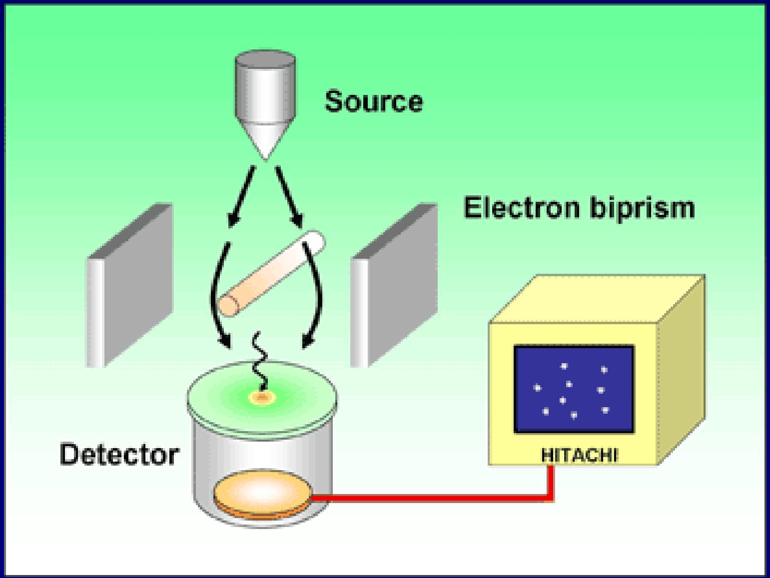
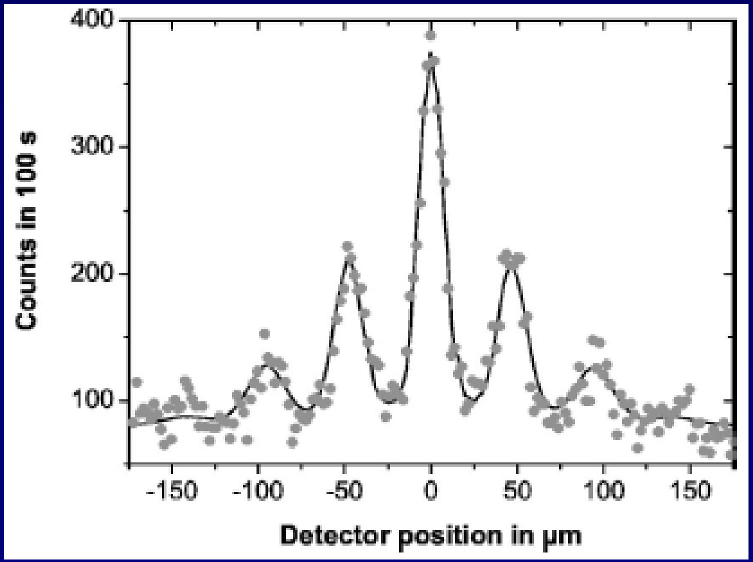
"We choose to examine a phenomenon which is impossible, absolutely *impossible*, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery."

--Richard P. Feynman

Lecture 8:
Introduction to Quantum Mechanics
Matter Waves and the Uncertainty Principle





This week and next are critical for the course:

Week 3, Lectures 7-9:
- Light as Particles
- Particles as waves
- Probability
- Uncertainty Principle

Week 4, Lectures 10-12:
- Schrödinger Equation
- Particles in infinite wells, finite wells

Midterm Exam Monday, Feb. 14.

It will cover lectures 1-10 and some aspects of lectures 11-12.

Practice exams: Old exams are linked from the course web page.

Review Sunday, Feb. 13, 3-5 PM in 141 Loomis

Office hours: Feb. 13 and 14

Last Time

The important results from last time:

Quantum mechanical entities can exhibit either wave-like or particle-like properties, depending on what one measures.

We saw this phenomenon for photons, and claimed that it is also true for matter (*e.g.*, electrons).

The wave and particle properties are related by these universal equations:

$E = hf$ Energy-frequency (= hc/λ only for photons)

$p = h/\lambda$ Momentum-wavelength

Today

Interference, the 2-slit experiment revisited

- Only indistinguishable processes can interfere

Wave nature of particles

- Proposed by DeBroglie in 1923, to explain atomic structure.
- Demonstrated by diffraction from crystals – just like X-rays!

Matter-wave Interference

- Double-slit interference pattern, just like photons
- Electron microscopy

Heisenberg Uncertainty Principle

- An object cannot have both position and momentum simultaneously.

Implications for measurements in QM

- Measuring one destroys knowledge of the other.

Two Slit Interference: Conclusions

Photons (or electrons …) can produce interference patterns even one at a time !

With one slit closed, the image formed is simply a single-slit pattern.
We "know" (*i.e.*, we have constrained) which way the particle went.

With both slits open, a particle interferes with itself to produce the observed two-slit interference pattern.

This amazing interference effect reflects, in a fundamental way, the indeterminacy of which slit the particle went through. We can only state the probability that a particle would have gone through a particular slit, if it had been measured.

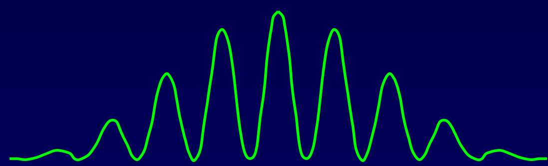
Confused? You aren't alone! We do not know how to understand quantum behavior in terms of our everyday experience. Nevertheless - as we will see in the next lectures – we know how to use the QM equations and make definite predictions for the probability functions that agree with careful experiments!

The quantum wave, ψ, is a probability amplitude. The intensity,
$P = |\psi|^2$, tells us the probability that the object will be found at some position.

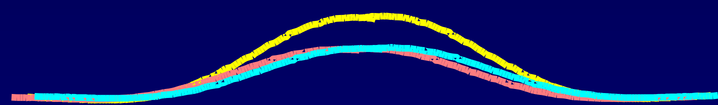
Act 1

Suppose we measure with the upper slit covered for half the time and the lower slit covered for the other half of the time. What will be the resulting pattern?

a. $|\psi_1 + \psi_2|^2$



b. $|\psi_1|^2 + |\psi_2|^2$



Solution

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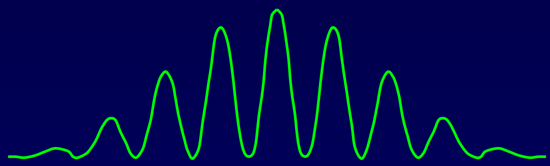
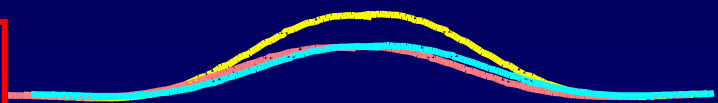


b. $|\psi_1|^2 + |\psi_2|^2$



At any given time, there is only one contributing amplitude (ψ_1 or ψ_2, but not both). Therefore, for half the time pattern P1 will build up, and for the other half we'll get P2. There is no interference. The result will be the sum of the two single-slit diffraction patterns.

In order for waves to interfere, they must both be present at the same time.

Interference – What Really Counts

We have seen that the amplitudes from two or more physical paths interfere if nothing else distinguishes the two paths.

Example: (2-slits)

ψ_{upper} is the amplitude corresponding to a photon traveling through the upper slit and arriving at point y on the screen.

ψ_{lower} is the amplitude corresponding to a photon traveling through the lower slit and arriving at point y on the screen.

If these processes are distinguishable (*i.e.*, if there's some way to know which slit the photon went through), add the probabilities:

$$P(y) = |\psi_{upper}|^2 + |\psi_{lower}|^2$$

If these processes are indistinguishable, add the amplitudes and take the absolute square to get the probability:

$$P(y) = |\psi_{upper} + \psi_{lower}|^2$$

What does "distinguishable" mean in practice?

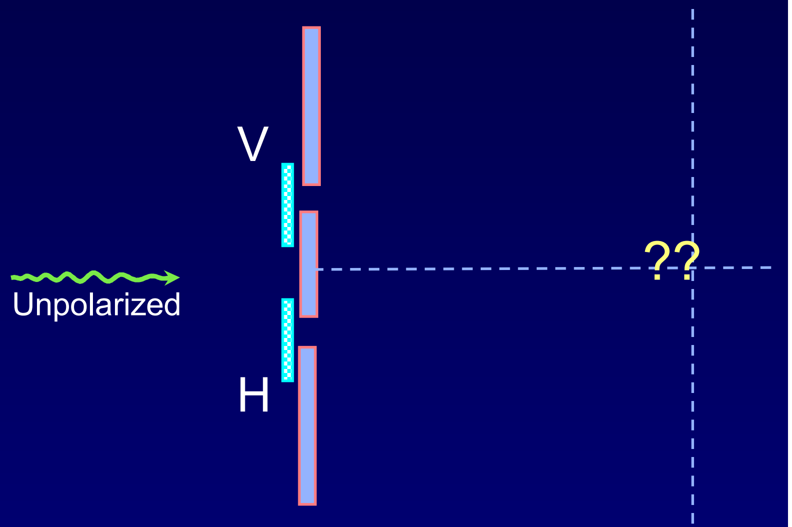
Act 2

Let's modify the 2-slit experiment a bit. Recall that EM waves can be polarized – electric field in the vertical or horizontal directions.

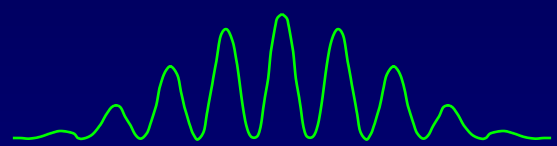
Send in unpolarized photons.

Cover the upper slit with a vertical polarizer
and cover the lower slit with a horizontal polarizer

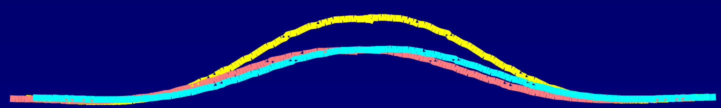
Now the resulting pattern will be:



a) $|\psi_1 + \psi_2|^2$



b) $|\psi_1|^2 + |\psi_2|^2$



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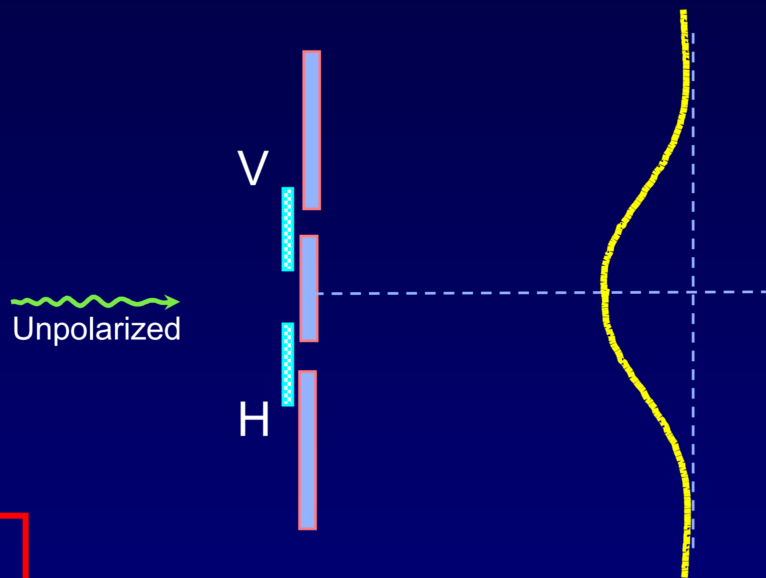

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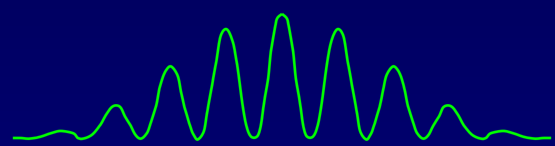

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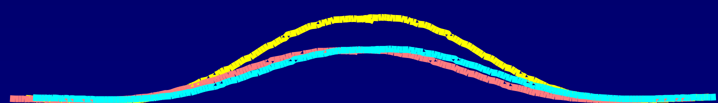


The photon's polarization labels which way it went.
Because the two paths are in principle distinguishable there is no interference.

Note, that we don't actually need to measure the polarization.
The mere possibility that one could measure it destroys the interference.

Bonus Question: How could we recover the interference?

Matter Waves

We described one of the experiments (the photoelectric effect) which shows that light waves also behave as particles. The wave nature of light is revealed by interference - the particle nature by the fact that light is detected as quanta: "photons".

Photons of light have energy and momentum given by:

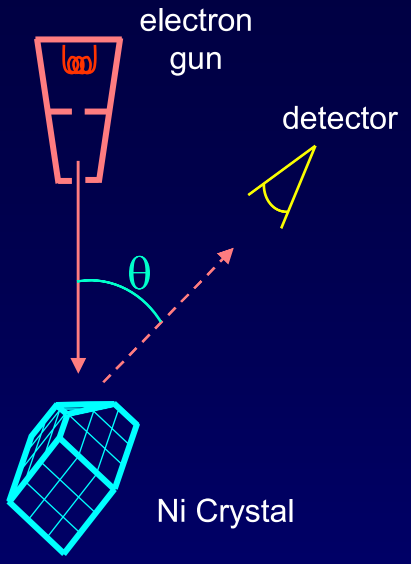
$$E = hf \quad \text{and} \quad p = h/\lambda$$

Prince Louis de Broglie (1923) proposed that particles also behave as waves; i.e., for <u>all particles</u> there is a quantum wave with frequency and wavelength given by the same relation:

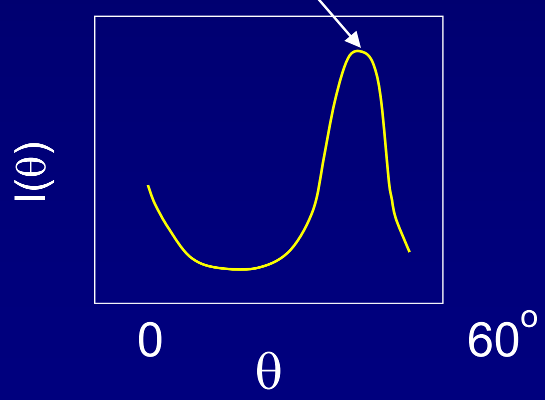
$$f = E/h \quad \text{and} \quad \lambda = h/p$$

Matter Waves

Interference demonstrates that matter (electrons) can act like waves. In 1927-8, Davisson & Germer* showed that, like x-rays, electrons can diffract off crystals !



Electrons can act like waves, just like photons!



You'll study electron diffraction in discussion.

*Work done at Bell Labs, Nobel Prize

Act 3: Matter Wavelengths

What size wavelengths are we talking about? Consider a photon with energy 3 eV, and therefore momentum p = 3 eV/c.* Its wavelength is:

$$\lambda = \frac{h}{p} = \frac{4.14 \times 10^{-15}\ eV \cdot s}{3\ eV} \times c = \left(1.4 \times 10^{-15}\ s\right) \times \left(3 \times 10^{8}\ m/s\right) = 414\ nm$$

What is the wavelength of an electron with the same momentum?

a) $\lambda_e < \lambda_p$ b) $\lambda_e = \lambda_p$ c) $\lambda_e > \lambda_p$

*It is an unfortunate fact of life that there is no named unit for momentum, so we're stuck with this cumbersome notation.

Solution

What size wavelengths are we talking about? Consider a photon with energy 3 eV, and therefore momentum p = 3 eV/c. Its wavelength is:

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What is the wavelength of an electron with the same momentum?

a) $\lambda_e < \lambda_p$ b) $\lambda_e = \lambda_p$ c) $\lambda_e > \lambda_p$

λ = h/p for all objects, so equal p means equal λ.

Note that the kinetic energy of the electron does not equal the energy of a photon with the same momentum (and wavelength):

$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{\left(6.625\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2(9.11\times10^{-31}\text{kg})(414\times10^{-9}\ \text{m})^2}$$

$$= 1.41\times10^{-24}\ \text{J} = 8.8\times10^{-6}\ \text{eV}$$

Wavelength of an Electron

The DeBroglie wavelength of an electron is inversely related to its momentum:

$$\lambda = h/p$$

$h = 6.626\times10^{-34}$ J-sec

Frequently we need to know the relation between the electron's wavelength λ and its kinetic energy E. Because the electron has v << c, p and E are related through the Physics 211 formula:

$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

Valid for all (non-relativistic) particles

For $m = m_e$: (electrons)

$h = 4.14\times10^{-15}$ eV-sec

$m_e = 9.11\times10^{-31}$ kg

$$E_{\text{electron}} = \frac{1.505\ \text{eV}\cdot\text{nm}^2}{\lambda^2}$$

(E in eV; λ in nm)

Don't confuse this with $E_{photon} = \dfrac{1240\ eV\cdot nm}{\lambda}$ for a photon.

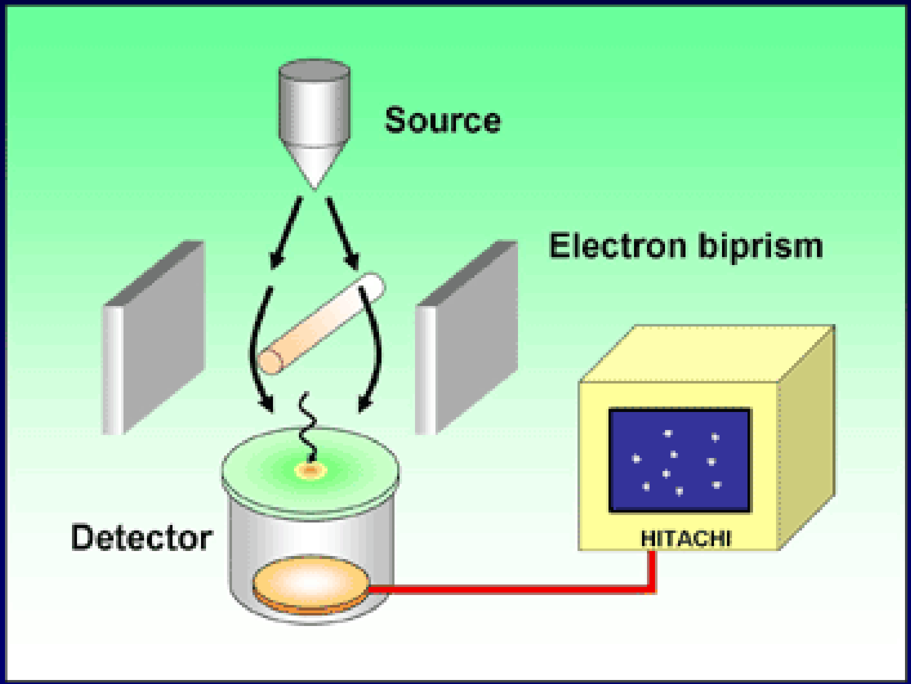
“Double-slit” Experiment for Electrons

Electrons are accelerated to 50 keV
→ λ = 0.0055 nm

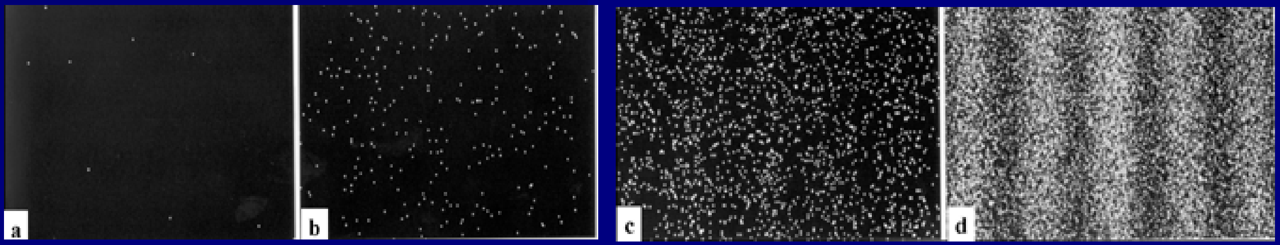
Central wire is positively charged → bends electron paths so they overlap.

A position-sensitive detector records where they appear.

<< 1 electron in system at any time



Video by A. TONOMURA (Hitachi) --pioneered electron holography.
http://www.hqrd.hitachi.co.jp/rd/moviee/doubleslite.wmv



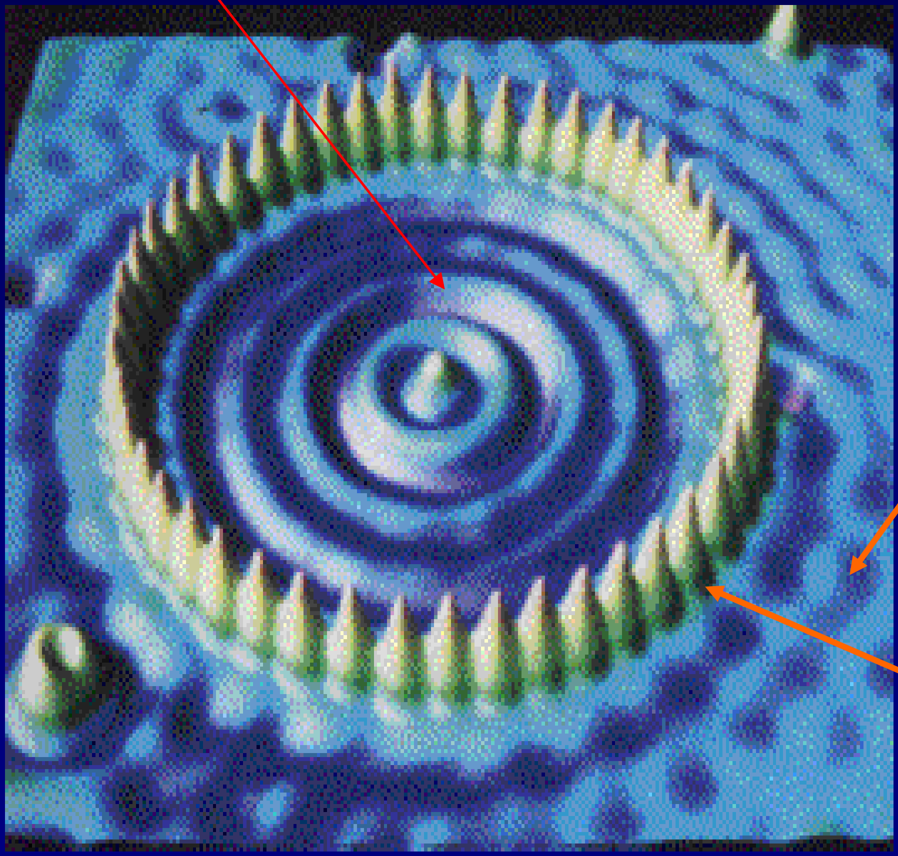
Exposure time: 1 s 10 s 5 min 20 min

See also this Java simulation: http://www.quantum-physics.polytechnique.fr/index.html

Observation of an electron wave "in a box"

Image taken with a scanning tunneling microscope (more later)
(Note: the color is not real! – it is a representation of the electrical current observed in the experiment)

Real <u>standing waves</u> of electron density in a "quantum corral"



Cu

Single atoms (Fe)

IBM Almaden

Wavelengths of Various "Particles"

Calculate the wavelength of

a. an electron that has been accelerated from rest across a 3-Volt potential difference ($m_e = 9.11\times10^{-31}$ kg).

b. Ditto for a proton ($m_p = 1.67\times10^{-27}$ kg).

c. a major league fastball ($m_{baseball} = 0.15$ kg, $v = 50$ m/s).

Solution

Calculate the wavelength of

a. an electron that has been accelerated from rest across a 3-Volt potential difference ($m_e = 9.11\times10^{-31}$ kg).

b. Ditto for a proton ($m_p = 1.67\times10^{-27}$ kg).

c. a major league fastball ($m_{baseball} = 0.15$ kg, v = 50 m/s).

a. $E = e\cdot V = 4.8\times10^{-19}$ J
$p = \sqrt{(2m_eE)} = 9.35\times10^{-25}$ kg m/s
$\lambda = h/p = 7.1\times10^{-10}$ m = 0.71 nm

Physics 212
Physics 211
Physics 214

b. $p = \sqrt{(2m_pE)} = 4.00\times10^{-23}$ kg m/s
$\lambda = h/p = 1.7\times10^{-11}$ m

E is the same.
Mass is bigger $\Rightarrow \lambda$ is smaller.

c. p = mv = 7.5 kg m/s
$\lambda = h/p = 8.8\times10^{-35}$ m

SI units were designed to be convenient for macroscopic objects.

QM wave effects are negligible in the motion of macroscopic objects. 10^{-35} m is many orders of magnitude smaller than any distance that has ever been measured (10^{-19} m, at Fermilab).

Summary: Photon & Matter Waves

Everything

$E = hf$

$p = h/\lambda$

Light (v = c)

$E = pc$, so

$E = hc/\lambda$

$$E_{photon} = \frac{1240\ \text{eV} \cdot \text{nm}}{\lambda}$$

Slow Matter (v << c)

$KE = p^2/2m$, so

$KE = h^2/2m\lambda^2$

For electrons:

$$KE = \frac{1.505\ \text{eV} \cdot \text{nm}^2}{\lambda^2}$$