Physics 214 Common Formulae

SI Prefixes

Power	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^0		
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Physical Data and Conversion Constants

speed of light	$c = 2.998 \times 10^8$ m/s
Planck constant	$h = 6.626 \times 10^{-34}$ J·s $= 4.135 \times 10^{-15}$ eV·s
Planck constant / 2π	$\hbar = 1.054 \times 10^{-34}$ J·s $= 0.658 \times 10^{-15}$ eV·s
electron charge	$e = 1.602 \times 10^{-19}$ C
energy conversion	1 eV = 1.602×10^{-19} J
conversion constant	hc = 1240 eV·nm= 1.986×10^{-25} J-m
useful combination	$h^2/2m_e = 1.505$ eV nm^2
Bohr radius	$a_o = (4\pi\varepsilon_o)\hbar^2 / m_e e^2$ = 0.05292 nm
Rydberg energy	$hcR_\infty = m_e e^4 / 2(4\pi\varepsilon_o)^2 \hbar^2$ = 13.606 eV
Coulomb constant	$\kappa = 1/(4\pi\varepsilon_o) = 8.99 \times 10^9$ N·m^2 / C^2
Avagadro constant	$N_A = 6.022 \times 10^{23}$ / mole
electron mass	$m_e = 9.109 \times 10^{-31}$ kg = 0.511 MeV/c^2
proton mass	$m_p = 1.673 \times 10^{-27}$ kg = 938.3 MeV/c^2
neutron mass	$m_n = 1.675 \times 10^{-27}$ kg = 939.6 MeV/c^2
hydrogen atom mass	$m_H = 1.674 \times 10^{-27}$ kg
Electron magnetic moment	$\mu_\varepsilon = 9.2848 \times 10^{-24}$ J/T $= 5.795 \times 10^{-5}$ eV/T
Proton magnetic moment	$\mu_p = 1.4106 \times 10^{-26}$ J/T $= 8.804 \times 10^{-8}$ eV/T

Trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$A_1\sin(\omega t+\phi_1) + A_2\sin(\omega t+\phi_2) = A_3\sin(\omega t+\phi_3)$$

$A^2 + B^2 + 2AB\cos\phi = C^2$ (ϕ here is the *external* angle)

Waves, Superposition

$$k \equiv \frac{2\pi}{\lambda} \quad \omega \equiv 2\pi f \quad T \equiv \frac{1}{f} \quad v = \lambda f = \frac{\omega}{k}$$

General relation for I and A: $I \propto A^2$, $A = A_1 + A_2 + \ldots$

Two sources: $I_{max} = |A_1 + A_2|^2$, $I_{min} = |A_1 - A_2|^2$

Two sources, same I_1: $I = 4I_1\cos^2(\phi/2)$ where $\phi = 2\pi\delta/\lambda$

Interference: Slits, holes, etc.

Far-field path-length difference: $\delta \equiv r_1 - r_2 \approx d\sin\theta$

Phase difference: $\frac{\phi}{2\pi} \equiv \frac{\delta}{\lambda} = \frac{d\sin\theta}{\lambda} \approx \frac{d\,\theta}{\lambda} \approx \frac{d}{\lambda}\frac{y}{L}$ if θ small

Principal maxima: $d\sin\theta_{max} = \pm m\lambda \quad m = 0, 1, 2, \ldots$

N slit: $I_N = I_1\left\{\frac{\sin(N\phi/2)}{\sin(\phi/2)}\right\}^2$ where $\phi = 2\pi d\sin\theta/\lambda$

Single slit: $\delta_a = a\sin\theta \quad a\sin\theta_{min} = \pm m\lambda$ with $m = 1, 2, 3\ldots$

$$\frac{\beta}{2\pi} \equiv \frac{\delta_a}{\lambda} = \frac{a\sin\theta}{\lambda} \approx \frac{a\,\theta}{\lambda} \approx \frac{a}{\lambda}\frac{y}{L}$$

Single slit: $I_1 = I_0\left\{\frac{\sin(\beta/2)}{\beta/2}\right\}^2$ with $\beta = 2\pi a\sin\theta/\lambda$

slit: $\theta_0 \approx \lambda/a$ *or* hole: $\theta_0 \approx 1.22\lambda/D \approx \alpha_c$

Approx. grating resolution: $\frac{\Delta\lambda}{\lambda} \geq \frac{1}{Nm}$

Quantum laws, facts….

UNIVERSAL: $p = \hbar k = h/\lambda \qquad E = hf = \hbar\omega$

Light: $E = hf = \hbar\omega = hc/\lambda = pc$

Slow particle: $KE = mv^2/2 = p^2/2m = h^2/2m\lambda^2$

Photoelectric effect: $KE_{max} = eV_{stop} = hf - \Phi$

UNIVERSAL: $\Delta x\,\Delta p_x \geq \hbar \qquad \Delta E\,\Delta t \geq \hbar$

$$\psi^*(x)\psi(x) \equiv |\psi(x)|^2$$

$$P_{ab} = \int_a^b |\psi(x)|^2\,dx, \quad a \leq x \leq b$$

(Slow) particle in fixed potential U:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + U(x)\psi(x) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

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Quantum stationary states (energy eigenstates):

$$\Psi(x,t)=\psi(x)e^{-i\omega t} \qquad \text{where } E=\hbar\omega$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+U(x)\psi(x)=\hbar\omega\,\psi(x)=E\,\psi(x)$$

In 1-D box: $n\lambda=2L$ where $n=1,2...$

$$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \qquad \text{for} \quad 0\le x\le L$$

$$E_n=\frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2=\left(\frac{h^2}{8mL^2}\right)n^2=E_1n^2 \qquad \text{(*last part*)}$$

Box, 3-D:

$$\psi(x,y,z)=\sqrt{\frac{8}{abc}}\sin\left(\frac{n_1\pi}{a}x\right)\sin\left(\frac{n_2\pi}{b}y\right)\sin\left(\frac{n_3\pi}{c}z\right)$$

$$E(n_1,n_2,n_3)=\frac{h^2}{8m}\left(\frac{n_1^2}{a^2}+\frac{n_2^2}{b^2}+\frac{n_3^2}{c^2}\right)$$

Simple Harmonic Oscillator (SHO):

$$E_n=\left(n+\tfrac{1}{2}\right)\hbar\omega \quad \text{where} \quad n=0,1,2...$$

$$\omega=\sqrt{k/m}$$

Free slow particle with definite p:

$$\Psi(x,t)=Ae^{i(kx-\omega t)} \;\; with\; \hbar\omega=\hbar^2k^2/2m$$

H-like atom

potential $U(r)=-\dfrac{\kappa Ze^2}{r}$

$$E_n=\frac{-1}{4\pi\varepsilon_o}\frac{(Ze)^2}{2a_o}\frac{1}{n^2}=-\frac{1}{(4\pi\varepsilon_o)^2}\frac{me^4Z^2}{2\hbar^2n^2}$$

$$=-13.606\,\text{eV}\,\frac{Z^2}{n^2}$$

Ground state: $\psi_{1s}(r,\theta,\phi)=\dfrac{1}{\sqrt{\pi a_o^3}}e^{-r/a_o}$

Radial density for s-state: $P(r)\,dr=4\pi r^2\,|\psi(r)|^2\,dr$

Form of *n, l, m* eigenstate:

$$\psi_{n\ell m}(r,\theta,\phi)=R_{n\ell}(r)\,Y_{\ell m}(\theta,\phi)$$

$$Y_{00}=\frac{1}{\sqrt{4\pi}}, \quad Y_{10}=\sqrt{\frac{3}{4\pi}}\cos\theta,$$

$$Y_{1\pm1}=\mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\varphi}$$

Tunneling

$$T\approx Ge^{-2KL} \qquad \text{where} \qquad G=16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)$$

$$K=\sqrt{\frac{2m}{\hbar^2}(U_0-E)}=2\pi\sqrt{\frac{2m}{h^2}(U_0-E)}$$

Angular momentum and magnetism

Orbital: $L_z=m\hbar$ where $m=0,\pm1,\pm2,...\pm\ell$

$L^2=\ell(\ell+1)\hbar^2$ where $\ell=0,1,2,...$

Spin: $S_z=m_s\hbar$ where $m_s=\pm 1/2$

Magnetic energy: $U=-\vec{\mu}\cdot\vec{B}$

Force: $F_z=\mu_z\dfrac{dB_z}{dz}$ where $\mu_z\approx-\dfrac{e}{m_e}S_z$

Atomic orbital filling