Einstein's paper is a "bold, not to say reckless, hypothesis…which flies in the face of thoroughly established facts of interference."

--R. Millikan

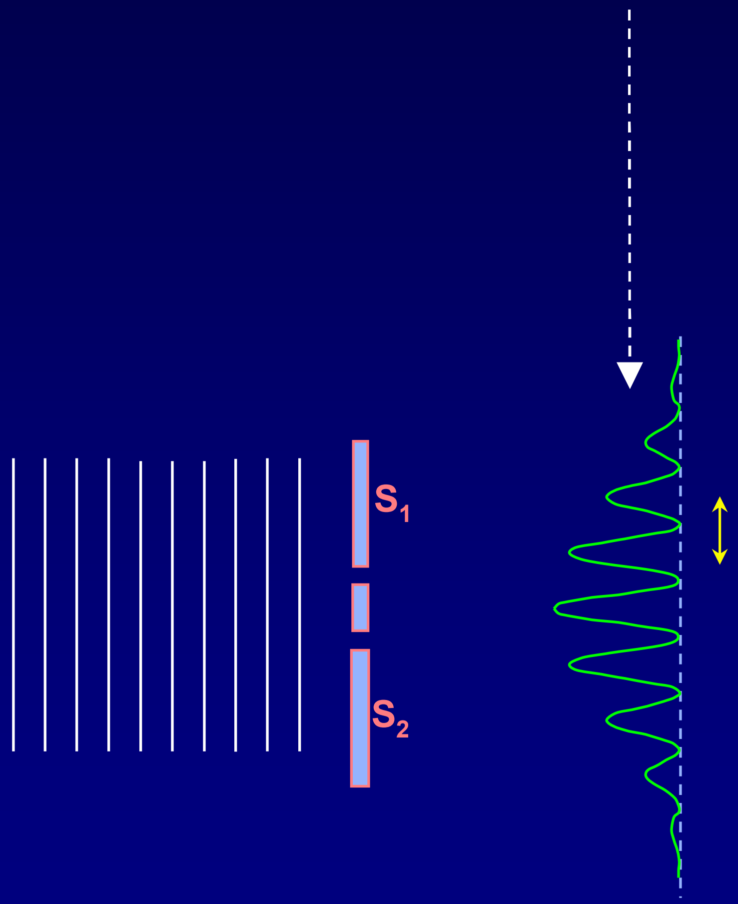
(discover of electric charge)

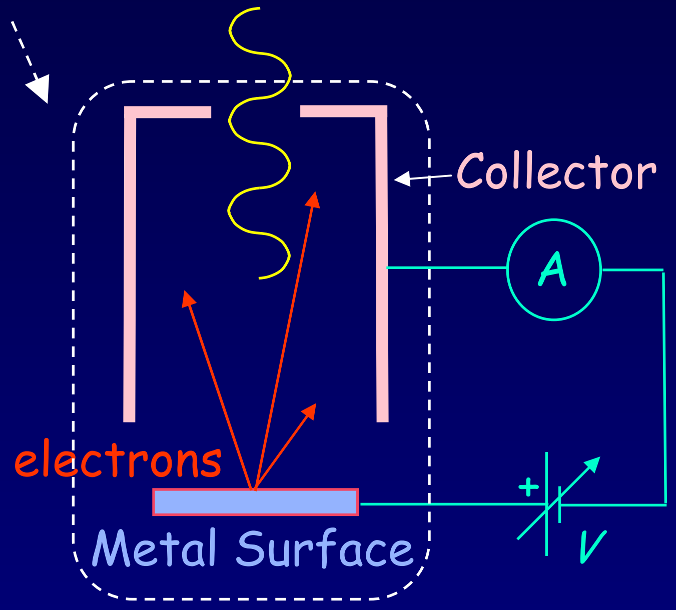
"Its unambiguous experimental verification in spite of its unreasonableness since it seems to violate everything that we knew about the interference of light."

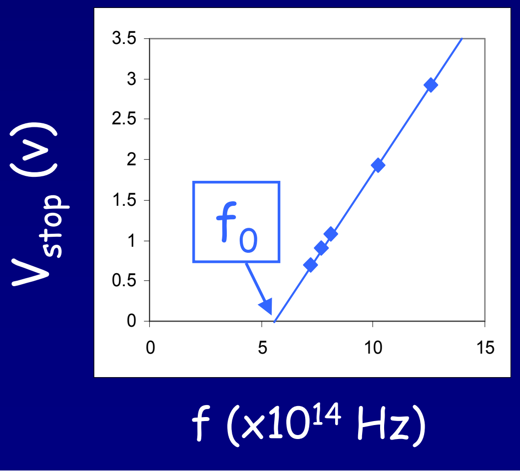
--R. Millikan

(after testing Einstein's photoelectric effect predictions)

Lecture 7:
Introduction to Quantum Mechanics:
Light as Particles







This week and next are critical for the course:

Week 3, Lectures 7-9:

- Light as Particles
- Particles as waves
- Probability
- Uncertainty Principle

Week 4, Lectures 10-12:

- Schrödinger Equation
- Particles in infinite wells, finite wells

Midterm Exam Monday, Sep. 24.

It will cover lectures 1-12.

Practice exams: Old exams are linked from the course web page.
Review Sunday, Sep. 23, 3-5 PM in 141 Loomis.
Office hours: Sep. 23 and 24.

Thumbnail Summary of Waves

Wave relationships:

Wavelength, frequency, speed, amplitude, intensity

$v = f\lambda$, $I = A^2$, *etc*.

2-slit interference:

Phase difference depends on source phases and path lengths.

$A_{tot} = 2A_1\cos(\phi/2)$, *etc*.

N-slit interference:

Diffraction gratings, Rayleigh's criterion.

1-slit diffraction:

Circular apertures, Rayleigh's criterion, limits on optics.

Interferometers.

We'll use many of these results when we study quantum mechanics.

Crystal diffraction

How do we know the atomic scale structure of matter around us?

A crystal is a very large number of atoms or molecules arranged in a periodic fashion. It acts like a grating with an extremely large number (~Avagadro's number) of units that diffract waves coherently.

Every crystal has its own "signature" of the various spacings between atoms.

By measuring the diffraction, we can determine the atomic scale structure.

Typical distances between atoms are of order 0.1-0.3 nm ($1\text{-}3\times10^{-10}$ m).
What characteristic wavelengths are needed to study crystals?

We want:

- λ < spacing (so that we can get $\delta > \lambda$).
- λ not too small (so that θ isn't too small).

That is: $\lambda \sim 10^{-10}$m. $\Rightarrow$ **X-rays!**

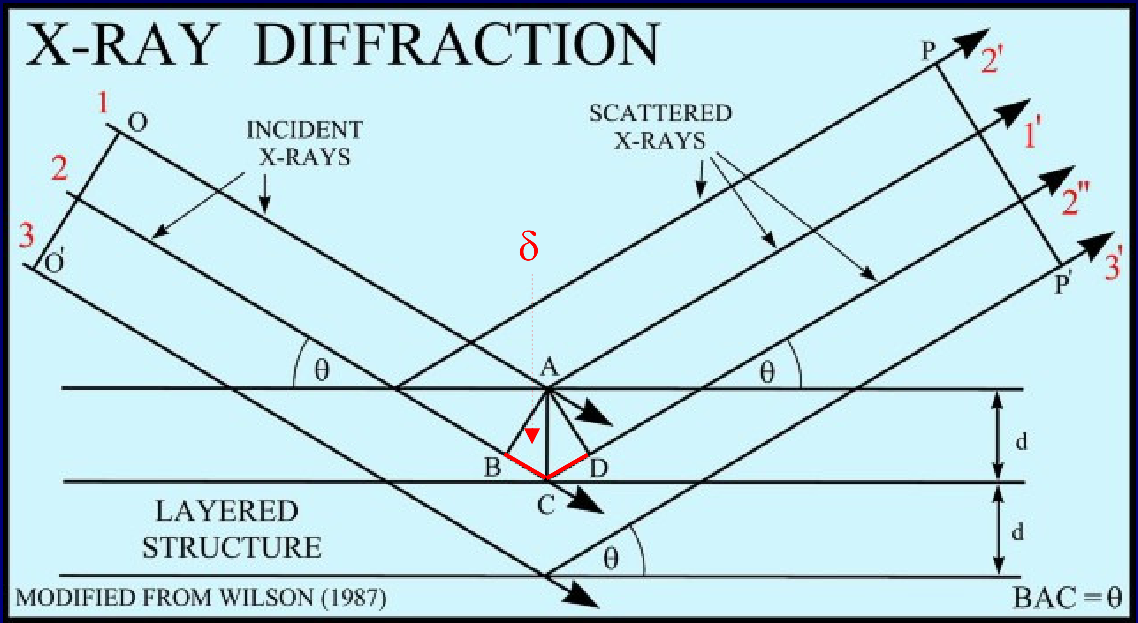
FYI: Crystal diffraction

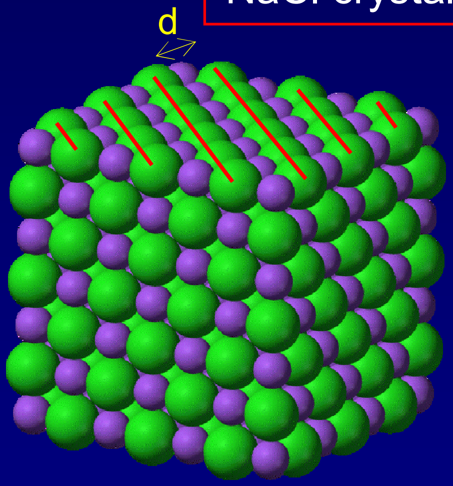
The structure of the crystal can be found using almost the same law we have for optical gratings!

Bragg Law for constructive interference: $\delta = 2d \sin\theta = m\lambda$

d = lattice spacing, λ = x-ray wavelength

θ = x-ray angle (with respect to plane of crystal)





Example of planes in a NaCl crystal

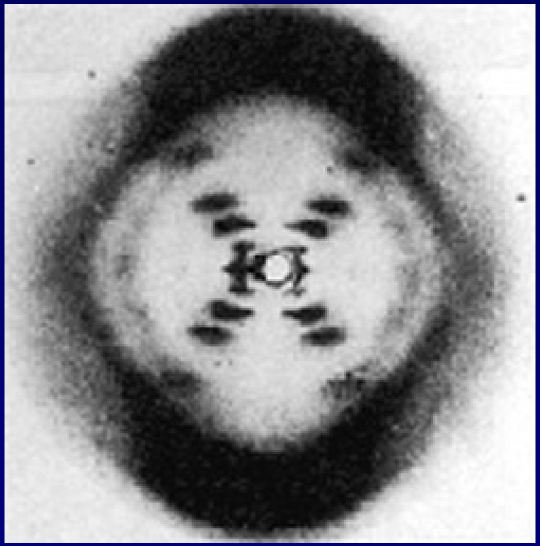
Each crystal has many values of d - the distances between different planes. For a known wavelength λ the observed angles θ can be used to determine the crystal structure.

FYI: Application: Structure of DNA

How do we know the structures of DNA, proteins and other biological molecules? X-ray diffraction!

The molecules are crystallized to create a crystal in which the molecules are arranged in a periodic lattice. By using the sharp Bragg diffraction from many molecules, the structure of each molecule is determined - the positions of thousands of atoms

The original diffraction image of DNA taken by Rosalind Franklin in 1953



Actually the DNA was not crystallized for the first DNA images, but the DNA was dehydrated in a fiber, and formed a “quasi-crystalline structure that showed the helical structure.

The dark bands arranged in a cross were the first evidence of the helical structure.

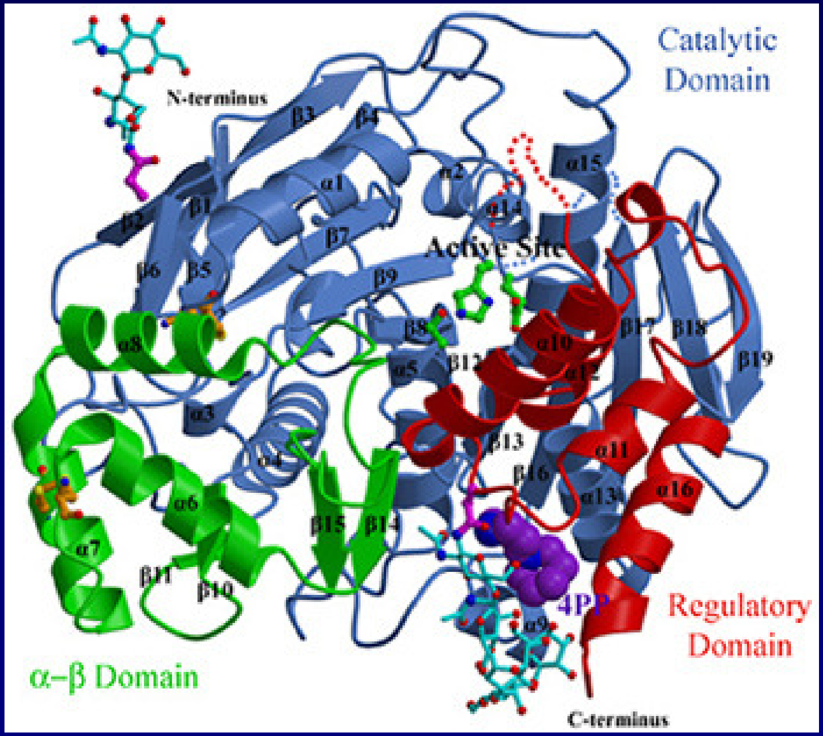
See figure and more details on this and other X-ray scattering in the text, sec. 36.6

FYI: Modern Applications in Biology

X-rays remain the primary methods for establishing the atomic scale structures of complex molecules.

Example: Rabbit liver carboxylesterase (one molecule showing atomic groups and attached large scale structures with atoms not shown).

“Alternative strategies to improve the antitumor efficacy have concentrated upon the design of novel camptothecin analogs. To effect this, *we have determined the x-ray crystal structures* of the rabbit liver carboxylesterase …”



Source: St. Jude Children Research Hospital
http://www.stjude.org/faculty/0,2512,407_2030_4278,00.html

Today

Photoelectric Effect:

Light as particles

Photons - Quanta of electromagnetic waves

The fundamental QM relations:

Energy: $E = hf$ or $E = \hbar\omega$ ($\hbar = h/2\pi$ and $\omega = 2\pi f$)

Momentum: $p = h/\lambda$

These equations relate the wave and particle properties of all quantum mechanical entities.

Reading in the text: 38.1-2,9 and 39.1-5

Note: All reading assignments are listed on the syllabus page.

Wave-particle Duality for Light and Matter

In Physics 212 and the first 4 Lectures of Physics 214, we considered "light" to be a wave.

This was established by experiment in the 19th century (cf. Poisson spot) Electromagnetic waves exhibit interference and diffraction.

Surprise:

In the early 20th century, it was discovered that light has particle-like properties (*e.g.*, localized lumps of energy) in some situations!

Furthermore, matter exhibits wave-like properties (*e.g.*, electrons, protons, *etc.*) under certain circumstances.

It may seem surprising that an entity might exhibit both "wave-like" and "particle-like" properties!

Let's look at some of the evidence.

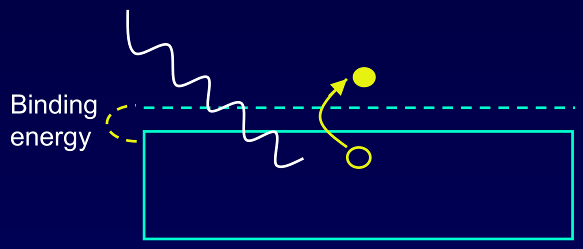
Photoelectric Effect (1)

Not phase difference!

Electrons in a metal are bound by an energy Φ, called the work function.

If you shine light on a clean metal surface, electrons can emerge → the light gives the electrons enough energy (> Φ) to escape.

Measure the flow of electrons with an ammeter.



Φ is the minimum energy needed to liberate an electron from the metal.

Φ is defined to be positive.

How will the current depend on intensity and frequency?

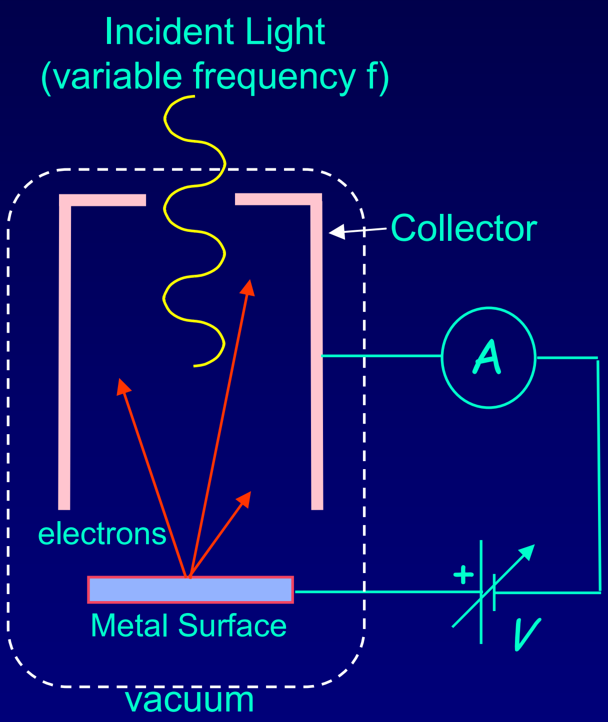
We might expect:

- Increasing the intensity should increase the current. Or maybe the energy of the electrons.
- Increasing the frequency shouldn't matter much. Perhaps a decrease in current due to rapid oscillations.
- With low intensity, there should be a time delay before current starts to flow, to build up enough energy.

This follows from the idea that light is a continuous wave that consists of an oscillating E and B field. The intensity is proportional to E^2.

Photoelectric Effect (2)

Experiment 1: Measure the maximum energy (KE_{max}) of ejected electrons



Bias the collector with a negative voltage to repel ejected electrons.

Increase bias voltage until flow of ejected electrons decreases to zero.
Current = 0 at $V = V_{stop}$. (the definition of V_{stop})

V_{stop} tells us the maximum kinetic energy:

$$\boxed{KE_{max} = eV_{stop}}$$

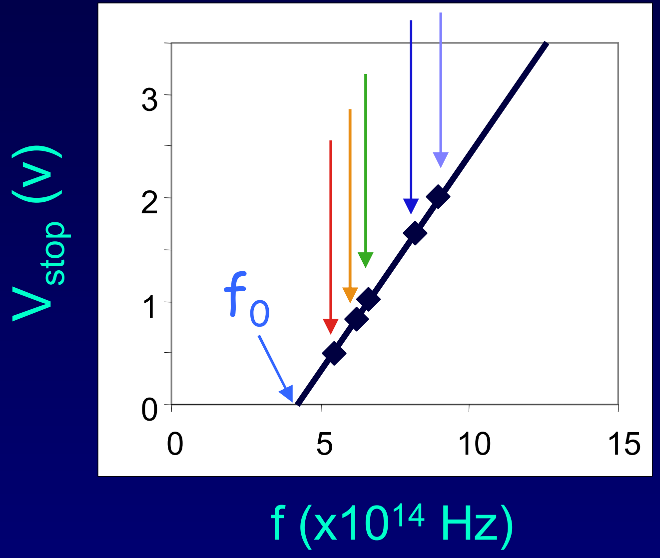
The result:

The stopping voltage is independent of light intensity.

Therefore, increasing the intensity does <u>not</u> increase the electron's KE!

Photoelectric Effect (3)

Experiment 2: Measure V_{stop} *vs* f



$$\boxed{KE_{max} = e \cdot V_{stop} = h(f - f_0) = hf - \Phi}$$

The slope: h, is Planck's constant.

$h = 6.626 \times 10^{-34}$ J•s

The -y intercept: Φ, is the work function.

Note that $\Phi = hf_0$. Φ is positive.

The results:

The stopping voltage V_{stop} (the maximum kinetic energy of electrons) increases linearly with frequency.

Below a certain frequency f_o, <u>no</u> electrons are emitted, even for intense light! This makes no sense classically. Increasing the electric field should have an effect.

Photoelectric Effect (4)

Summary of Results:

- Electron energy depends on frequency, not intensity.
- Electrons are not ejected for frequencies below f_0.
- Electrons have a probability to be emitted immediately.

Conclusions:

- Light arrives in “packets” of energy (photons).
- $E_{photon} = hf$ ←

We will see that this is valid for all objects. It is the fundamental QM connection between an object’s wave and particle properties.

- Increasing the power increases # photons, not the photon energy. Each photon ejects (at most) one electron from the metal.

Recall: For EM waves, frequency and wavelength are related by: $f = c/\lambda$.
Therefore: $E_{photon} = hc/\lambda$

Beware: This is only valid for EM waves,
as evidenced by the fact that the speed is c.

Convenient Units for Quantum Mechanics

Because most of the applications we will consider involve atoms,
it is useful to use units appropriate to those objects.

We will express wavelength in nanometers (nm).

We will express energy in electron volts (eV).

1 eV = energy an electron gains moving across a one volt potential difference:
1 eV = (1.6022 x 10^{-19} Coulomb)(1 volt) = 1.6022 x 10^{-19} Joules.

Therefore, SI units: h = 6.626 x 10^{-34} J-s and hc = 1.986 x 10^{-25} J-m
eV units: h = 4.14 x 10^{-15} eV-s, and hc = 1240 eV-nm.

$$\boxed{E_{photon} = \frac{hc}{\lambda} = \frac{1240\ eV \cdot nm}{\lambda}}$$

E_{photon} in electron volts

λ in nanometers

Example: A red photon with λ = 620 nm has E = 2 eV.

Photoelectric Effect Example

1. When light of wavelength λ = 400 nm shines on lithium, the stopping voltage of the electrons is V_{stop} = 0.21 V. What is the work function of lithium?

2. What is the maximum wavelength that can cause the photoelectric effect in lithium? Hint: What is V_{stop} at the maximum wavelength (minimum frequency)?

Work out yourself

Answer: l_{max} = 429 nm

Photoelectric Effect: Solution

1. When light of wavelength λ = 400 nm shines on lithium, the stopping voltage of the electrons is V_{stop} = 0.21 V. What is the work function of lithium?

Φ = hf - eV_{stop}
= 3.1eV - 0.21eV
= 2.89 eV

Instead of hf, use hc/λ: 1240/400 = 3.1 eV

For V_{stop} = 0.21 V, eV_{stop} = 0.21 eV

2. What is the maximum wavelength that can cause the photoelectric effect in lithium? Hint: What is V_{stop} at the maximum wavelength (minimum frequency)?

V_{stop} = 0, so E = Φ = 2.89 eV.
Use E = hc / λ .
So, λ = hc / E = 1240 eV·nm / 2.89 eV = 429 nm

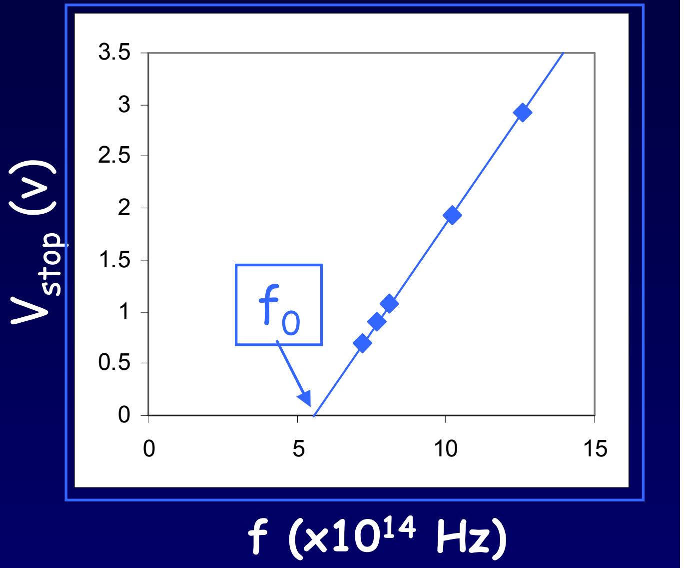
Act 1: Work Function

- Calculating the work function Φ.

$$KE_{max} = e \cdot V_{stop} = hf - \Phi$$

If $f_0 = 5.5 \times 10^{14}$ Hz, what is Φ?
(h = 4.14×10^{-15} eV•s)

a) -1.3 V   b) -5.5 eV   c) +2.3 eV



Act 1: Work Function - Solution

- Calculating the work function Φ?

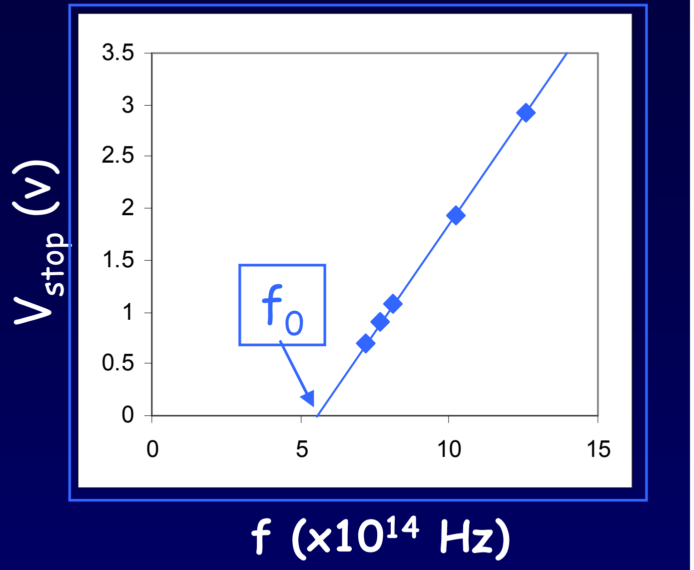
$$KE_{max} = e \cdot V_{stop} = hf - \Phi$$

If $f_0 = 5.5 \times 10^{14}$ Hz, what is Φ?
($h = 4.14 \times 10^{-15}$ eV•s)

a) -1.3 V

b) -5.5 eV

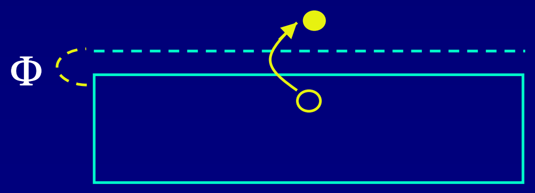
[✓ c) +2.3 eV]



When $V_{stop} = 0$, $hf_0 = \Phi = 4.1 \times 10^{-15}$ eV•s x 5.5×10^{14} Hz = 2.3 eV

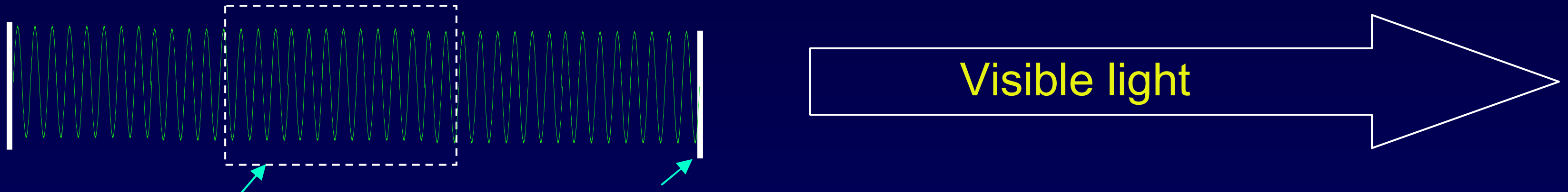
Physical interpretation of the work function:

- Φ is the *minimum* energy needed to strip an electron from the metal.
- Φ is defined as <u>positive</u> and is usually given in eV units.
- Not all electrons will leave with the maximum kinetic energy (due to losses)



Discrete *vs* Continuous

Can we reconcile the notion that light comes in ‘packets’ with our view of an electromagnetic wave, *e.g.*, from a laser?



Visible light

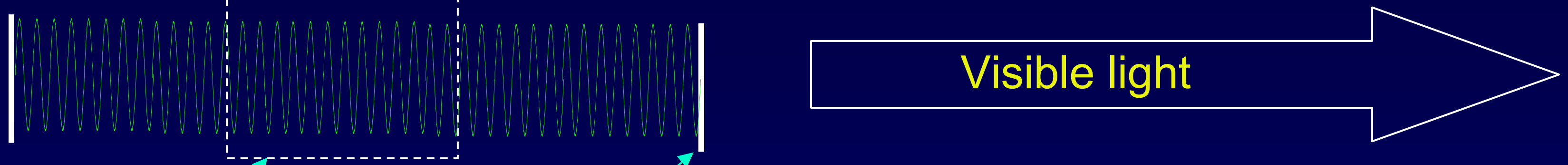
Power input

Partially transmitting mirror

How many photons per second are emitted from a 1-mW laser (λ=635 nm)?

Solution

Can we reconcile the notion that light comes in ‘packets’ with our view of an electromagnetic wave, *e.g.*, from a laser?



Visible light

Power input

Partially transmitting mirror

How many photons per second are emitted from a 1-mW laser (λ=635 nm)?

$$E_{photon} = \frac{hc}{\lambda} = \frac{1240\,\text{eV-nm}}{635\,\text{nm}} \approx 2\,\text{eV}$$

Power output: P = (# photons/sec) x E_{photon}

$$\#\text{ photons/sec} = \frac{P}{E_{photon}} = \frac{10^{-3}\,\text{J}}{\text{s}} \times \frac{1\text{eV}}{1.6\times10^{-19}\,\text{J}} \times \frac{1\text{photon}}{2\,\text{eV}} = 3\times10^{15}\,\text{s}^{-1}$$

This is an incredibly huge number. Your eye cannot resolve photons arriving every femtosecond (though the rods can detect single photons!).

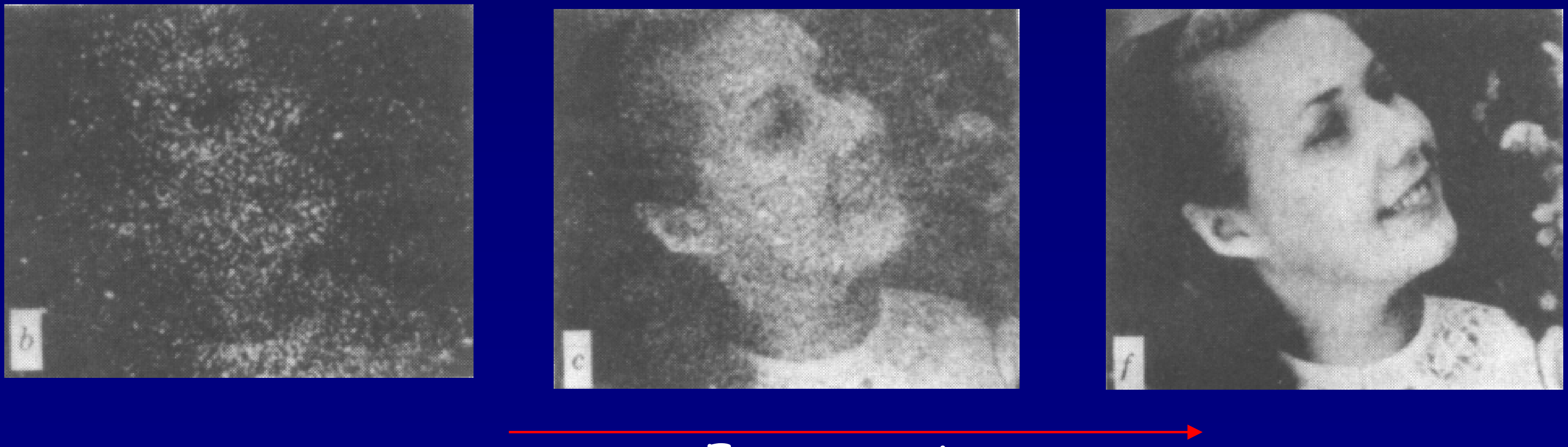
Formation of Optical Images

The point:
Processes that seem to be continuous may, in fact, consist of many microscopic “bits”.
(Just like water flow.)

For large light intensities, image formation by an optical system can be described by classical optics.

For very low light intensities, one can see the statistical and random nature of image formation. Use a sensitive camera that can detect single photons.

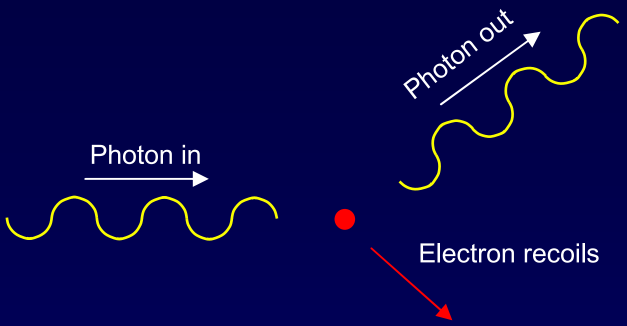
A. Rose, J. Opt. Sci. Am. 43, 715 (1953)



Exposure time

Momentum of a Photon (1)

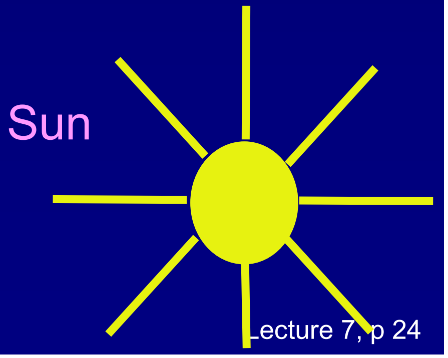
Between 1919 and 1923, A.H. Compton showed that x-ray photons collide elastically with electrons in the same way that two particles would elastically collide! “Compton Scattering”



Comet’s tail

Photons carry momentum!
Perhaps this shouldn’t surprise us:
Maxwell’s equations also predict
that light waves have $p = E/c$.

Why not $E_{photon} = p^2/2m$? (Physics 211) Because photons have no mass.
$E_{photon} = pc$ comes from special relativity, which more generally says $E^2 = m^2c^4 + p^2c^2$. For the photon, $m = 0$.



Momentum of a Photon (2)

What is the momentum of a photon?

Combine the two equations:

$E_{photon} = hf = hc/\lambda$ – quantum mechanics

$p = E/c$ – Maxwell's equations, or special relativity

This leads to the relation between momentum and wavelength:

$p_{photon} = hf/c = h/\lambda$

These are the key relations of quantum mechanics:

$$\boxed{\begin{array}{l} E = hf \\ p = h/\lambda \end{array}}$$

They relate an object's particle properties (energy and momentum) to its wave properties (frequency and wavelength).

Remember:
$E = hc/\lambda$
$p = hf/c$
are only valid for photons

So far, we discussed the relations only for light.

But they hold for all matter! We'll discuss this next lecture.

Wave-Particle "Duality"

Light sometimes exhibits wave-like properties (interference),
and sometimes exhibits particle-like properties (trajectories).

We will soon see that matter particles (electrons, protons, etc.)
also display both particle-like and wave-like properties!

An important question:

When should we expect to observe wave-like properties,
and when should we expect particle-like properties?

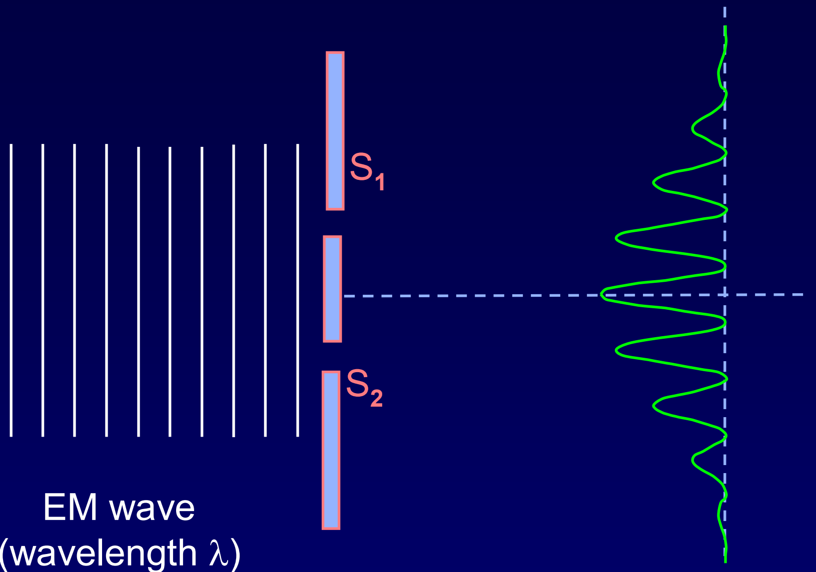
To help answer this question, let's reconsider the 2-slit experiment.

2-slits Revisited (1)

Recall 2-slit interference:

We analyzed it this way

(Wave view):

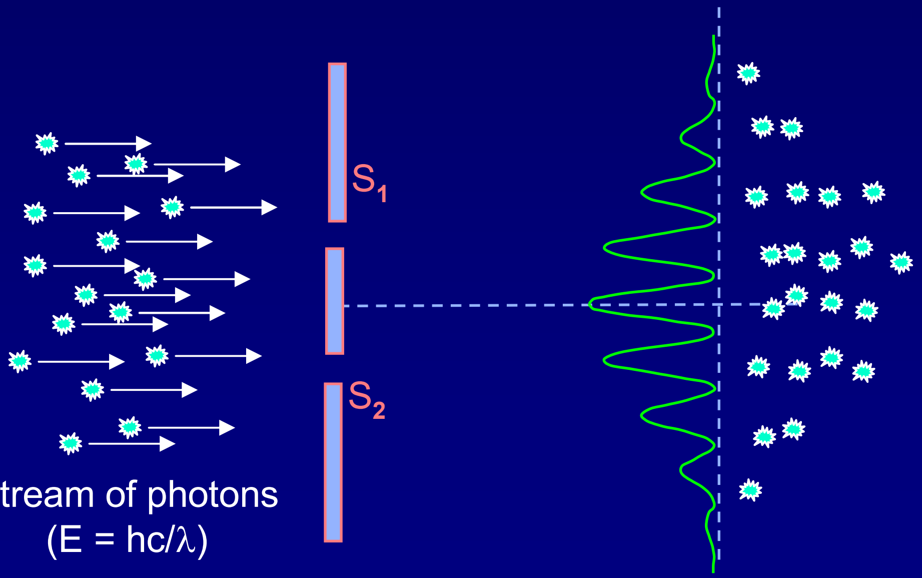


EM wave
(wavelength λ)

Waves interfere, creating intensity maxima and minima.

Can we also analyze it this way?

(Particle view):



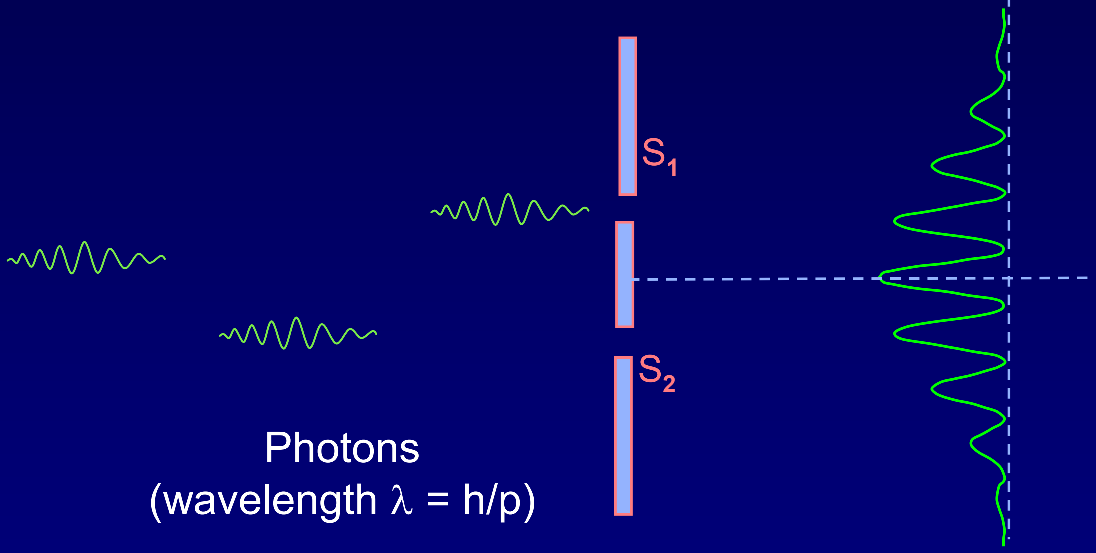
Stream of photons
($E = hc/\lambda$)

Photons hit the screen at discrete points.

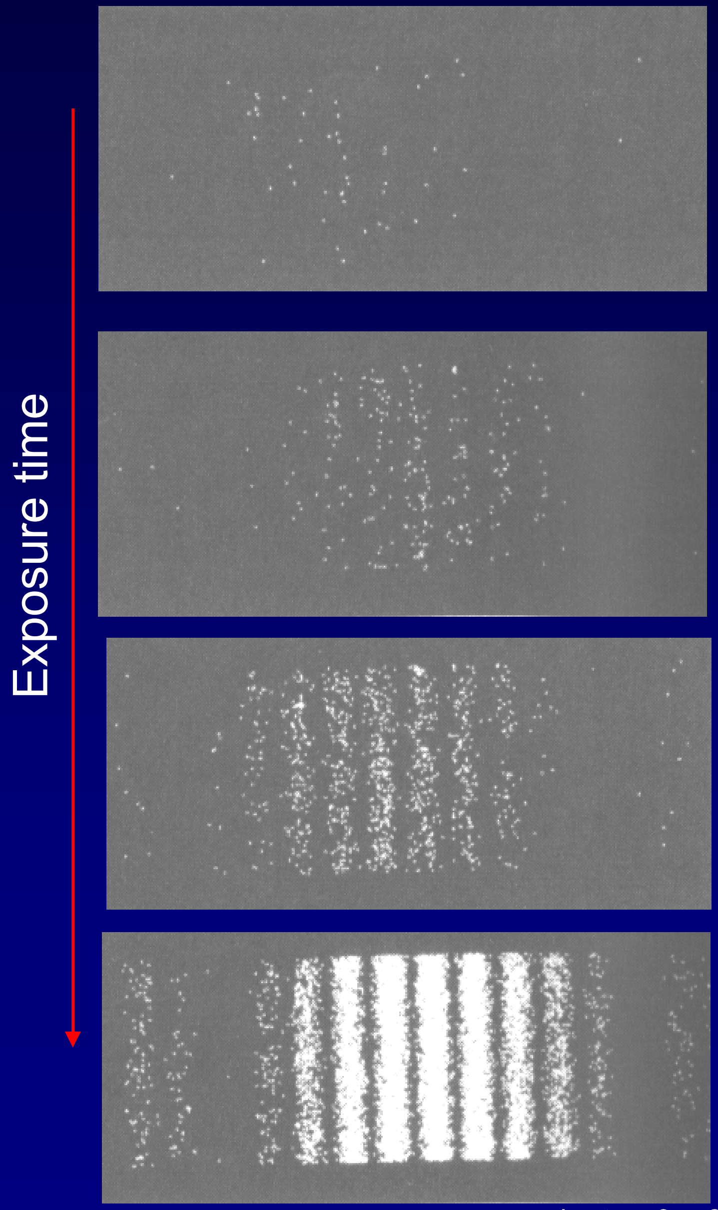
How can particles yield an interference pattern?

2-slits Revisited (2)

It's just like the formation of a photographic image. More photons hit the screen at the intensity maxima.



Photons
(wavelength $\lambda = h/p$)



The big question …

What determines where an individual photon hits the screen?

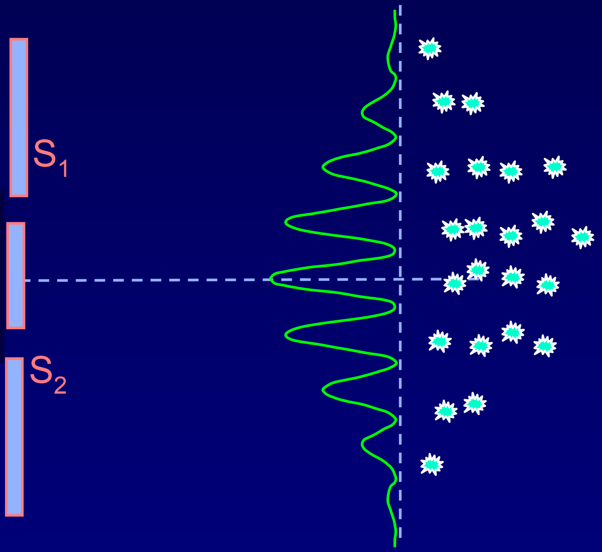
2-slits Revisited (3)

The quantum answer:

The intensity of the wave pattern describes the probability of arrival of quanta. The wave itself is a "probability amplitude", usually written as ψ.

Light consists of quantum "entities".

(neither waves nor particles)



One observes a random arrival of photons.

Randomness is intrinsic to QM.

Quantum mechanical entities are neither particles nor waves separately, but both simultaneously. Which properties you observe depends on what you measure.

Very large number of quanta $\Rightarrow$ classical wave pattern

2-slits Revisited (4)

Hold on! This is kind of weird!

How do we get an interference pattern from single "particles" going through the slits one at a time?

Q: Doesn't the photon have to go through either slit 1 or slit 2?
A: No! Not unless we actually measure which slit !

The experimental situation:

- With only one slit open: You get arrival pattern P_1 or P_2 (see next slide).

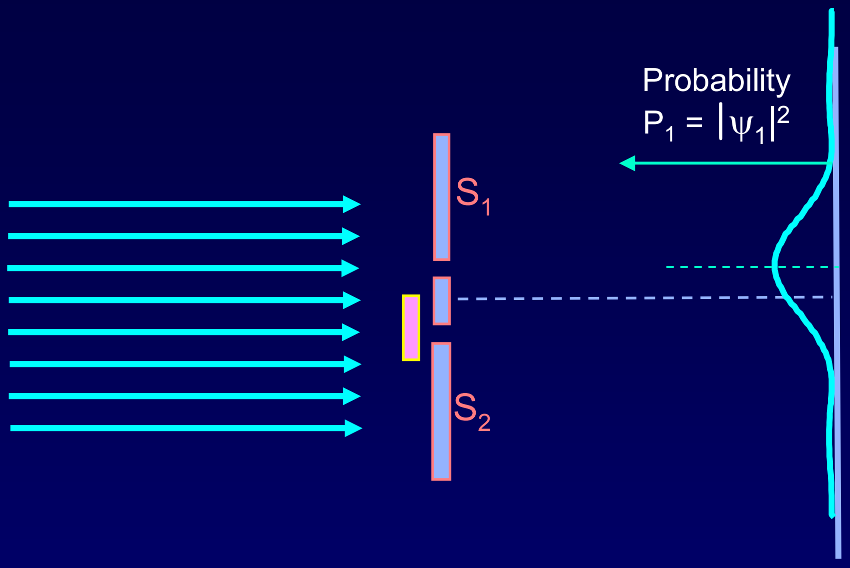
- With both slits open:
- If something 'measures' which slit the photon goes through, there is no interference: $P_{tot} = P_1 + P_2$.
- If nothing 'measures' which slit the light goes through, P_{tot} shows interference, as if the photon goes through both slits!

Each individual photon exhibits wave behavior!

QM waves are not a collective phenomenon.

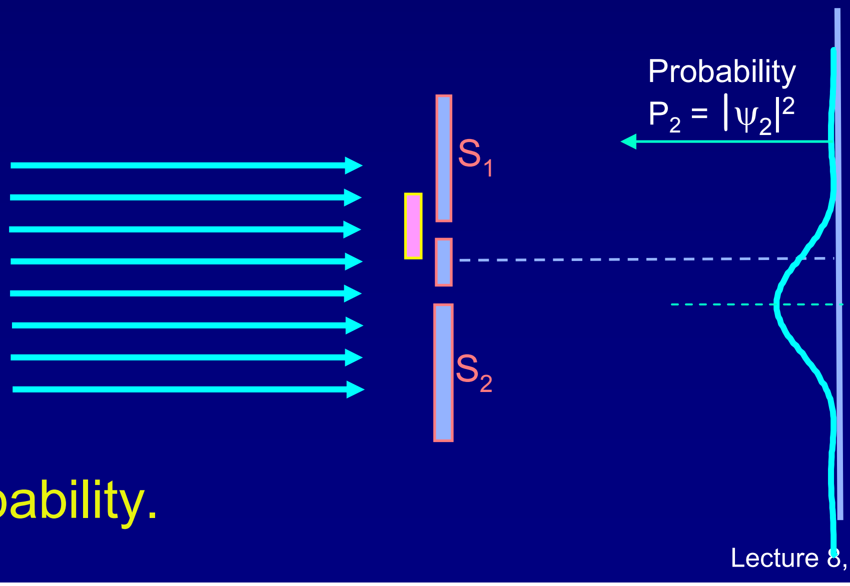
2-slits Revisited (5)

First, cover slit 2; *i.e.*, only light that goes through slit 1 is transmitted. What do we see on the screen?



We get a single-slit diffraction pattern.

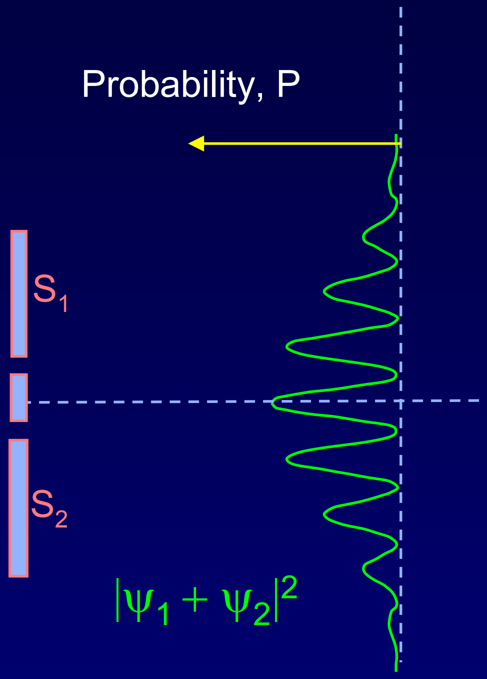
Probability amplitude = ψ_1
Probability density = $|\psi_1|^2 = P_1$



Similar results when slit 1 is covered. $|\psi_2|^2 = P_2$

Changing the wave changes the probability.

2-slits Revisited (6)



Now, open both slits. We see interference!

The **probability amplitude** is now $\psi_1 + \psi_2$, because you don't know which slit the photon went through.

P_{tot} = Probability density

$= |\psi_1 + \psi_2|^2$

$= |\psi_1|^2 + |\psi_2|^2$ + interference term

$$\boxed{P \neq P_1 + P_2}$$

Add amplitudes not intensities.

The interference term will depend on phase differences, just like the wave calculations we did before.

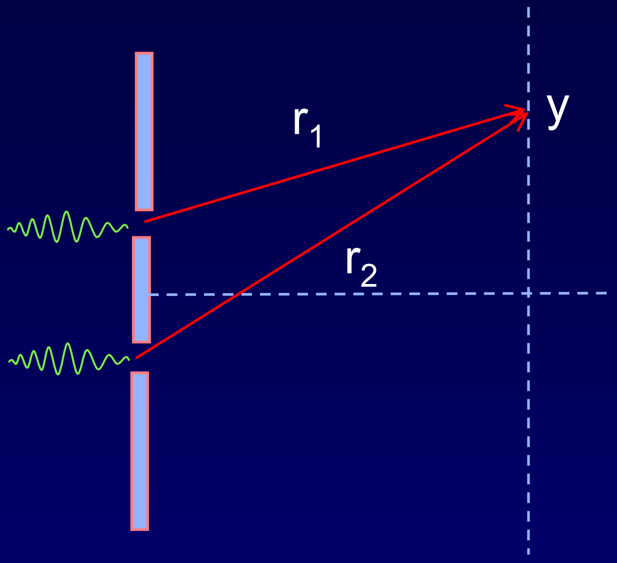
FYI: Two-Slit Experiment, More Carefully

ψ_1: amplitude to pass through upper slit, and travel to y

$$\psi_1 \sim e^{ikr_1}$$

ψ_2: amplitude to pass through lower slit, and travel to y

$$\psi_2 \sim e^{ikr_2}$$



Assume that the only difference between ψ_1 and ψ_2 is a result of the difference between r_1 and r_2.

$$P = \left|\psi_1+\psi_2\right|^2 \sim \left|e^{ikr_1}+e^{ikr_2}\right|^2 = \left(e^{+ikr_1}+e^{+ikr_2}\right)\left(e^{-ikr_1}+e^{-ikr_2}\right)$$

$$= e^{+ikr_1}e^{-ikr_1} + e^{+ikr_2}e^{-ikr_2} + e^{+ikr_1}e^{-ikr_2} + e^{-ikr_1}e^{+ikr_2}$$

$$= 1 + 1 + e^{+i\phi} + e^{-i\phi}$$

$$= \boxed{2+2\cos(\phi)}$$

$$\boxed{\phi = 2\pi\frac{r_1-r_2}{\lambda} = kr_1 - kr_2}$$

Summary

Photoelectric Effect → light as particles

Measurement shows diractly that light (all electromagnetic waves) are quantized with quanta denoted by “Photons”

Energy and momentum of a photon

Key relations of quantum mechanics:
$E=hf$ or $E = \hbar\omega$ and $p=h/\lambda$ where $\hbar = h/2\pi$
Derived here for light – but valid for all matter – more later!

Wave-particle Duality

Two slit experiment reveals the profoundly different nature of the quantum world from our everyday experiences

Probability is a part of nature!

Next Lecture

Interference, revisited
- Only indistinguishable processes can interfere

Wave nature of particles
- Proposed by DeBroglie in 1923, to explain atomic structure.
- Demonstrated by diffraction from crystals – just like X-rays!

Matter-wave Interference
- Double-slit interference pattern, just like photons
- Electron microscopy

Heisenberg Uncertainty Principle
- An object cannot have both position and momentum simultaneously.

Implications for measurements in QM
- Measuring one destroys knowledge of the other.

FYI: The origins of quantum mechanics

- 1900 Planck “solves” the blackbody problem by postulating that the oscillators that emit light have quantized energy levels.

  “Until after some weeks of the most strenuous work of my life, light came into the darkness, and a new undreamed-of perspective opened up before me…the whole procedure was an act of despair because a theoretical interpretation had to be found at any price, no matter how high that might be.”
- 1905 Einstein proposes that light energy is quantized with quanta called ”photons” - waves behave like particles
  - Photoelectric electric effect for which he got the Nobel Prize
- 1913 Bohr proposes that electron orbits are quantized
  - Idea that electrons act like waves - “explained” H atom, but wrong in crucial ways
- 1923 de Broglie proposes that particles behave like waves
  - The step that paved the way for understanding all of nature
- 1925 Pauli introduces “exclusion principle” – only 2 electrons/orbital
  - The step that leads to understanding of electrons in atoms, molecules, solids
- 1926 Schrödinger introduces the wave-formulation of QM
  - The fundamental equation that predicts the nature of matter
- 1927 Heisenberg uncertainty principle
  - The principle that shows the fundamental uncertainty in any one measurement
- 1928 Dirac combines quantum mechanics and special relativity
  - The step that made QM “the most successful theory in the history of physics” – description of atoms, nuclei, elementary particles, prediction of antimatter, . . .