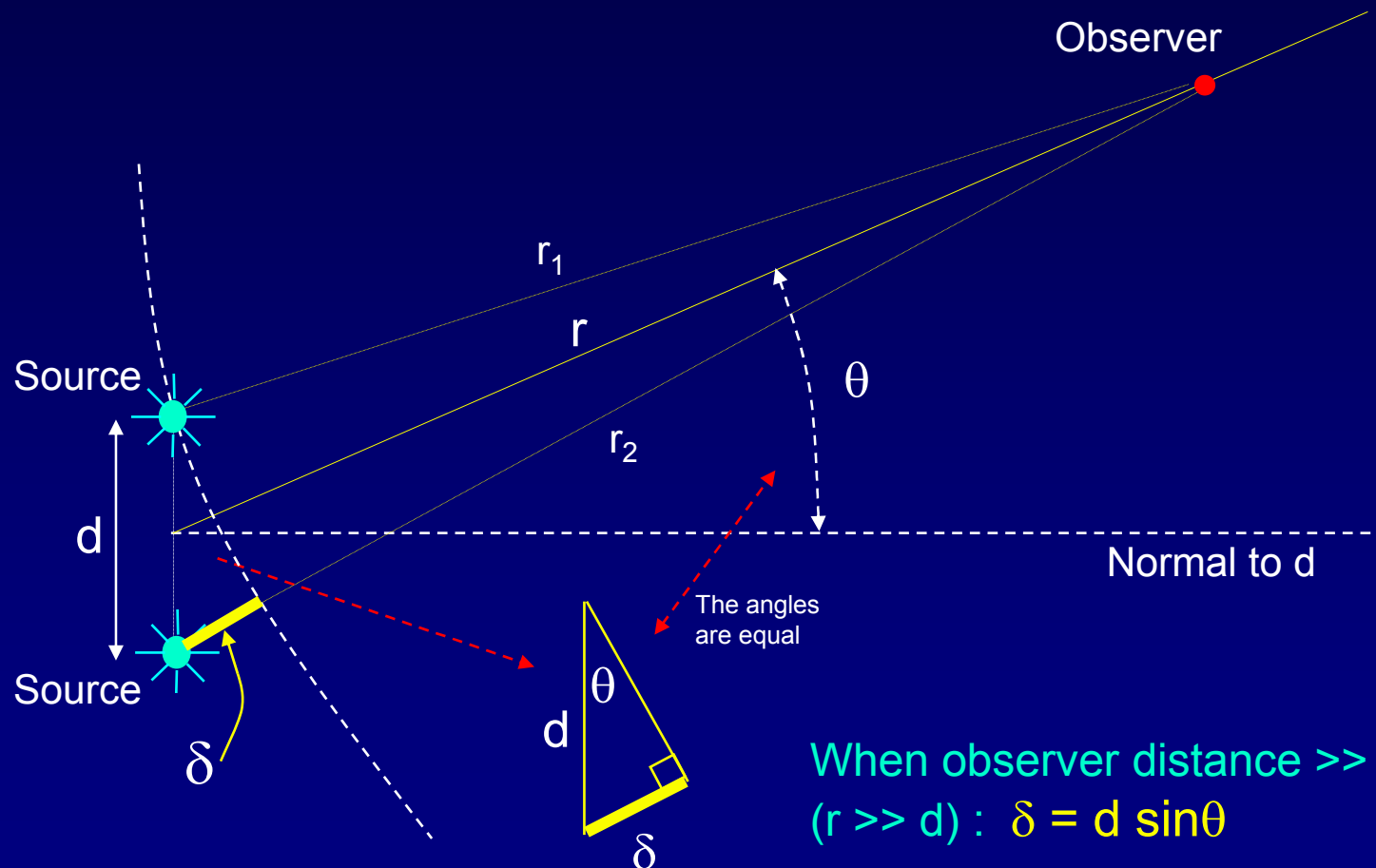


In both cases, $I = 4 I_1 \cos^2(\phi/2)$ with $\phi = 2\pi(\delta/\lambda)$, $\delta = r_2 - r_1$

However, **for light**, the distance L is generally much greater than the wavelength λ and the slit spacing d : $L \gg \lambda$, $L \gg d$.

Simple formula for the path difference, δ , when the observer is far from sources.

Assume 2 sources radiating in phase:



When observer distance \gg slit spacing ($r \gg d$): $\delta = d \sin\theta$

Therefore: $\phi = 2\pi(\delta/\lambda) = 2\pi(d \sin\theta / \lambda)$

Two-Slit Interference

Constructive interference:

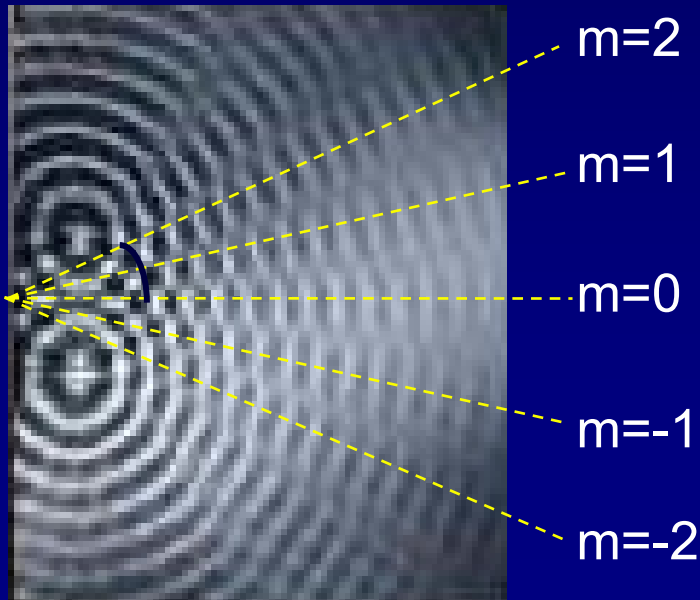
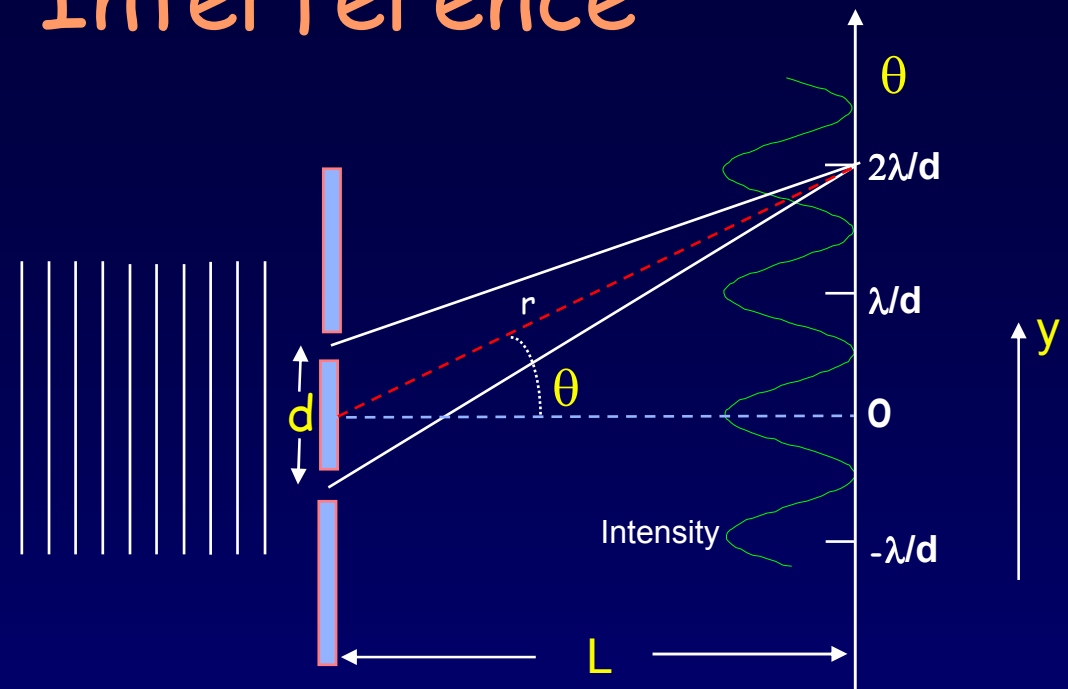
$$\delta = d \sin \theta = m \lambda$$

$$m = 0, \pm 1, \pm 2, \dots$$

Destructive Interference:

$$\delta = d \sin \theta = (m + \frac{1}{2}) \lambda$$

$$m = 0, \pm 1, \pm 2, \dots$$



Lines of constructive interference

Usually we care about the linear displacement y of the pattern (because our screens are flat):

$$y = L \tan \theta$$

Two-Slit Interference, small angles:

Often, $d \gg \lambda$, so that θ is small.

Then we can use the **small angle approximation** to simplify our results:

For small angles: ($\theta \ll 1$ radian):

$\sin\theta \approx \theta \approx \tan\theta$ (only in radians!)

$$y = L \tan\theta \approx L\theta$$

Constructive interference:

$$\theta \approx m(\lambda/d)$$

$$y \approx m(\lambda/d)L$$

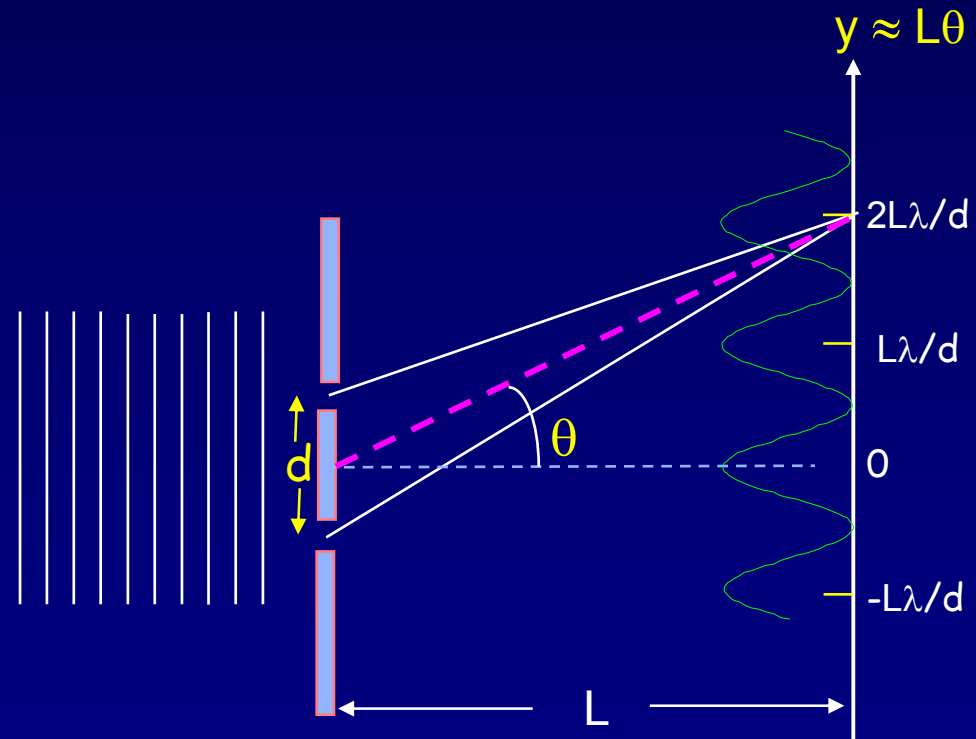
$$m = 0, \pm 1, \pm 2, \dots$$

Destructive interference:

$$\theta \approx (m + \frac{1}{2})(\lambda/d)$$

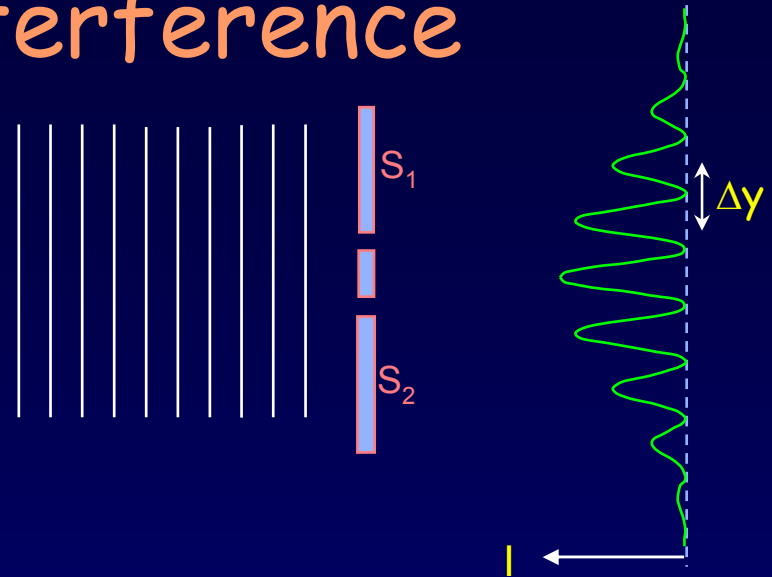
$$y \approx (m + \frac{1}{2})(\lambda/d)L$$

$$m = 0, \pm 1, \pm 2, \dots$$



Act 2: 2-slit interference

A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.

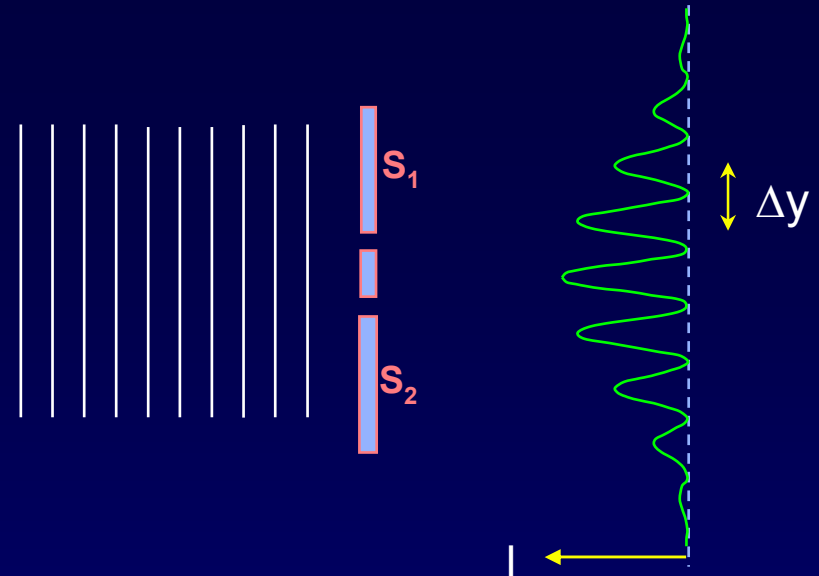


1. If we increase the spacing between the slits, what will happen to Δy ?
 - a. decrease
 - b. stay the same
 - c. increase

2. If we instead use a green laser (smaller λ), Δy will?
 - a. decrease
 - b. stay the same
 - c. increase

Solution

A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.



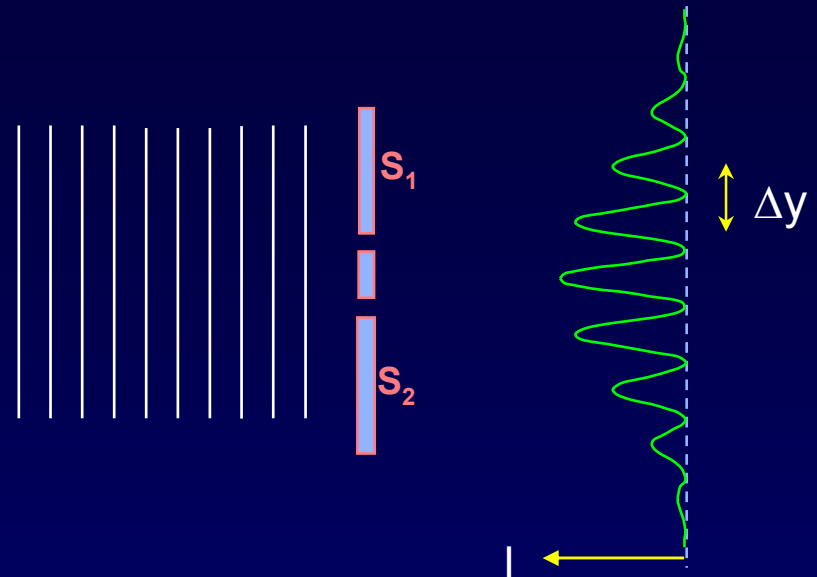
1. If we increase the spacing between the slits, what will happen to Δy ?
a. decrease b. stay the same c. increase

$\Delta y \propto 1/d$, so it decreases. This is a general phenomenon: the more spread out the sources are, the narrower the interference pattern is.

2. If we instead use a green laser (smaller λ), Δy will?
a. decrease b. stay the same c. increase

Solution

A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.



1. If we increase the spacing between the slits, what will happen to Δy ?
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2. If we instead use a green laser (smaller λ), Δy will?
- a. decrease b. stay the same c. increase

$\Delta y \propto \lambda$, so it decreases.

Next time

Phasors, review, examples, examples, examples

Next week

Diffraction from a single slit

Multiple-slit interference

Diffraction and Spectroscopy

Text – Ch. 36 + added material

Applications – resolution of telescopes and
microscopes, interferometers, crystallography, etc...