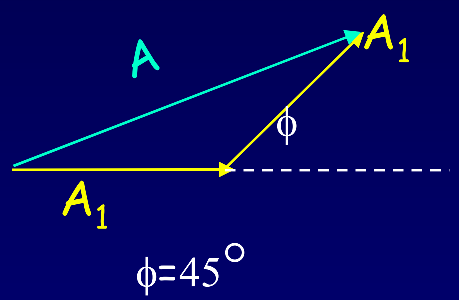
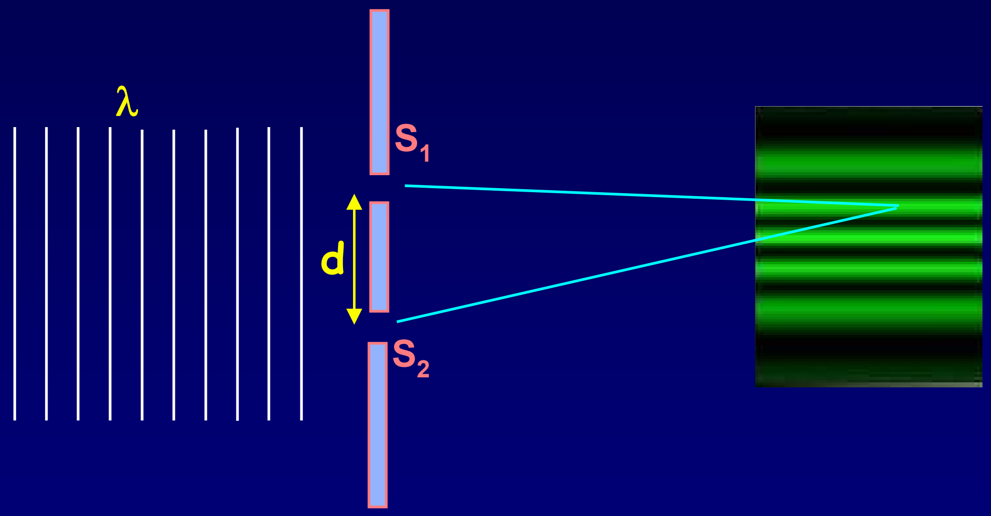
Lecture 3:
Review, Examples and Phasors



$$\delta = \lambda / 8$$



The Many "Fathers" of QM

- 1900 Planck "solves" the blackbody problem by postulating that the oscillators in the walls have quantized energy levels.

"Until after some weeks of the most strenuous work of my life, light came into the darkness, and a new undreamed-of perspective opened up before me…the whole procedure was an act of despair because a theoretical interpretation had to be found at any price, no matter how high that might be."

- 1905 Einstein proposes that light energy is quantized – "photons"
- 1913 Bohr proposes that electron orbits are quantized
- 1923 de Broglie proposes that particles behave like waves
- 1925 Pauli introduces "exclusion principle" – only 2 electrons/orbital
- 1925 Heisenberg introduces matrix-formulation of QM
- 1926 Schrödinger introduces the wave-formulation of QM

HOMEWORK #1

- Monday is a holiday
- The University is closed all weekend
- There will be no office hours this weekend
- Homework #1 will be due on Thursday at 8 am.
- There will be extra office hours on Wednesday
- See the webpage

Lab #1

- Lab #1 is based on things from Lects 1-4
- If you have lab Tuesday, or Wednesday 8am, you should review Prelecture 2, and look ahead at Lect. 4.
- Prelab due when you walk in...

Review: The Harmonic Waveform

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right) \equiv A\cos(kx - 2\pi ft) \equiv A\cos(kx - \omega t)$$

y is the displacement from equilibrium.

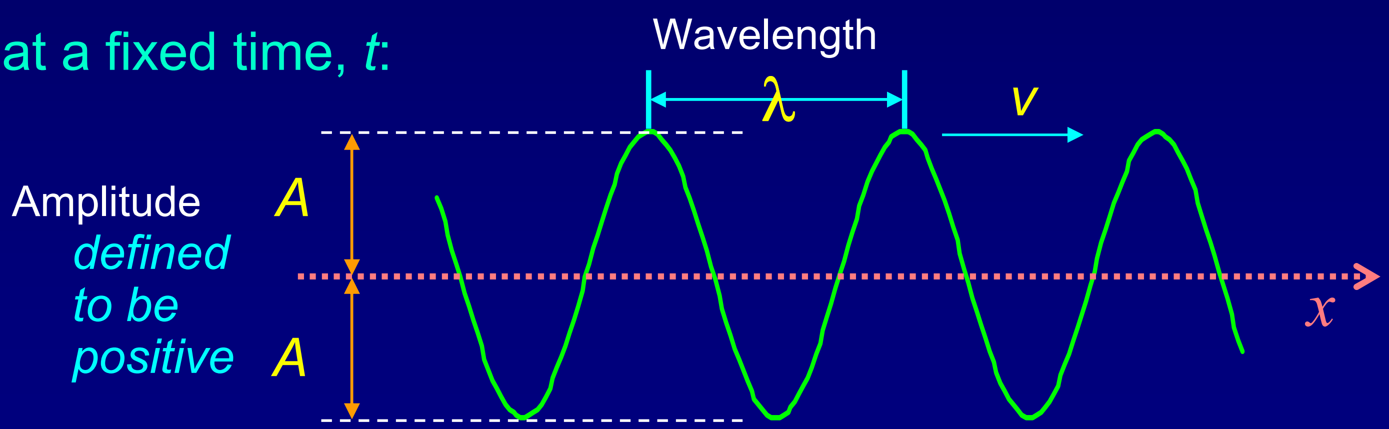
A function of two variables: x and t.

$v \equiv$ speed $\quad A \equiv$ amplitude (defined to be positive)

$\lambda \equiv$ wavelength $\quad k \equiv \frac{2\pi}{\lambda} \equiv$ wavenumber

$f \equiv$ frequency $\quad \omega \equiv 2\pi f \equiv$ angular frequency

A snapshot of y(x) at a fixed time, t:



This is review from Physics 211/212.
For more detail see Lectures 26 and 27 on the 211 website.

Act 1

The speed of sound in air is a bit over **300 m/s**, and the speed of light in air is about **300,000,000 m/s**.

Suppose we make a sound wave and a light wave that both have a wavelength of **3 meters**.

1. What is the ratio of the frequency of the light wave to that of the sound wave?

   (a) About **1,000,000** (b) About **0.000001** (c) About **1000**

2. What happens to the **frequency** if the light passes under water?

   (a) Increases (b) Decreases (c) Stays the same

3. What happens to the **wavelength** if the light passes under water?

   (a) Increases (b) Decreases (c) Stays the same

Act 1 - Solution

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(a) About **1,000,000** (b) About **0.000001** (c) About **1000**

$$f = \frac{v}{\lambda} \quad \text{and} \quad \frac{v_{light}}{v_{sound}} \cong 1{,}000{,}000 \quad \Rightarrow \quad \frac{f_{light}}{f_{sound}} \cong 1{,}000{,}000$$

Act 1 - Solution

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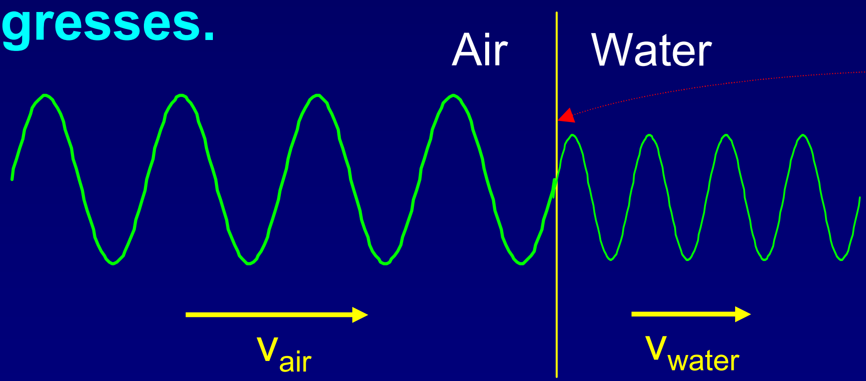
3. What happens to the **wavelength** if the light passes under water?

   (a) Increases (b) Decreases (c) Stays the same

Act 1 - Discussion

Why does the wavelength change but not the frequency?

The frequency does not change because the time dependence in the air must match the time depencence at the air/water boundary. Otherwise, the wave will not remain continuous at the boundary as time progresses.



Continuity of the wave at the air-water interface (at all times) requires that the frequencies be the same.

Question: Do we 'see' frequency or wavelength?

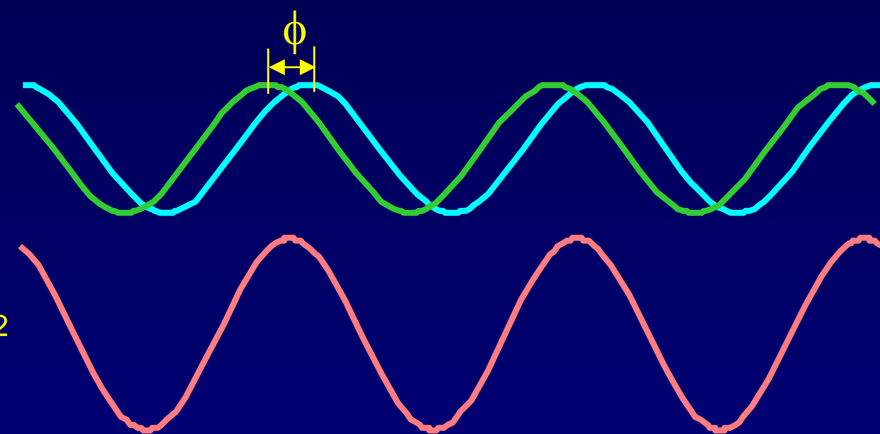
Review: Adding Sine Waves

Suppose we have two sinusoidal waves with the same A_1, ω, and k.
Suppose one starts at phase ϕ after the other:

$$y_1 = A_1 \cos(kx - \omega t) \quad \text{and} \quad y_2 = A_1 \cos(kx - \omega t + \phi)$$

Spatial dependence of 2 waves at t = 0:

Resultant wave: $y = y_1 + y_2$



Use this trig identity:

$$A_1(\cos\alpha + \cos\beta) = 2A_1 \cos\left(\frac{\beta-\alpha}{2}\right)\cos\left(\frac{\beta+\alpha}{2}\right)$$

$$y_1 + y_2 \qquad (\phi/2) \qquad (kx - \omega t + \phi/2)$$

$$y = 2A_1\cos(\phi/2)\,\cos(kx - \omega t + \phi/2)$$

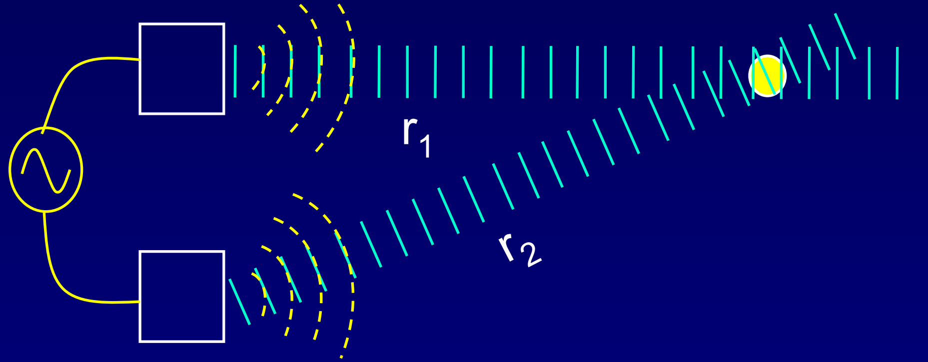
Amplitude Oscillation

Example: Path-Length Dependent Phase

Each speaker alone produces intensity I_1 = 1W/m^2 at the listener, and f = 300 Hz.

Sound velocity: v = 330 m/s

Drive speakers in phase. Compute the intensity I at the listener in this case:



Procedure:

1) Compute path-length difference: δ =

2) Compute wavelength: λ =

3) Compute phase difference: ϕ =

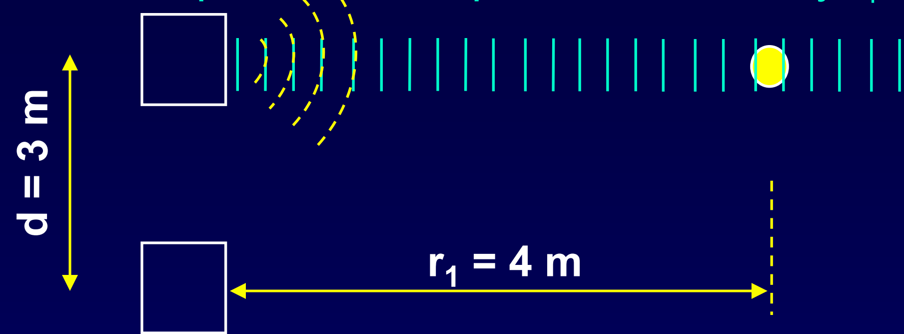
4) Write formula for resultant amplitude: A =

5) Compute the resultant intensity: $I = A^2$ =

Nice demo on web: www.falstad.com/interference

Solution

Each speaker alone produces intensity I_1 = 1W/m^2 at the listener, and f = 300 Hz.



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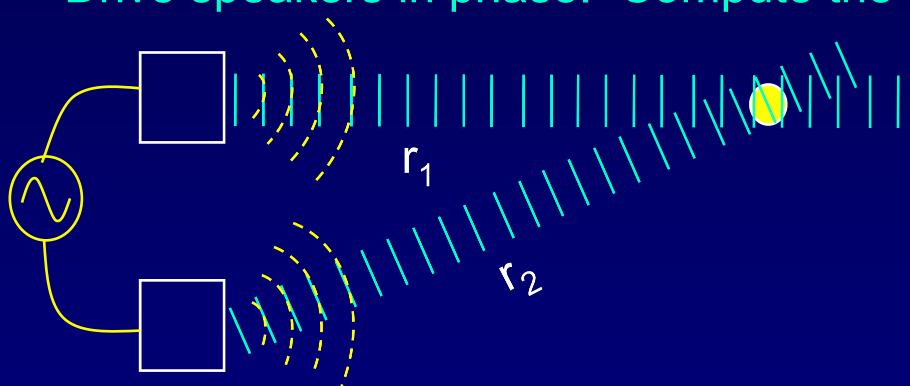


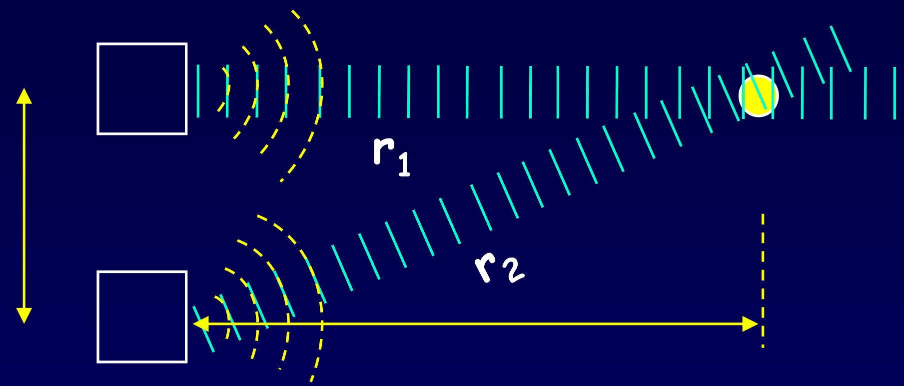
Procedure:

1) Compute path-length difference: δ = 5 m - 4 m = 1 m
2) Compute wavelength: λ = v/f = 330 m/s / 300 Hz = 1.1 m
3) Compute phase difference: ϕ = 2π(1 m / 1.1 m) = 5.71 rad = 327°
4) Write formula for resultant amplitude: A = $2A_1\cos(\phi/2)$ = 2*1*cos(2.86) = -1.92
5) Compute the resultant intensity: I = A^2 = 3.69 W/m^2

The - sign is not significant. We care about |A|.

Nice demo on web: www.falstad.com/interference

Act 2: Speaker interference

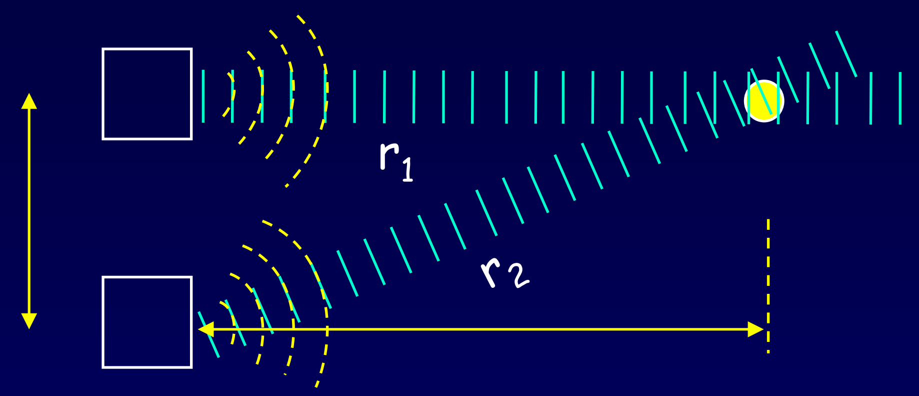


What happens to the intensity at the listener if we decrease the frequency f by a small amount?

a. decrease   b. stay the same   c. increase

Hint: How does intensity vary with ϕ when ϕ = 327°?

Solution

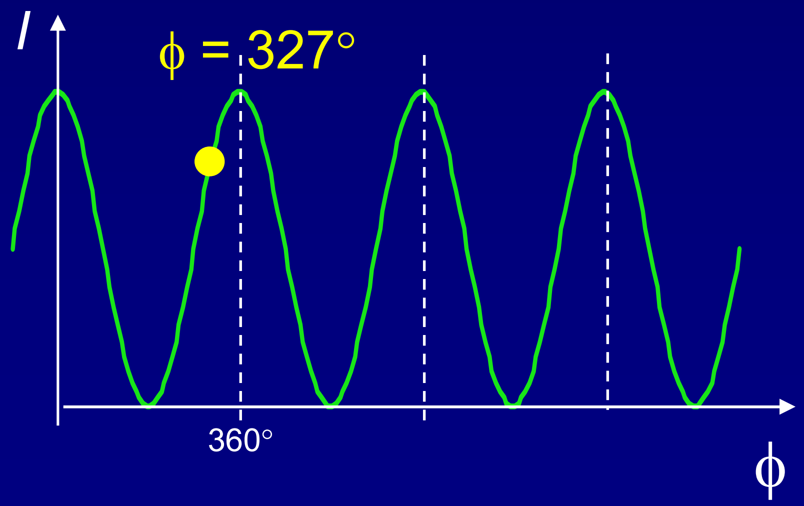


What happens to the intensity at the listener if we decrease the frequency f by a small amount?

a. decrease   b. stay the same   c. increase

Hint: How does intensity vary with ϕ when $\phi = 327°$?

Draw the graph of $I(\phi)$:



f decreases:

→ λ increases

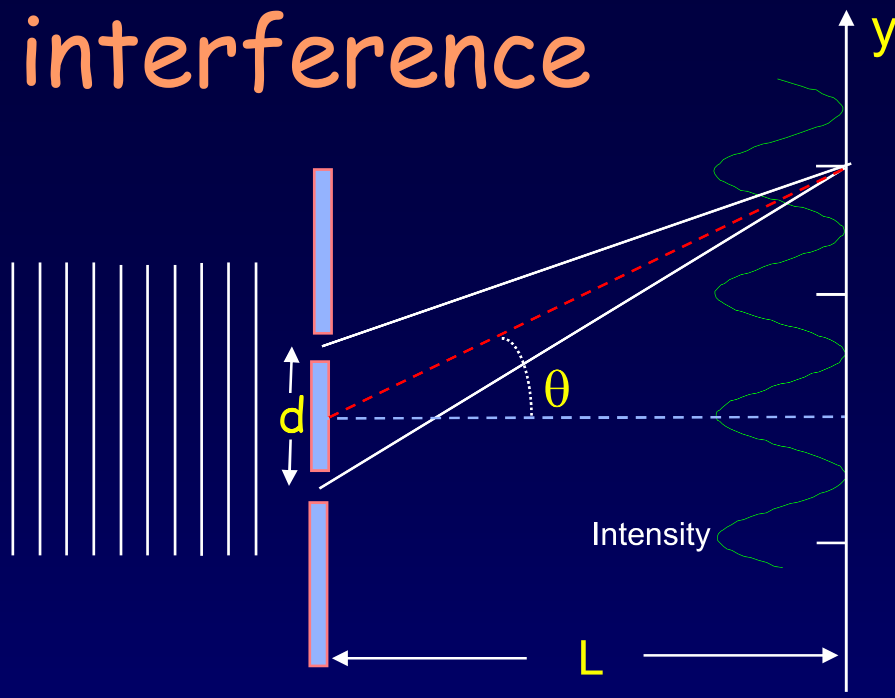
→ δ/λ decreases

→ ϕ decreases

→ I decreases

Example: 2-slit interference

A laser of wavelength 532 nm is incident on two slits separated by 0.125 mm.

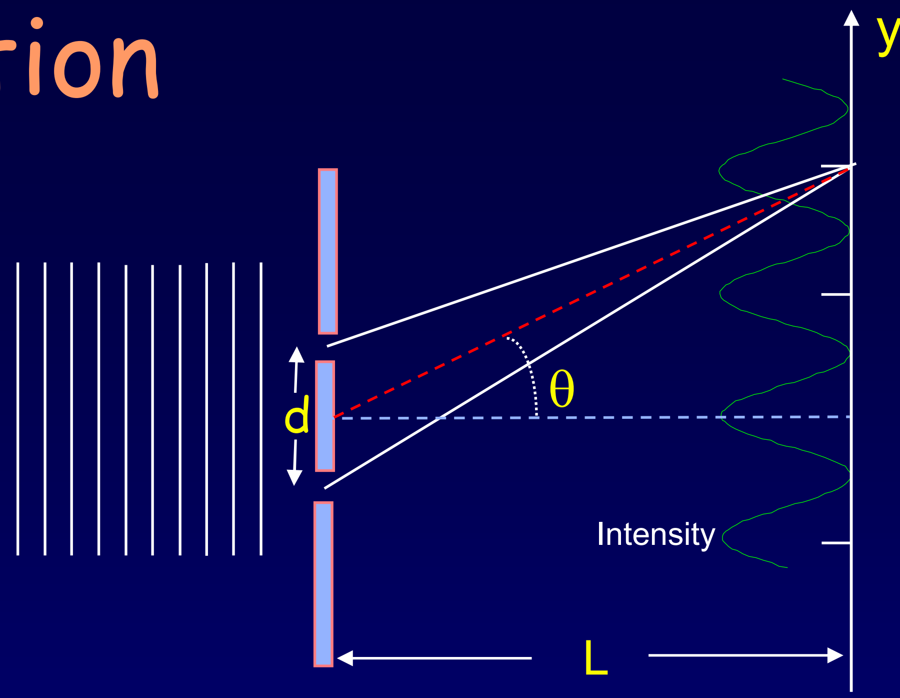


1. What is the angle of the second principle maximum?

2. What is the spacing Δy between adjacent fringe maxima (*i.e.*, $\Delta m = 1$) on a screen 2m away?

Solution

A laser of wavelength 532 nm is incident on two slits separated by 0.125 mm.



1. What is the angle of the <u>second</u> principle maximum?

First: Can we use the small angle approximation?

$d = 125\ \mu m$; $\lambda = 0.532\ \mu m$ → $d \gg \lambda$ → θ is small.

$d \sin\theta_m = m\lambda \sim d\theta_m$ → $\theta_m \approx m(\lambda/d) = 2\ (0.532/125) = 0.0085\ \text{rad} = 0.49°$ (small!)

2. What is the spacing Δy between adjacent fringe maxima (*i.e.*, $\Delta m = 1$) on a screen 2m away?

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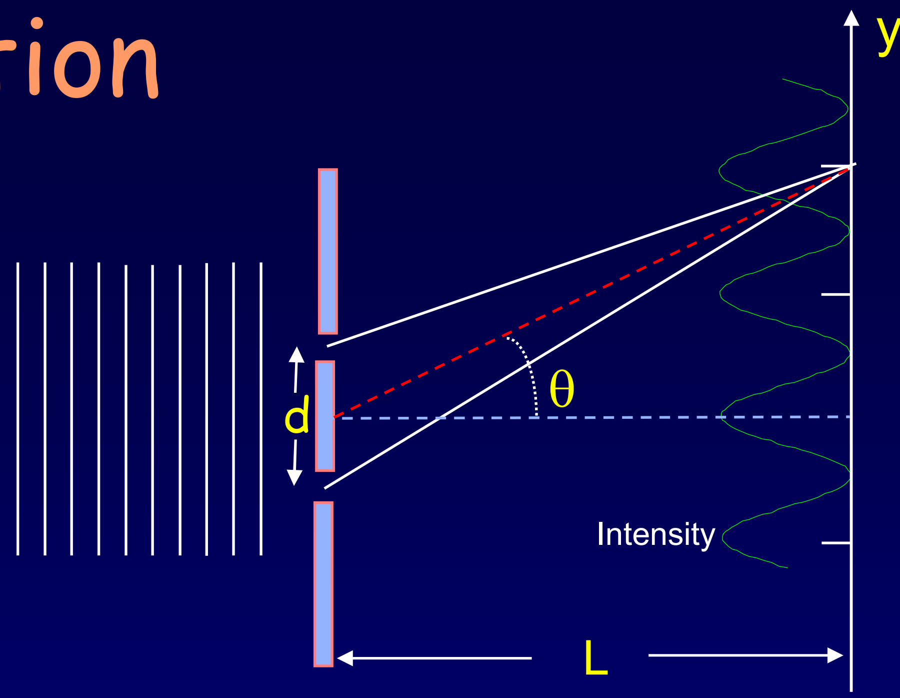

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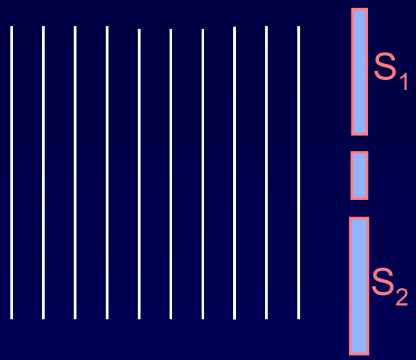
2. What is the spacing Δy between adjacent fringe maxima (*i.e.*, $\Delta m = 1$) on a screen 2m away?

$\Delta y \approx L(\theta_2 - \theta_1) \approx L(2 - 1)(\lambda/d) = L\lambda/d = (2\ m)(0.532\ \mu m)/125\ mm = 0.0085\ m \sim 1\ cm$

Could have also used 1 – 0 (or 6 – 5).

Act 3: 2-slit interference

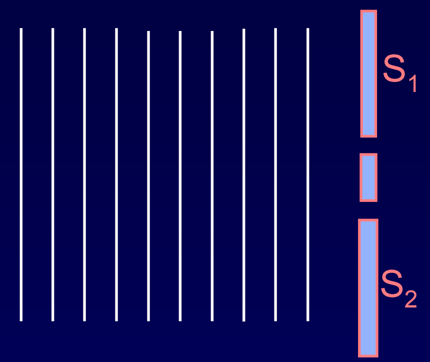


We now increase the wavelength by 20 and decrease the slit spacing by 10, i.e., direct a 10.6-μm laser onto two slits separated by 12.5 μm.

How *many* interference peaks may be observed?
(Hint: Does the small angle approximation hold?)

a. 0 b. 1 c. 3 d. 4 e. ∞

Solution



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(Hint: Does the small angle approximation hold?)

a. 0 b. 1 **c. 3** d. 4 e. ∞

First: Can we use the small angle approximation?

$d = 12.5\ \mu m$; $\lambda = 5.32\ \mu m$ → $d \sim \lambda$ → θ is *not* small.

$d \sin\theta_m = m\lambda$ Because $\sin\theta_m \leq 1$, $m < d/\lambda = 12.5/10.6 = 1.17$

$\therefore\ m_{max} = 1$ $(\theta_1 = 58°)$

Note: This ALWAYS has a solution for m = 0 → there's *always* a central peak

Note: The pattern is symmetric, so there's a peak corresponding to m = -1 too.

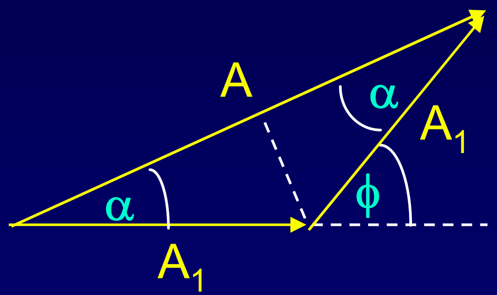
"Beam me up Scotty –
It ate my phasor!"

Phasors

- See the supplementary slide.
- See text: 35.3, 36.3, 36.4.
- See Physics 212 lecture 20.
- Phasors make it easier to solve other problems later.

Lets find the resultant amplitude of two waves using phasors.

Suppose the amplitudes are the same. Represent each wave by a vector with magnitude (A_1) and direction (ϕ). One wave has $\phi = 0$.

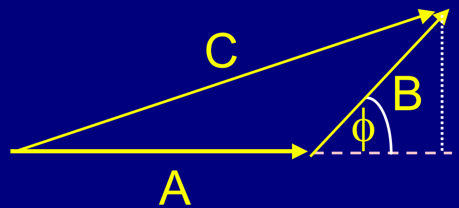


Isosceles triangle: $\alpha = \phi/2$. So, $A = 2A_1 \cos\left(\frac{\phi}{2}\right)$

This is identical to our previous result !

More generally, if the phasors have different amplitudes A and B:

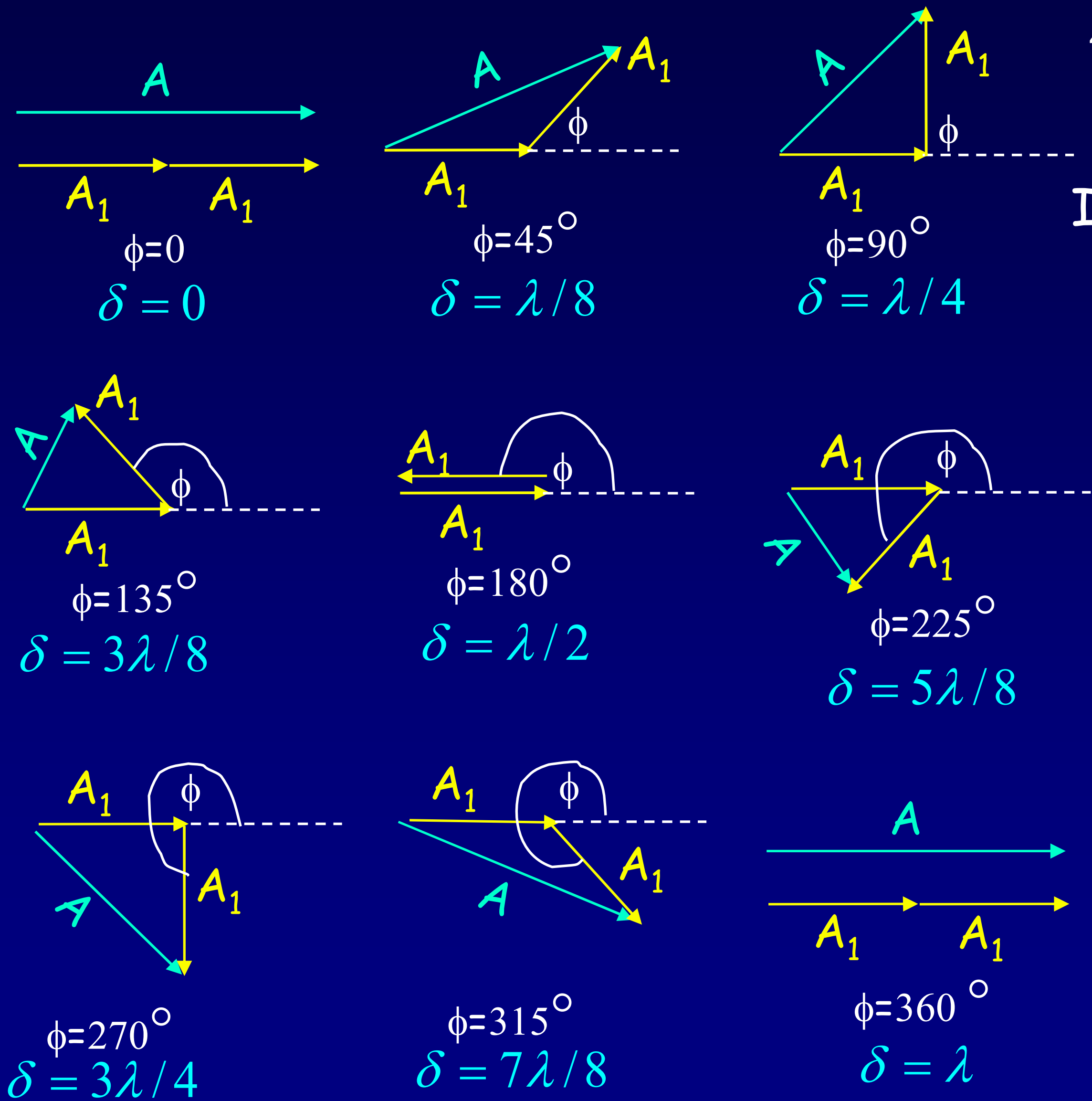
$$C^2 = A^2 + B^2 + 2AB\cos\phi$$

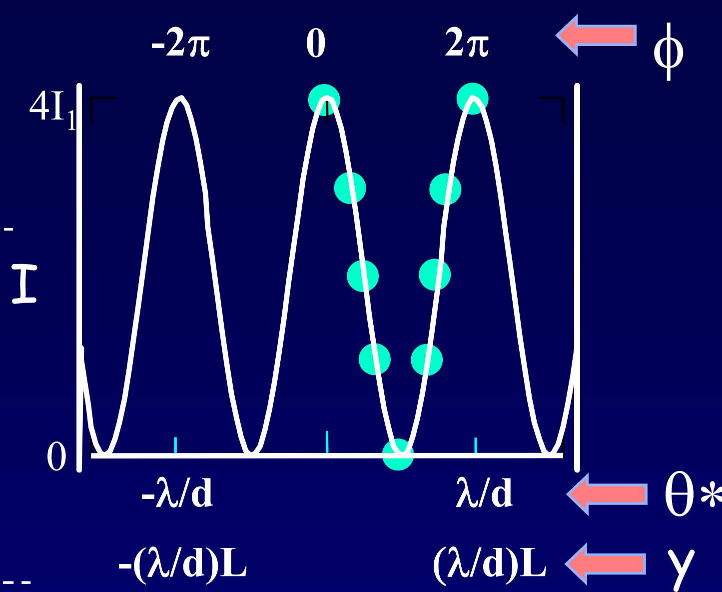


Here ϕ is the *external* angle.

Phasors for 2-Slits

- **Plot the phasor diagram for different ϕ:**

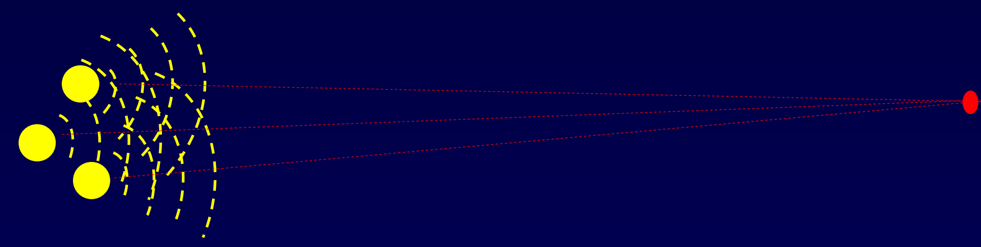




***Small-angle approx. assumed here**

Act 4: Multiple sources

Consider light now coming from *three* openings, arranged in an equilateral triangle.



At given distance far from the openings, the light from each independently has intensity I_1. What is the total intensity when all three openings are open?

a. $3I_1$ b. $9I_1$ c. cannot be determined

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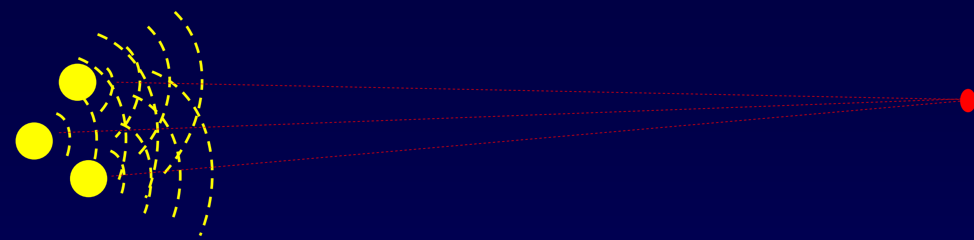

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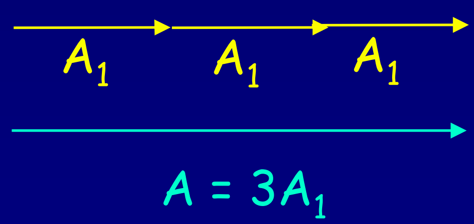


Therefore, $I_{tot} = A^2 = (3\ A_1)^2 = 9\ I_1$

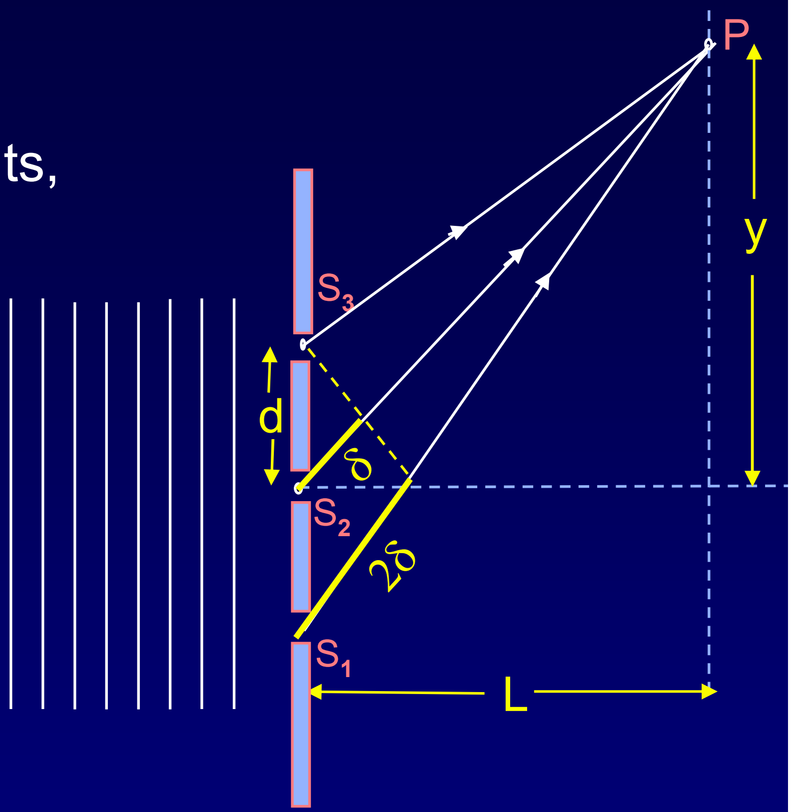
In general, , $I_{tot} = A^2 = (N\ A_1)^2 = N^2\ I_1$

Multiple-Slit Interference

What changes if we increase the number of slits,

e.g., N = 3, 4, 1000, . . .

(for now we'll go back to very small slits, so we can neglect diffraction from each of them)



First look at the principal maxima.

For equally spaced slits:

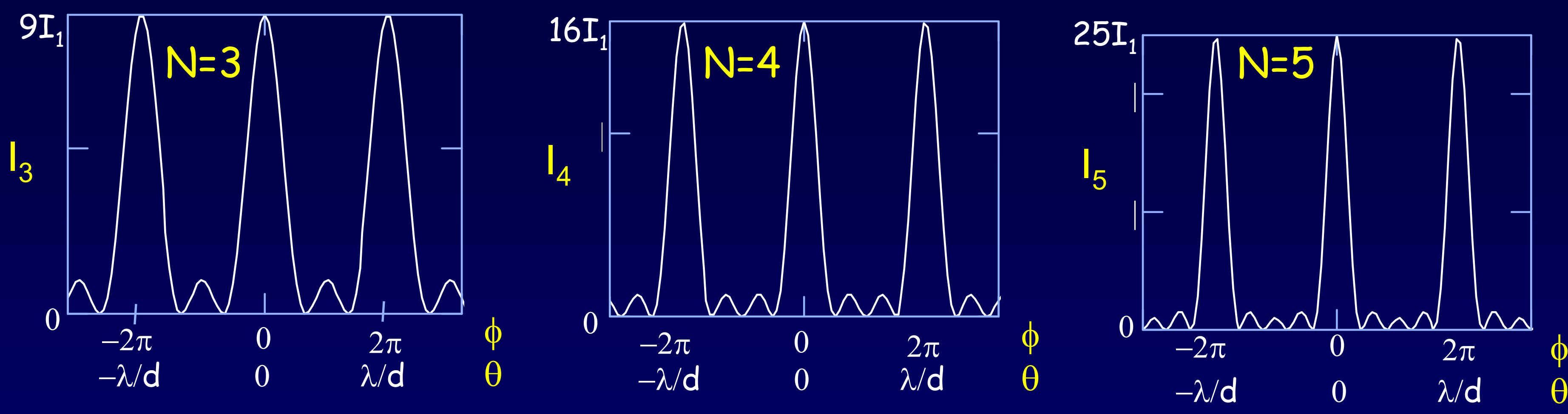
If slit 1 and 2 are in phase with each other, than slit 3 will also be in phase, etc.

The positions of the principal interference maxima are the *same* for any number of slits!

$$d \sin\theta = m\lambda$$

We will almost always consider equally spaced slits.

Multiple-Slit Interference (2)



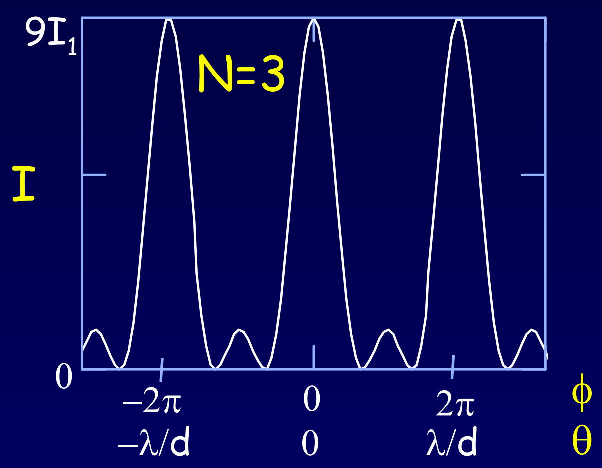
The positions of the principal maxima occur at $\phi = 0, \pm 2\pi, \pm 4\pi, ...$
where ϕ is the phase between adjacent slits. $\theta = 0, \pm\lambda/d, \pm 2\lambda/d, ...$

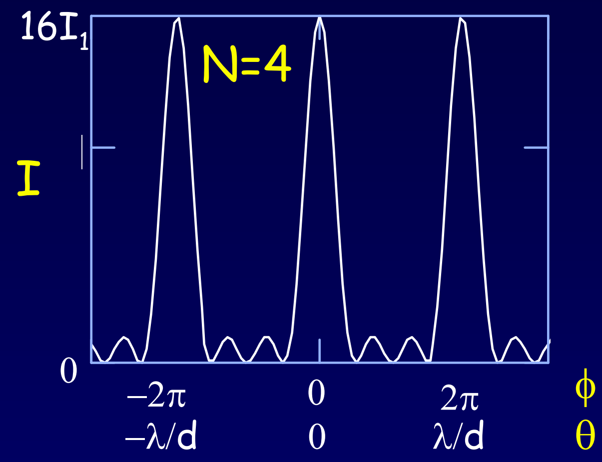
The intensity at the peak of a principal maximum goes as N^2.
3 slits: $A_{tot} = 3A_1 \Rightarrow I_{tot} = 9I_1$. N slits: $I_N = N^2 I_1$.

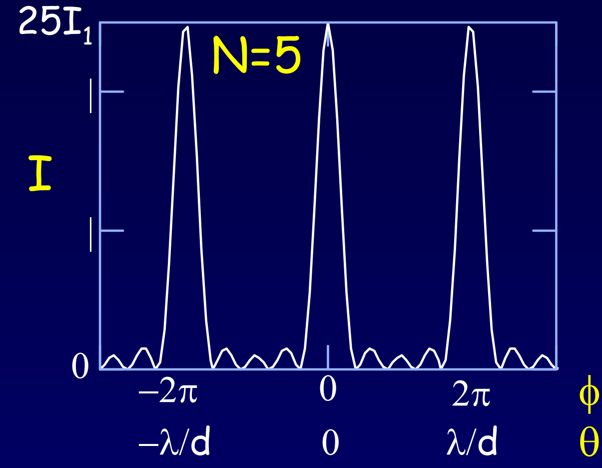
Between two principal maxima there are N-1 zeros and
N-2 secondary maxima $\Rightarrow$ The peak width $\propto$ 1/N.

The total power in a principal maximum is proportional to $N^2(1/N) = N$.

Phasors for N-Slit Interference



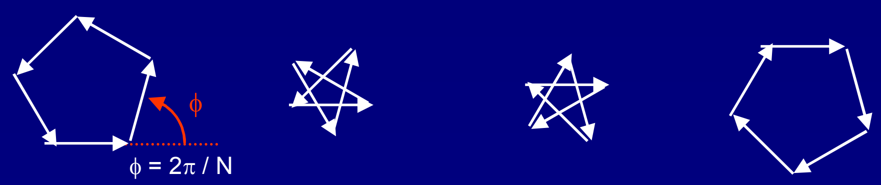




Drawn here for N = 5:

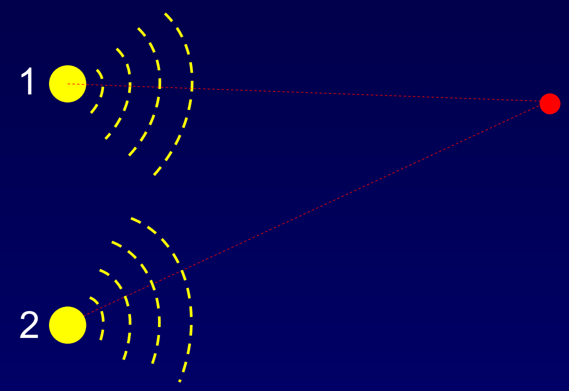
Principal maxima: $\phi = 0, \pm 2\pi$, *etc*.

Zeros. $\phi = m(2\pi/N)$, for m = 1 to N-1.



At Home: Phasor Exercise

Two speakers emit equal intensity (call the amplitude A = 1) sound of frequency f = 256 Hz. The waves are in phase at the source. Suppose that the path difference to the observer is δ = 0.3 m (speaker 1 is closer). v = 330 m/s.



Draw a phasor diagram that describes the two waves at the observer and the resulting wave. What is the resulting amplitude?

Solution

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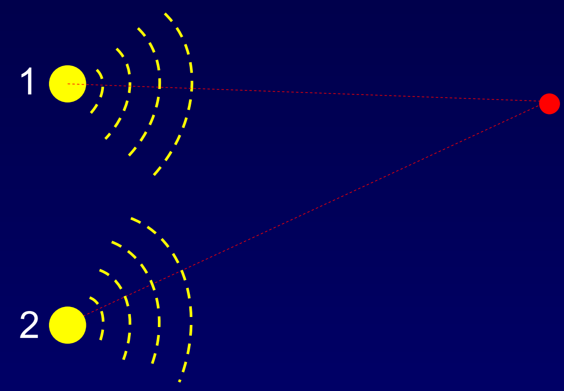
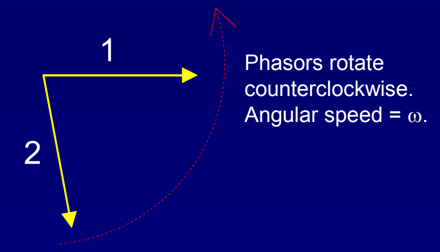


Draw a phasor diagram that describes the two waves at the observer and the resulting wave. What is the resulting amplitude?

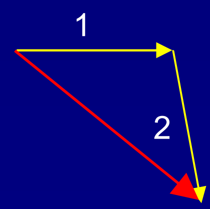
The wavelength is λ = v/f = 1.29 m, so the phase difference is $\phi = 2\pi(\delta/\lambda)$ = 1.46 radians = 83.7°.



Notes:

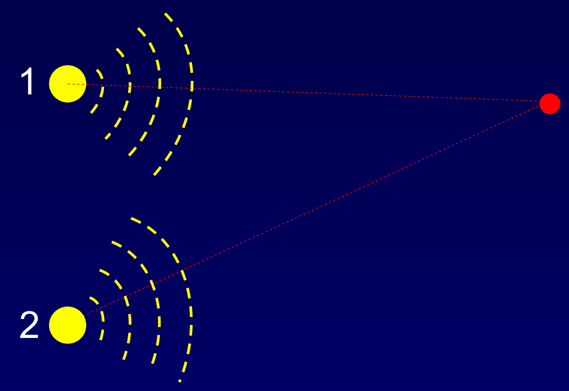
- The two phasors have the same length (amplitude).
- We can always pick one phasor to be horizontal.
- Source 2 is farther from the observer, so its phasor lags behind.

Find the resultant by adding the phasors. The resulting amplitude is approximately $\sqrt{2}$. You'll need to use the algebraic solution to get a more accurate answer.



At Home: Phasor Exercise 2

Suppose the intensity of speaker 2 is twice that of speaker 1. Everything else is the same as in the previous exercise. Draw the phasor diagram that describes this situation.



Solution

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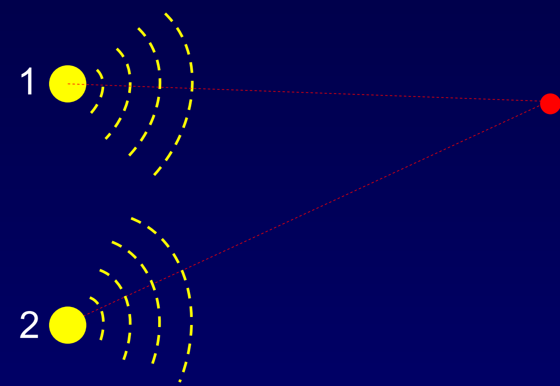
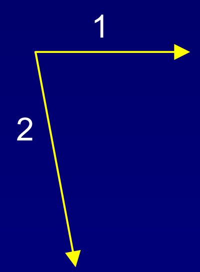
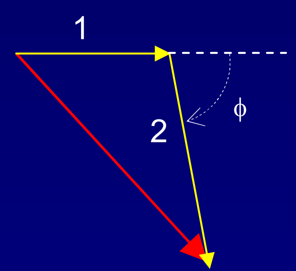


The phase difference is unchanged: $\phi = 83.7°$.
Now, the length of phasor 2 is $\sqrt{2}$ larger.
(Remember that phasors are amplitudes.)

So we have



and



Note that the algebraic solution we wrote before does not apply here, because the amplitudes aren't equal. You can use some trigonometry to calculate the length of the third side of the triangle.

Law of cosines: $c^2 = a^2 + b^2 + 2ab\cos\phi = 1 + 2 + 2\sqrt{2}\times 0.11 = 3.31$ ($c = 1.82$)

Supplement: Phase shift and Position or Time Shift

Because the wave is oscillating both in time and position, we can consider ϕ to be either a time or position shift:

Time:

$$\begin{aligned} y &= A_1 \cos(kx - \omega t + \phi) \\ &= A_1 \cos(kx - \omega(t - \phi/\omega)) \\ &= A_1 \cos(kx - \omega(t - \phi T/2\pi)) \\ &= A_1 \cos(kx - \omega(t - \delta t)) \end{aligned}$$

The time shift: $\delta t/T = \phi/2\pi$

Positive ϕ shifts to later times.

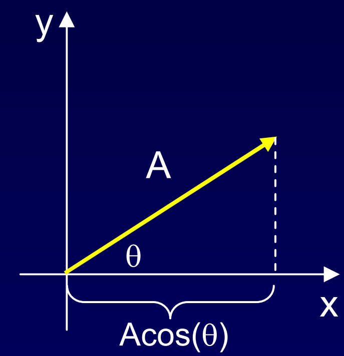
Position:

$$\begin{aligned} y &= A_1 \cos(kx - \omega t + \phi) \\ &= A_1 \cos(k(x+\phi/k) - \omega t) \\ &= A_1 \cos(k(x+\phi\lambda/2\pi) - \omega t) \\ &= A_1 \cos(k(x-\delta x) - \omega t) \end{aligned}$$

The position shift: $\delta x/\lambda = -\phi/2\pi$

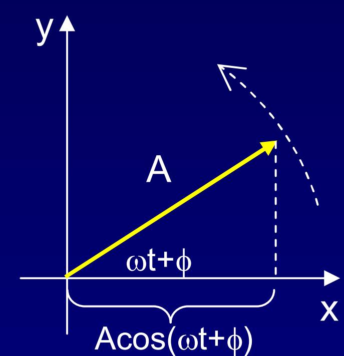
Positive ϕ shifts to negative position.

Supplement: Phasor Math



We want to manipulate Acos(ωt+ϕ). Use the fact that the x-component of a 2-dimensional vector is Acos(θ).

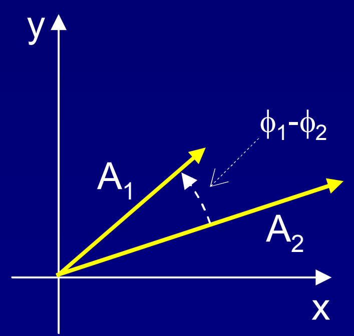
If θ is changing with time, θ = ωt, the vector is rotating, and the x component is Acos(ωt+ϕ). That's what we want.

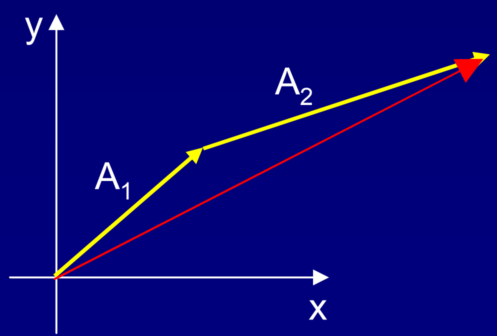


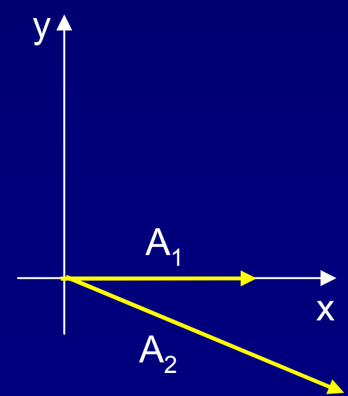
If we have two quantities that have the same frequency, but different amplitudes and phases:

$A_1\cos(\omega t+\phi_1)$ and $A_2\cos(\omega t+\phi_2)$

we can use vector addition to calculate their superposition.







It is conventional to draw one phasor horizontal. Because the phasors are rotating, this merely means we are looking at them at a particular time.