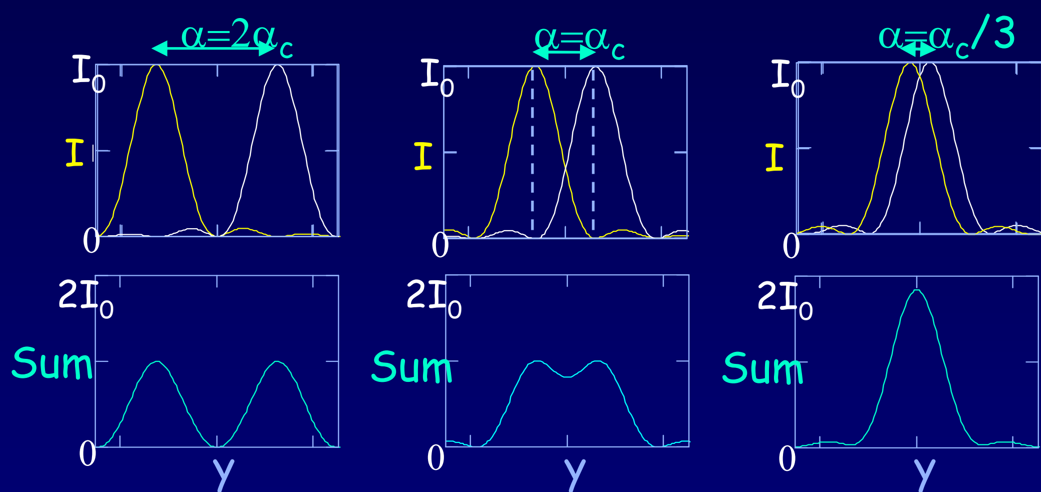
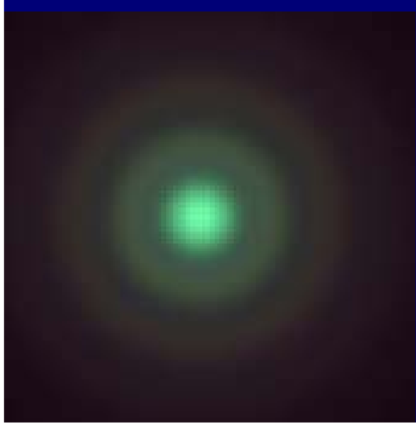
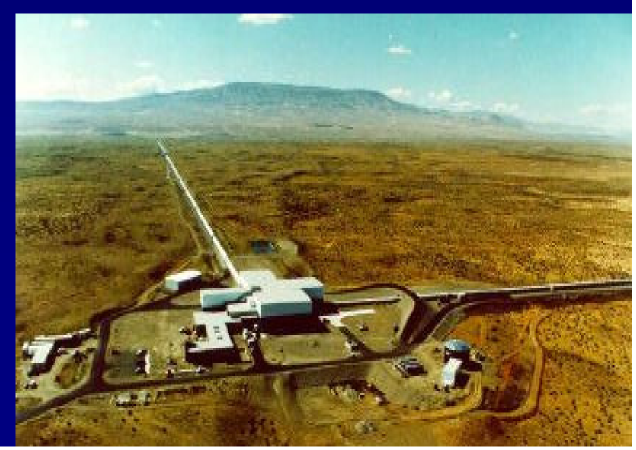
# Lecture 5:

## Applications of Interference and Diffraction





# Today

## Circular Diffraction

- Angular resolution (Rayleigh's criterion)
- Minimum spot size

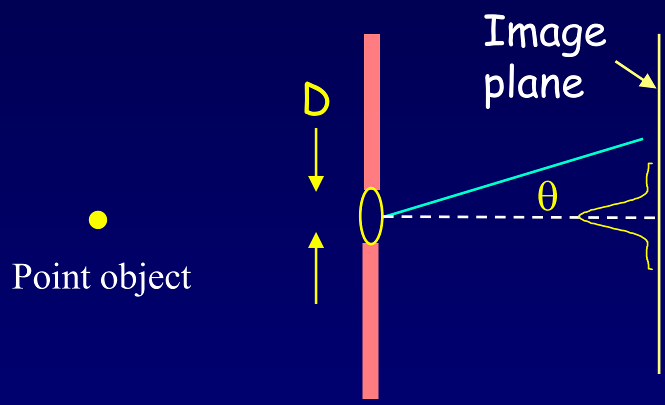
## Interferometers

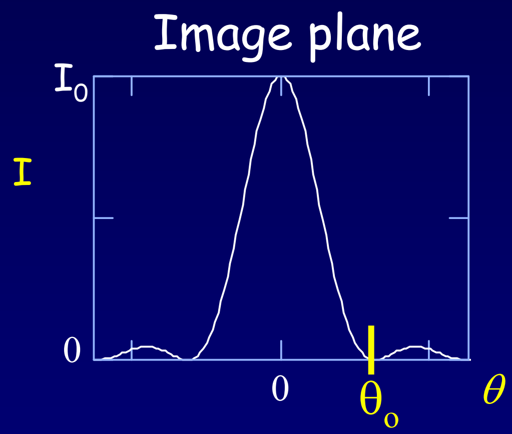
- Michelson
- Applications

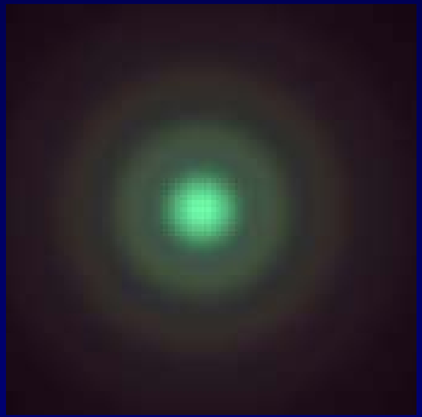
# Diffraction-limited Optics

Diffraction has important implications for optical instruments

Even for perfectly designed optics the image of a point source will be a little blurry - the circular aperture produces diffraction.



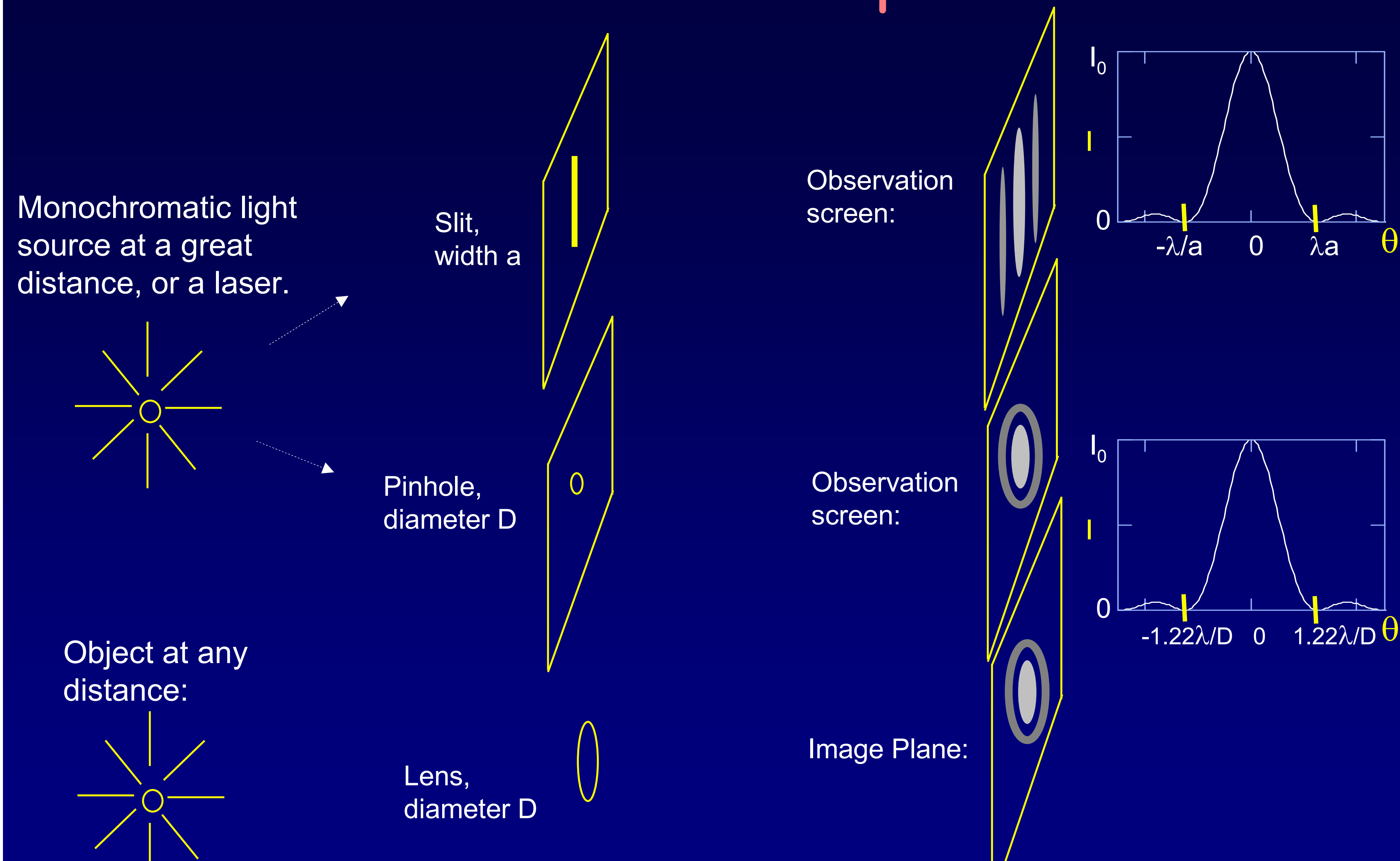
The "Airy disk".
The central lobe contains 84% of power.

The size of the spot is determined by the **diameter, D,** of the aperture, and **wavelength, λ,** of the incident light.

Diffraction by a circular aperture is similar to single-slit diffraction. But note the difference:

| | |
|---|---|
| Slit $\theta_0 \approx \dfrac{\lambda}{a}$ | Circular aperture $\theta_0 \approx 1.22\dfrac{\lambda}{D}$ |

# Slits and circular apertures

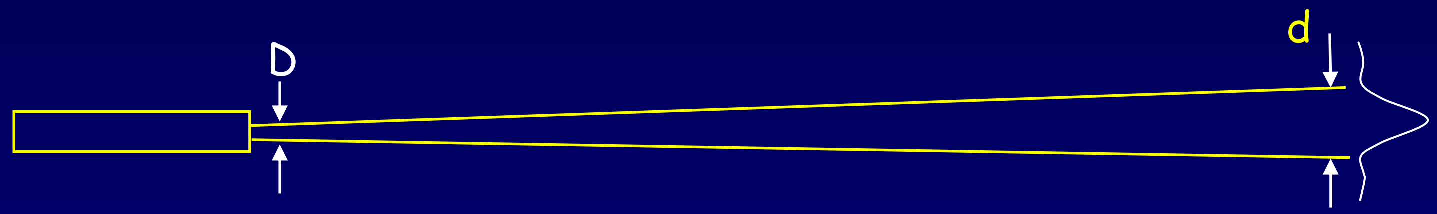



The focusing effect of the lens is independent of the diffraction effect due to the aperture

# Exercise: Expansion of a Laser beam

In 1985, a laser beam with a wavelength of $\lambda$ = 500 nm was fired from the earth and reflected off the space shuttle Discovery, in orbit at a distance of L = 350 km away from the laser.

If the circular aperture of the laser was D = 4.7 cm, what was the beam diameter d at the space shuttle?




# Solution

In 1985, a laser beam with a wavelength of $\lambda$ = 500 nm was fired from the earth and reflected off the space shuttle Discovery, in orbit at a distance of L = 350 km away from the laser.

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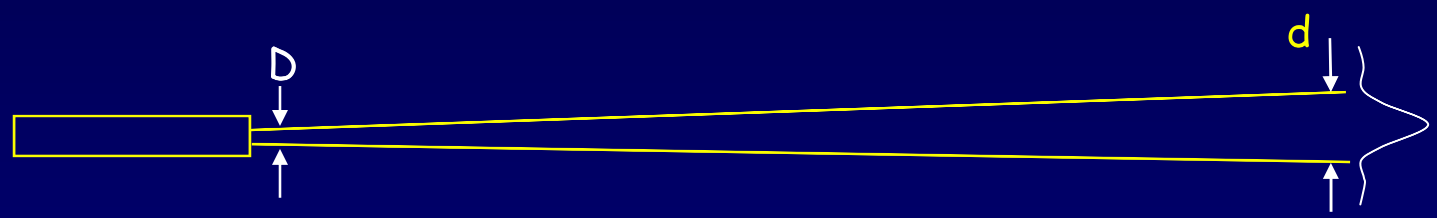


Half-angle-width of diffraction maximum:

$$\theta_o = 1.22\frac{\lambda}{D} = 1.22\frac{500\times10^{-9}}{4.7\times10^{-2}} = 1.3\times10^{-5} \text{ radians}$$

$$d \approx 2\theta_o L = 2(1.3\times10^{-5})(350\times10^{3}\,\text{m}) = \boxed{9.1\text{ m}}$$

# Act 1

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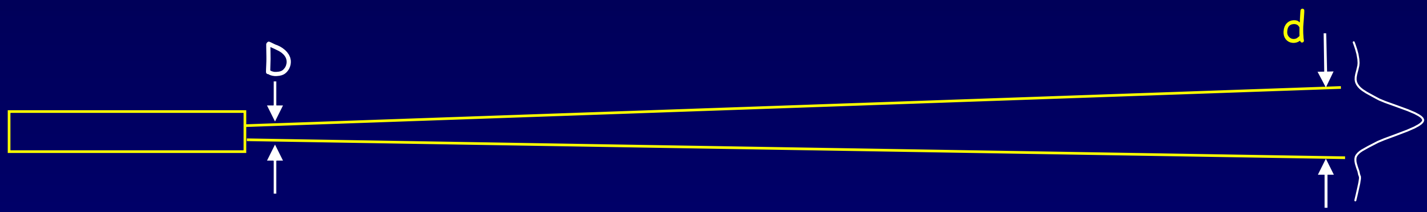


To make a smaller spot on the shuttle, what should we do to the beam diameter at the source?

a. reduce it  b. increase it  c. cannot be made smaller

# Solution

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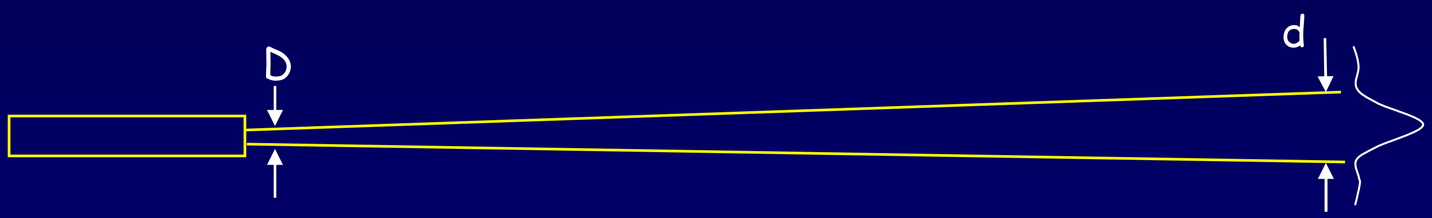


To make a smaller spot on the shuttle, what should we do to the beam diameter at the source?

a. reduce it b. increase it c. cannot be made smaller

Counter-intuitive as this is, it is correct – you reduce beam divergence by using a bigger beam. (Note: this will work as long as D < d.)

We'll see that this can be understood as a non-quantum version of the uncertainty principle: $\Delta x \, \Delta p_x > \hbar$.

# Exercise: Focusing a laser beam

There are many times you would like to focus a laser beam to as small a spot as possible. However, diffraction limits this.


The circular aperture of a laser ($\lambda$ = 780 nm) has $D_{laser}$ = 5 mm.
What is the spot-size d of the beam
after passing through a perfect lens with focal length f = 5mm
and diameter $D_{lens}$ = 6 mm?

# Solution

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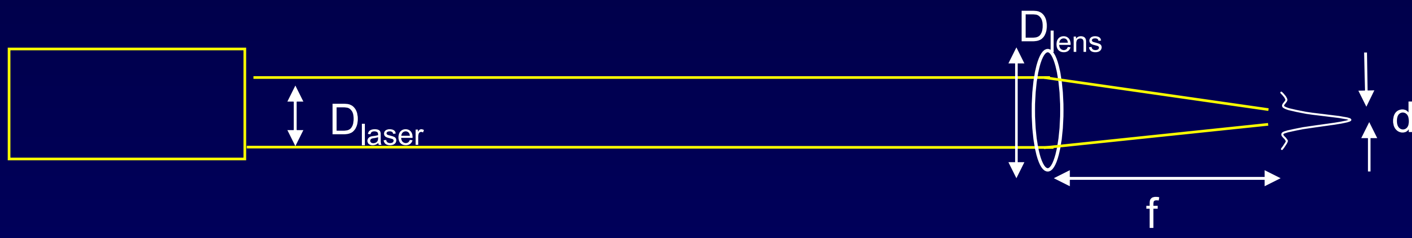


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What is the spot-size d of the beam
after passing through a perfect lens with focal length f = 5mm
and diameter $D_{lens}$ = 6 mm?

The angular spread of the beam is determined by the smaller of $D_{laser}$ and $D_{lens}$.  Here, it's $D_{laser}$.

$$\theta_o = 1.22\lambda / D_{laser}$$

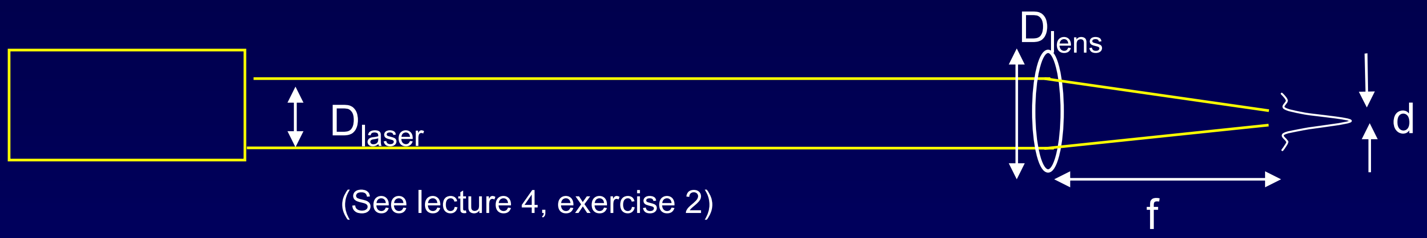
Light at this angle will intercept the focal plane at d/2 ~ f $\theta_o$.

$$d \approx 2\theta_o f = 2.44\lambda f / D_{laser}$$

$$= 2.44(0.78\mu m)(5mm)/(5mm) = 1.9\mu m$$

# Act 2

There are many times you would like to focus a laser beam to as small a spot as possible.  However, diffraction limits this.




(See lecture 4, exercise 2)

$\lambda$ = 780 nm, $D_{laser}$ = 5 mm, f = 5 mm, $D_{lens}$ = 6 mm.
Which of the following will reduce the spot size?

a. increase $\lambda$ b. decrease $\lambda$ c. increase $D_{lens}$ d. decrease $D_{lens}$

# Solution

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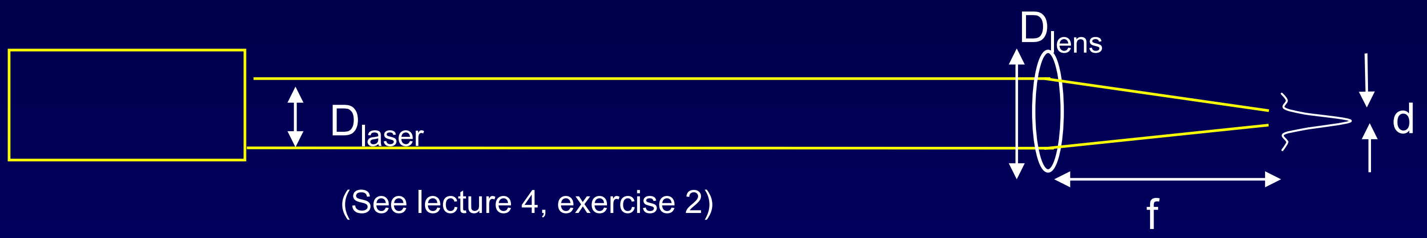


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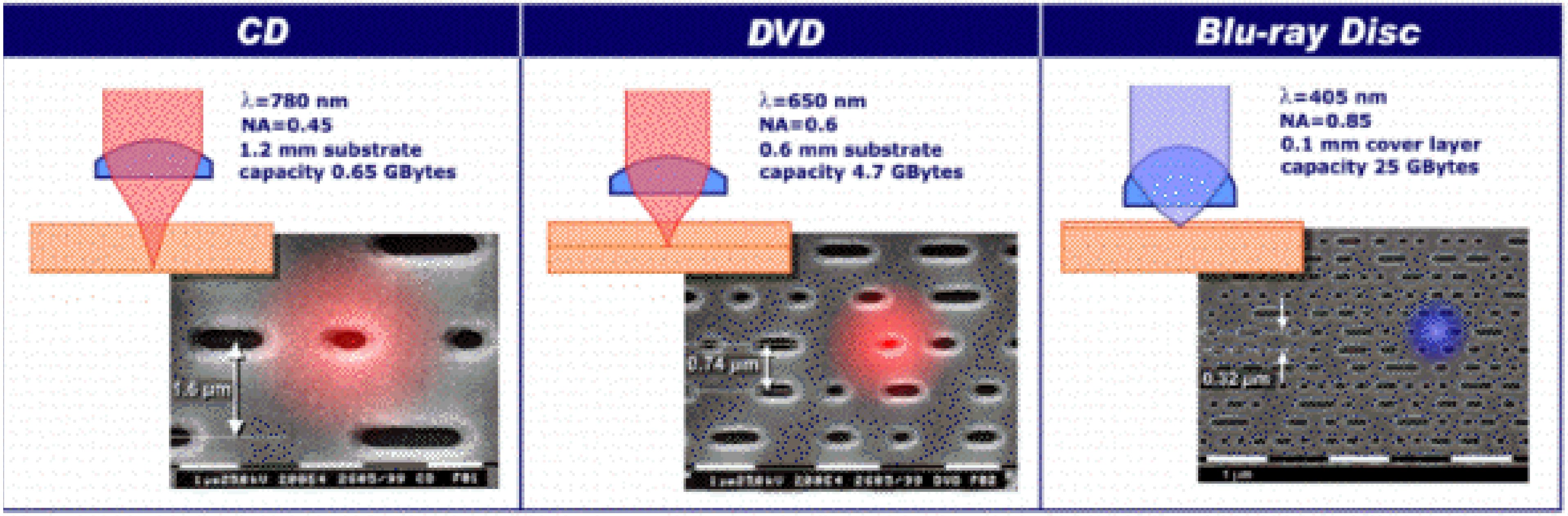
$\lambda$ = 780 nm, $D_{laser}$ = 5 mm, f = 5 mm, $D_{lens}$ = 6 mm.
Which of the following will reduce the spot size?

a. increase $\lambda$   **b. decrease $\lambda$**   c. increase $D_{lens}$   d. decrease $D_{lens}$

The diffraction is already limited by $D_{laser}$. Increasing $D_{lens}$ doesn't help.

There is a huge industry devoted to developing cheap blue diode lasers ($\lambda$ ~ 400 nm) for just this purpose, i.e., to increase DVD capacity.

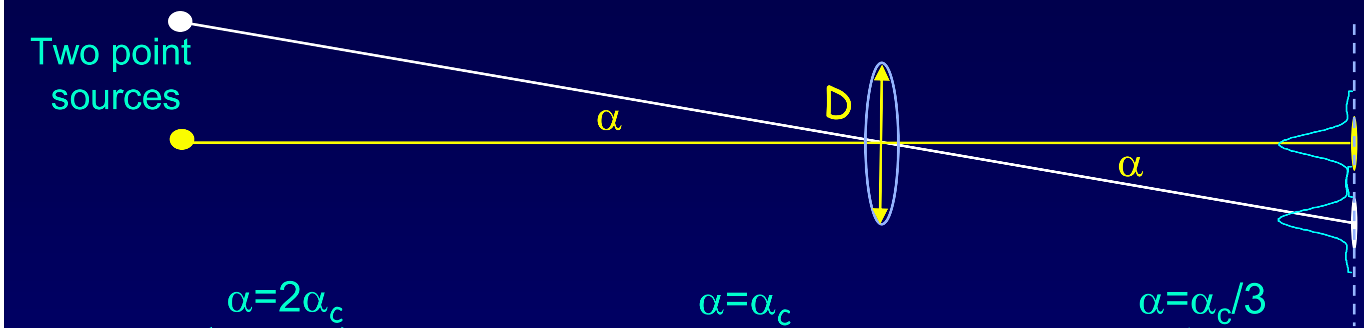
"Blue-Ray" technology!



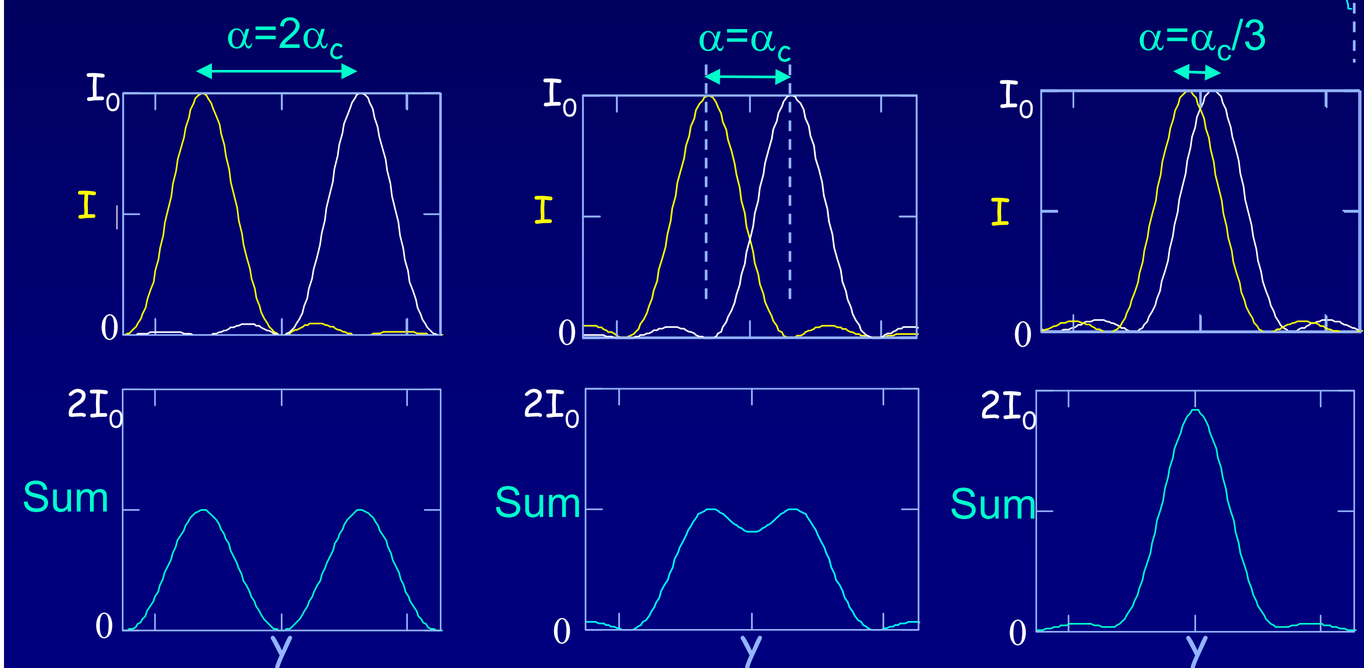
CD
λ=780 nm
NA=0.45
1.2 mm substrate
capacity 0.65 GBytes
1.6 μm
DVD
λ=650 nm
NA=0.6
0.6 mm substrate
capacity 4.7 GBytes
0.74 μm
Blu-ray Disc
λ=405 nm
NA=0.85
0.1 mm cover layer
capacity 25 GBytes
0.32 μm

# Angular Resolution

Diffraction also limits our ability to "resolve" (*i.e.*, distinguish) two point sources. Consider two point sources (e.g., stars) with angular separation $\alpha$ viewed through a circular aperture or lens of diameter D.




Rayleigh's Criterion defines the images to be resolved if the central maximum of one image falls on or further than the first minimum of the second image.




Two images resolvable

'Diffraction limit' of resolution

Two images not resolvable

$$\alpha_c = 1.22\frac{\lambda}{D}$$

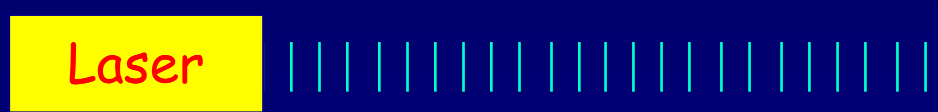
NOTE:
No interference!!
Why not?

# FYI:Coherent and Incoherent Waves

We only observe interference when the sources have a definite (usually constant) phase difference. In this case, the sources are said to be coherent.

Examples of coherent sources:

- Sound waves from speakers driven by electrical signals that have the same frequency and a definite phase.
- Laser light. In a laser, all the atoms emit light with the same frequency and phase. This is a quantum effect that we'll study later in the course.




The laser light is also all going the same direction.

Incoherent waves: The phase relation is random.

Waves from two unrelated sources.

- Examples: light from two points on the sun or two atoms on a light bulb filament, or two people singing the same note.
- Incoherent intensities add. The average of constructive and destructive interference is no interference!

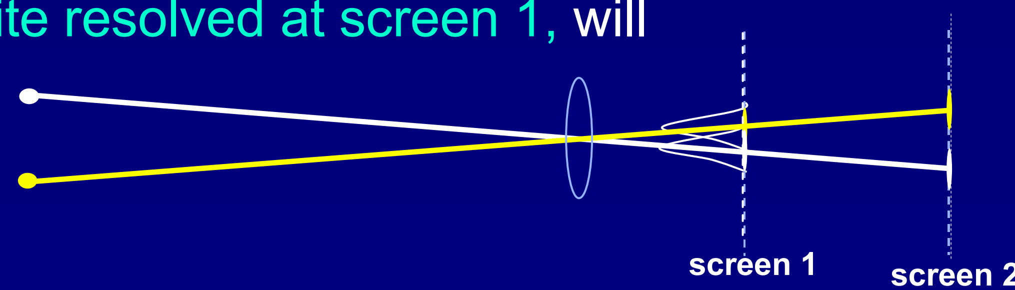
# Act 3: Resolving Stars


1. Assuming diffraction-limited optics, what is the minimum angular separation of two stars that can be resolved by a D = 5 m telescope using light of λ = 500 nm?

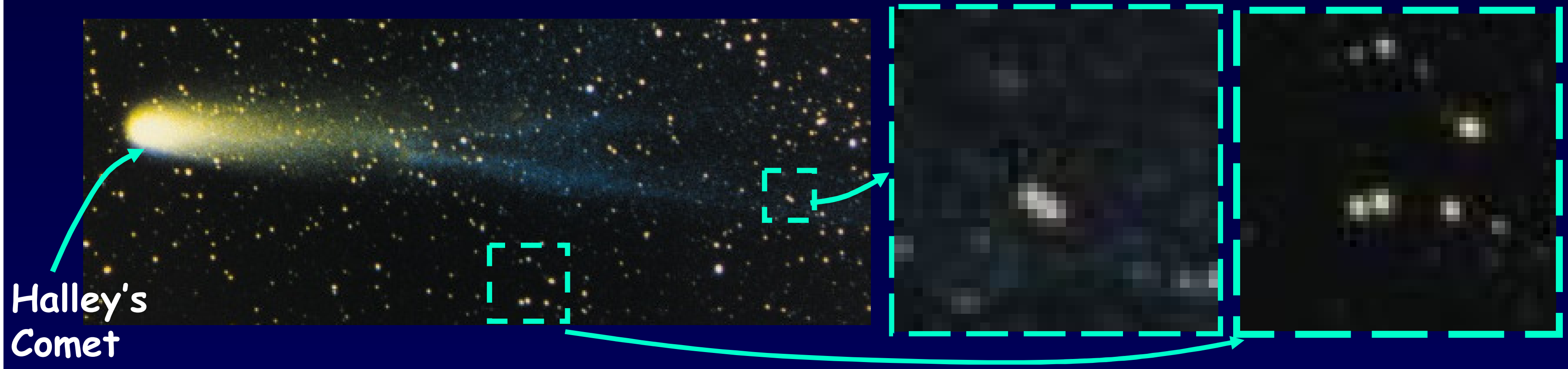
a. 0.1 μrad    b. 1 μrad    c. 10 μrad

2. If the two point sources are not quite resolved at screen 1, will they be resolved at screen 2?

a. Yes    b. No



# Solution

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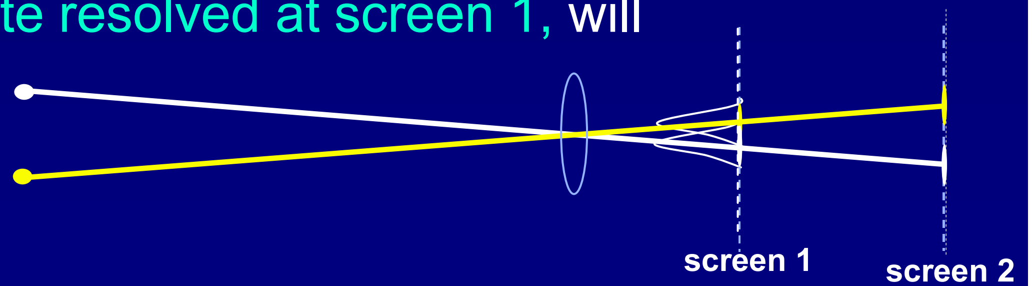
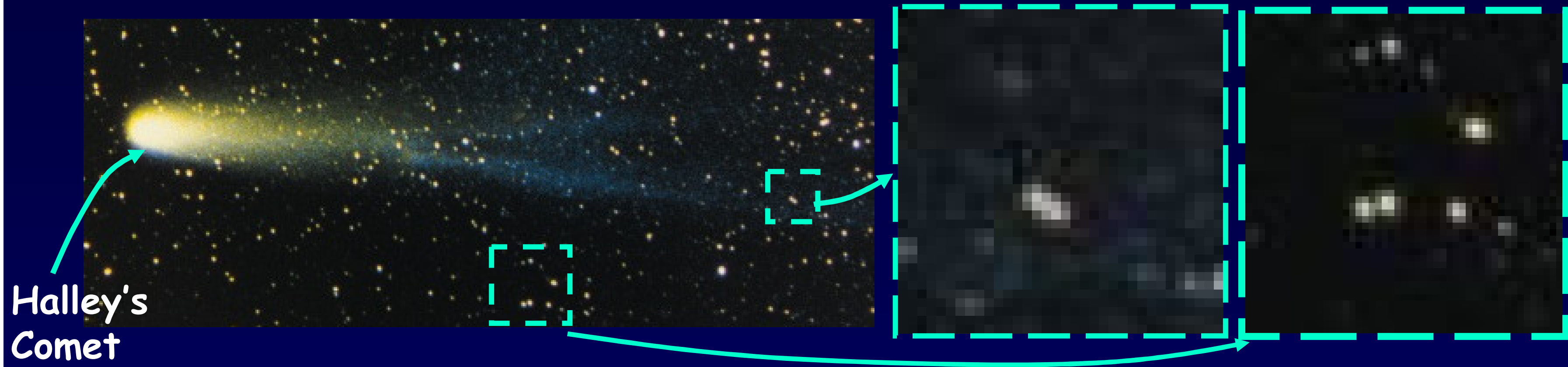
a. Yes    b. No



# Solution

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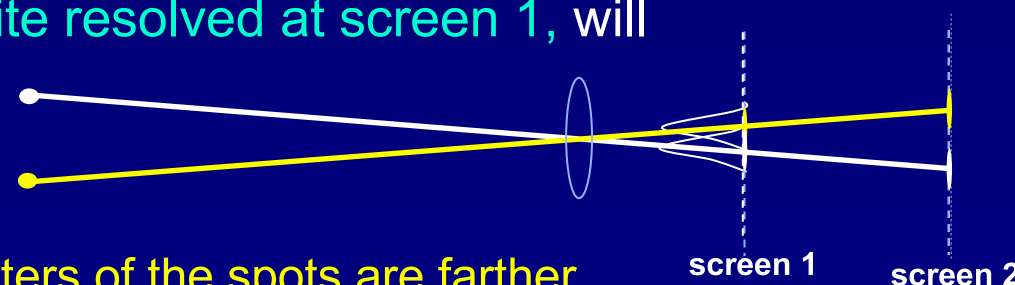


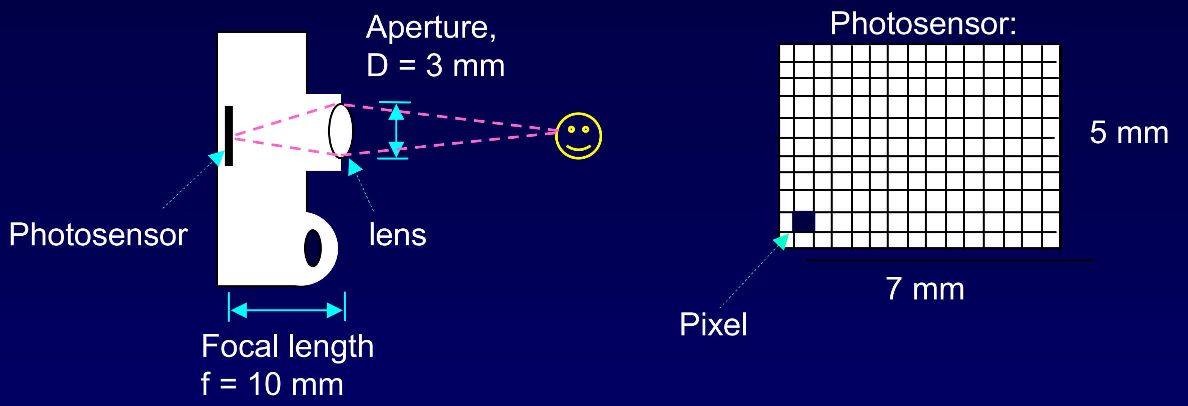

a. Yes   b. No

$\alpha_c$ only depends on $\lambda$ and D. The centers of the spots are farther apart, but the spots are also wider by the same amount.

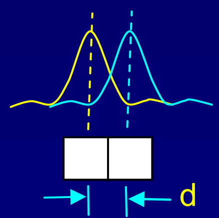
# Example: Camera resolution

## (Next week's discussion)

Digital cameras look something like this:




If the distance between adjacent pixels is less than the minimum resolvable separation due to diffraction, then diffraction limits the image quality.




The "f-number" of a lens is defined as f/D. To minimize diffraction, you want a small f-number, i.e., a large aperture*.

*This assumes a 'perfect lens'. In practice, lens aberrations limit the resolution if D is too big.

# Optical Interferometers

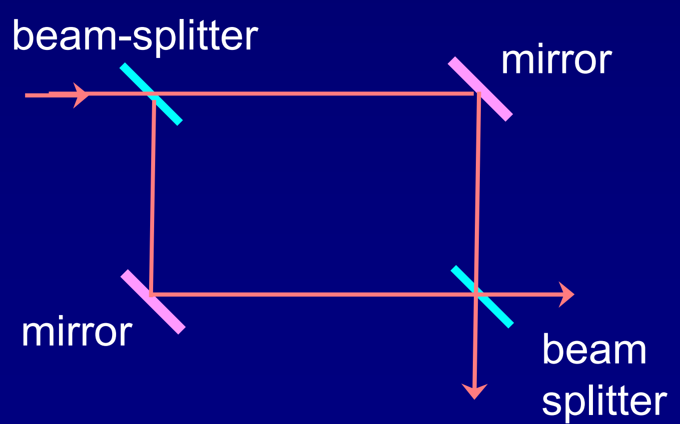
Interference arises whenever there are two (or more) ways for something to happen, *e.g.*, two slits for the light to get from the source to the screen.

$I = 4I_1\cos^2(\phi/2)$, with $\phi = 2\pi\delta/\lambda$, and path-length difference $\delta$

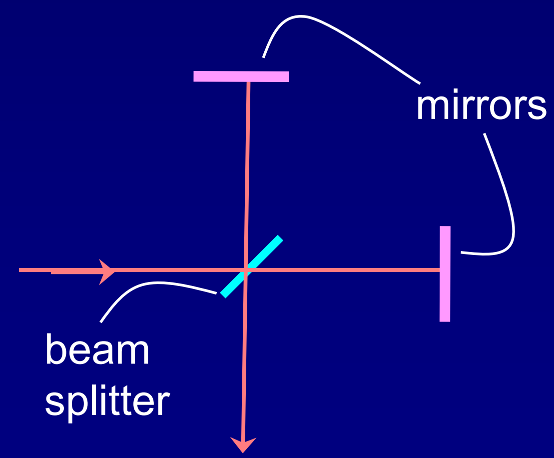
An interferometer is a device using mirrors and "beam splitters" (half of the light is transmitted, half is reflected) to give two separate paths from source to detector.

Two common types:

Mach-Zehnder:




Michelson :

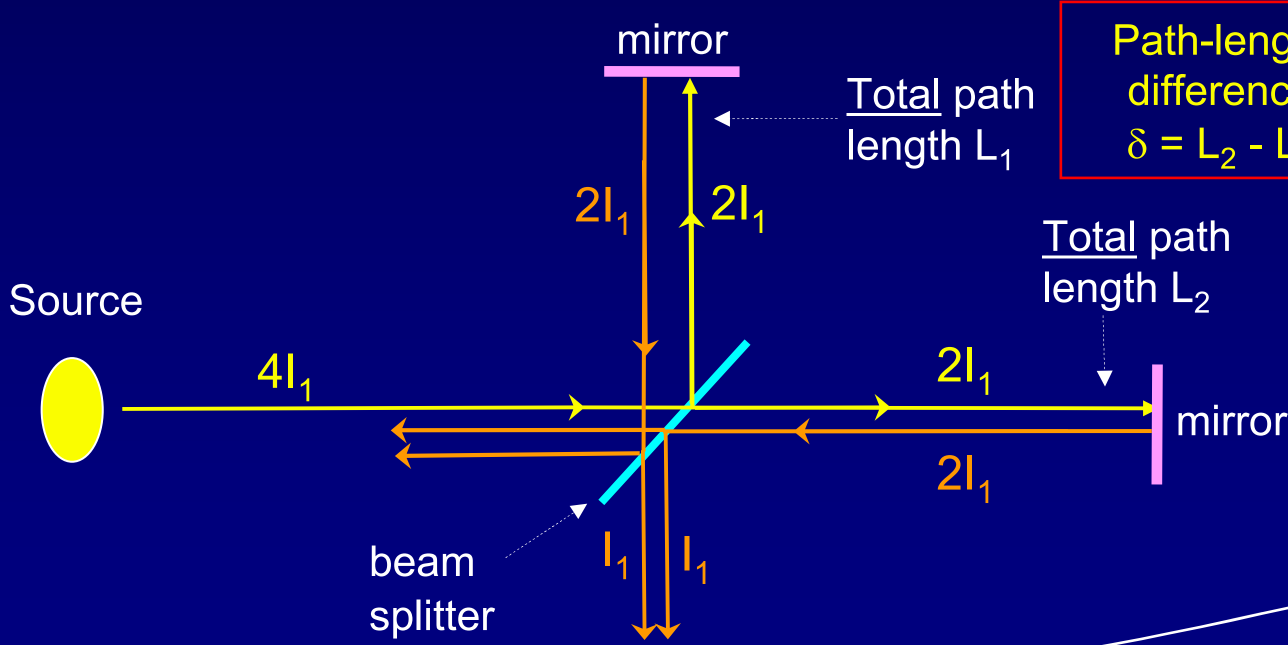


# Michelson Interferometer

The Michelson interferometer works by varying the phase difference between the two paths the light can take.

One possibility is to vary the lengths $L_1$ or $L_2$.

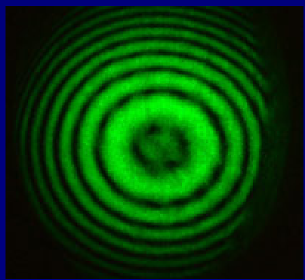
This makes possible very accurate measurements of displacements.




Path-length difference
$\delta = L_2 - L_1$

Be careful !!

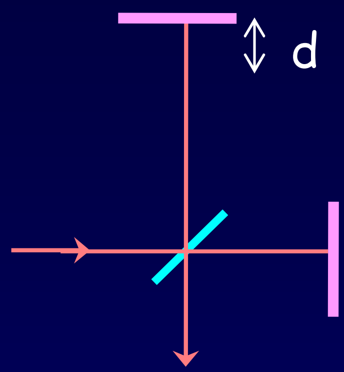
What's important is the total time to get back to the beam splitter...



$$I = 4I_1 \cos^2(\phi/2), \text{ with } \phi = 2\pi\,\delta/\lambda$$

# ACT 4

Consider the following Michelson interferometer. Suppose that for the setup shown, all the light (with $\lambda$ = 500 nm) comes out the bottom port.

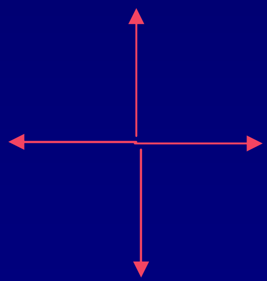



1. How much does the top mirror need to be moved so that none of the light comes out the bottom port?

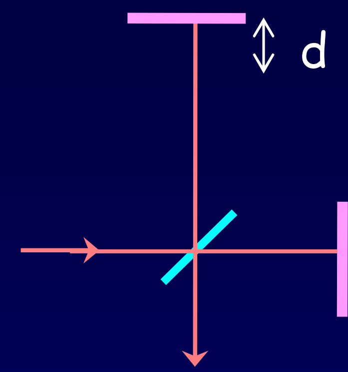
   a. 125 nm    b. 250 nm    c. 500 nm

2. Where does the light then go?

   a. down    b. up    c. left    d. right



# Solution




Consider the following Michelson interferometer. Suppose that for the setup shown, all the light (with $\lambda$ = 500 nm) comes out the bottom port.

1. How much does the top mirror need to be moved so that none of the light comes out the bottom port?

   a. 125 nm　　b. 250 nm　　c. 500 nm

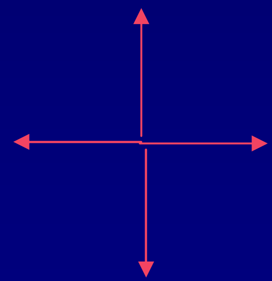
   We need to go from complete constructive to complete destructive interference → $\Delta\phi = 180^\circ$ → $\delta = \lambda/2$.

   However…when we move the mirror by d, we change $\delta$ by 2d.

   Therefore, $d = \delta/2 = \lambda/4 = 500/4 = 125$ nm.

2. Where does the light then go?

   a. down　　b. up　　c. left　　d. right



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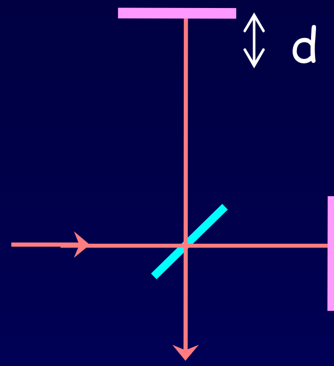


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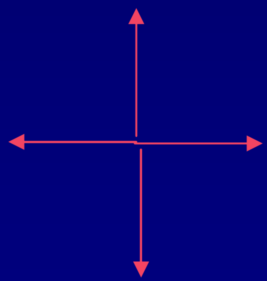


   The light goes out the way it came in.
   Energy is conserved --the light can't just disappear!

The Michelson interferometer is perhaps most famous for disproving the hypothesis that EM waves propagate through an "aether" – this result helped stimulate the Special Theory of Relativity

# Michelson Interferometer

Another possibility is to vary the phase by changing the speed of the waves in the two arms.

Recall v=c/n where n = index of refraction.

Using $\lambda = v/f$, the number of wavelengths in arm 1 is:

$$N_1 = \frac{L}{\lambda_1} = \frac{n_1 f L}{c}$$

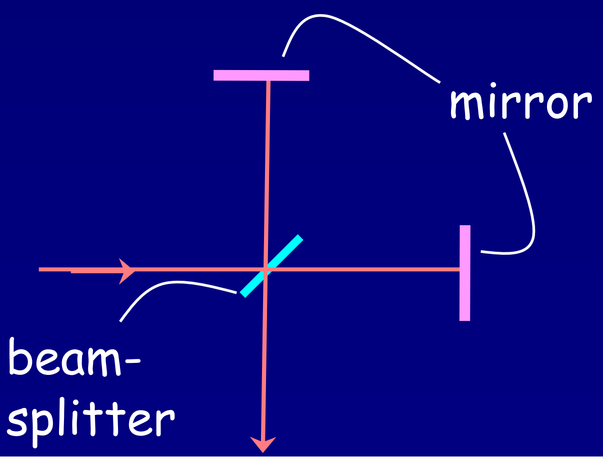
and similarly for arm 2.

(You can think of it as the path being longer by n.)

The phase difference is thus:

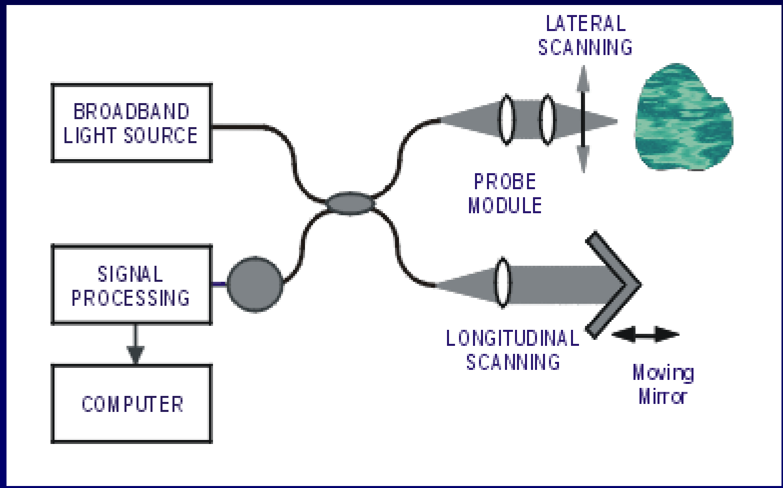
$$\phi = 2\pi(N_1 - N_2) = 2\pi(fL/c)(n_1 - n_2)$$

This makes possible very accurate measurement of changes in the speed of light in the two arms.



# FYI: Application
# Optical Coherence Tomography

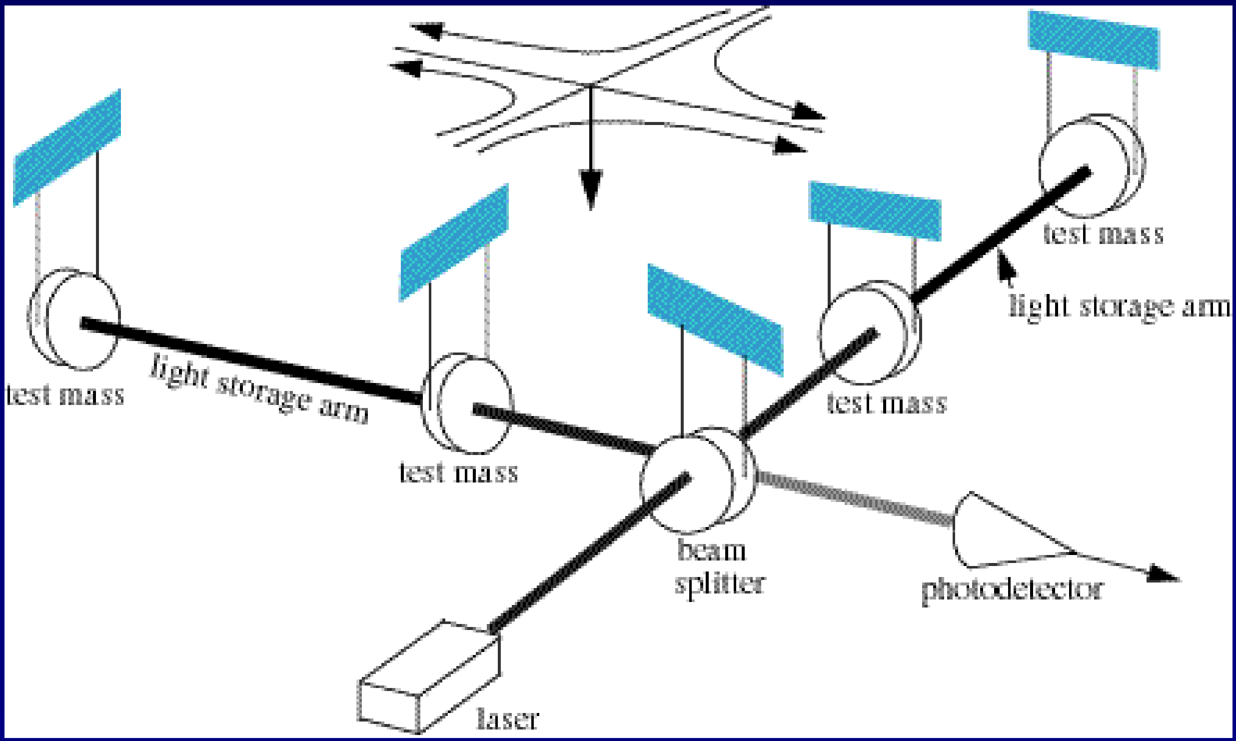
- One "mirror" of the Michelson is replaced by human tissue. The type of tissue controls the amount of reflection and the phase shift.
- By sending in many colors, one can learn about the density, composition, and structure of the tissue.
- Used for medical diagnostics – like a microscope, but you don't have to excise the sample from the body!
- Used to study
  - skin cancer
  - cardiovascular disease (detect bad plaques)
  - glaucoma and macular degeneration (incurable eye disease)



# FYI: Gravitational Wave Detection

General relativity predicts that when massive objects accelerate, they produce time-dependent gravitational fields – gravitational waves – that propagate as "warpings" of spacetime at the speed of light. (similar to EM radiation from accelerated charge)

The effect is very tiny: E.g., estimated $\Delta L/L$ of $\sim 10^{-21}$ for in-spiraling binary neutron stars. How to detect this???



# FYI: Application: Gravity Wave Detection

LIGO:

Laser

Interferometric

Gravitational wave

Observatory

-World's largest interferometers: 4-km

-2 in Hanford, WA; 1 in Livingston, LO

- >500 scientists

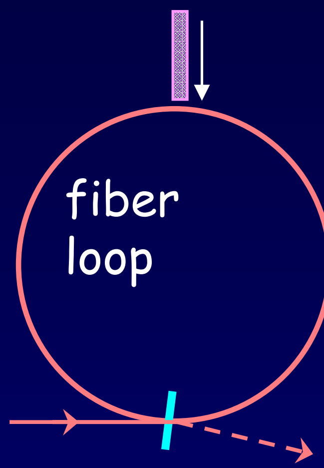
-Achieved sensitivity $\Delta L/L \sim 10^{-23} \rightarrow \Delta L \sim 10^{-20}$ m

-Six data runs completed.

-"Advanced LIGO" should improve sensitivity by another 10x.

# FYI: Modern Applications in Navigation

Consider the following "Sagnac" ["sahn-yack"] interferometer. Here the two possible paths are the clockwise and counter-clockwise circuits around the fiber loop.




1. If we insert an extra piece of glass as shown, how does the relative path length change?

2. How could we change the relative path-length difference, and thereby change how much light exits the bottom port?

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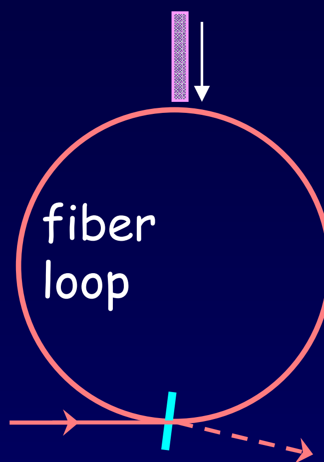


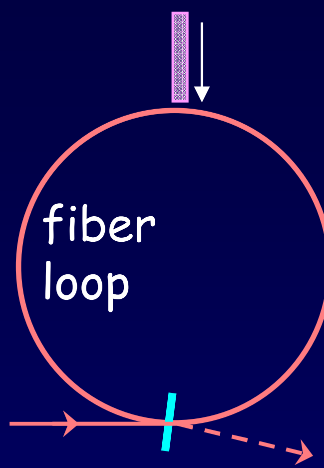

1. If we insert an extra piece of glass as shown, how does the relative path length change?

   It doesn't! Because the interference paths completely overlap, the Sagnac is a remarkably stable interferometer, e.g., to temperature fluctuations in the fiber.

2. How could we change the relative path-length difference, and thereby change how much light exits the bottom port?

# FYI: Modern Applications in Navigation

Consider the following “Sagnac” [“sahn-yack”] interferometer. Here the two possible paths are the clockwise and counter-clockwise circuits around the fiber loop.




1. If we insert an extra piece of glass as shown, how does the relative path length change?

It doesn’t! Because the interference paths completely overlap, the Sagnac is a remarkably stable interferometer, e.g., to temperature fluctuations in the fiber.

2. How could we change the relative path-length difference, and thereby change how much light exits the bottom port?

Rotate the entire interferometer (in the plane of the paper). For example, if we rotate it clockwise, the light making the clockwise circuit will have farther to go (the beamsplitter is “running away”), while the counterclockwise path will be shortened.

It is not difficult to show that

$$\phi \approx \frac{2\pi}{\lambda}\frac{4\left(\pi R^2\right)\omega}{c^2}$$

Monitor output intensity → determine $\phi$ → rate of rotation $\omega$ → “laser ring gyroscope”!

# Next Week

Introduction to quantum mechanics

Photoelectric effect

Relation between energy and frequency of a photon

Photon momentum

The key relations of quantum mechanics

Wave-particle duality

# FYI: Thin Films!

- Why do soap bubbles appear colored?  Oil films on water?
- Interference -- light reflected from the front and back surfaces interferes.
- However, light that reflects off a *higher-index* layer gets an extra $\pi$ phase-shift (from Maxwell's equations).
- For a film of thickness d, viewed at an angle $\theta$, the path length difference is $\delta = 2d\sin\theta$ and the phase difference between the light reflected from the front and back surfaces is $\phi = 2\pi\delta/\lambda + \pi$.

Destructive interference:

$$2d\sin\theta = m\lambda$$

Constructive interference:

$$2d\sin\theta = (m+1/2)\lambda$$