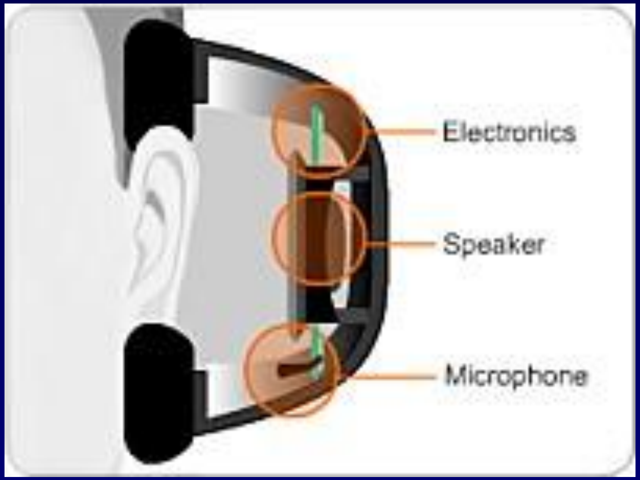
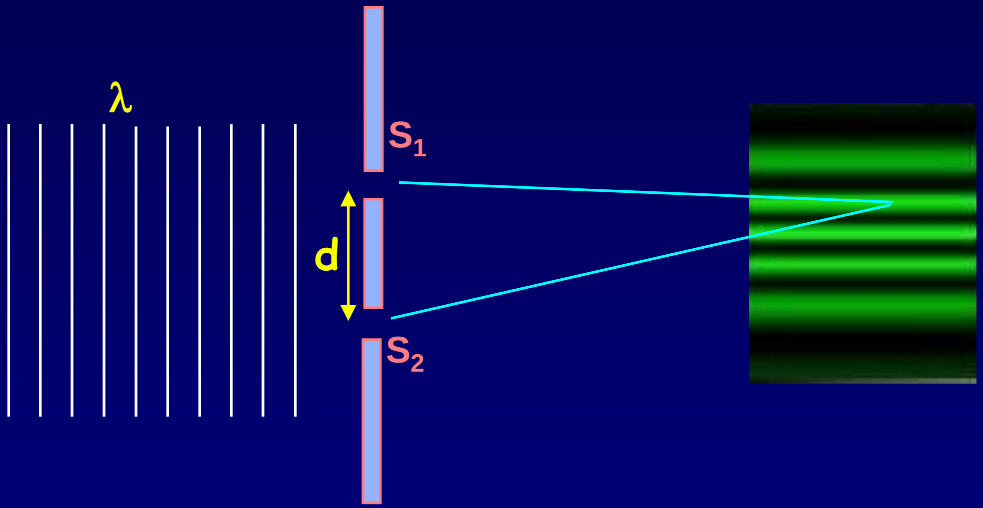
Lecture 2: Interference





- Interference of sound waves
- Two-Slit Interference
- Phasors

The Many "Fathers" of QM

- 1900 Planck "solves" the blackbody problem by postulating that the oscillators in the walls have quantized energy levels.

"Until after some weeks of the most strenuous work of my life, light came into the darkness, and a new undreamed-of perspective opened up before me…the whole procedure was an act of despair because a theoretical interpretation had to be found at any price, no matter how high that might be."

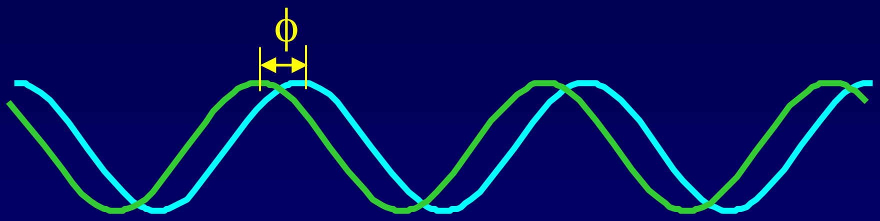
- 1905 Einstein proposes that light energy is quantized – "photons"
- 1913 Bohr proposes that electron orbits are quantized
- 1923 de Broglie proposes that particles behave like waves
- 1925 Pauli introduces "exclusion principle" – only 2 electrons/orbital
- 1925 Heisenberg introduces matrix-formulation of QM
- 1926 Schrödinger introduces the wave-formulation of QM

Adding Sine Waves with Different Phases

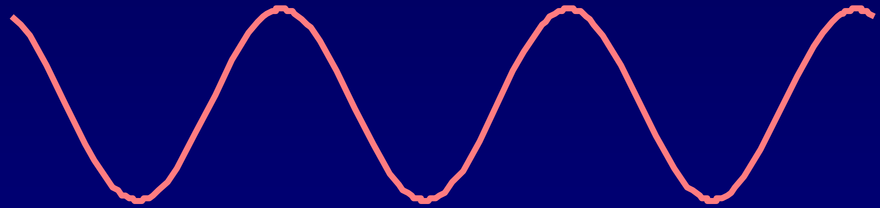
Suppose we have two sinusoidal waves with the same A_1, ω, and k: $y_1 = A_1 \cos(kx - \omega t)$ and $y_2 = A_1 \cos(kx - \omega t + \phi)$

One starts at phase ϕ after the other:

Spatial dependence of 2 waves at t = 0:



Resultant wave: $y = y_1 + y_2$



Use this trig identity:

$$A_1(\cos\alpha + \cos\beta) = 2A_1 \cos\left(\frac{\beta - \alpha}{2}\right)\cos\left(\frac{\beta + \alpha}{2}\right)$$

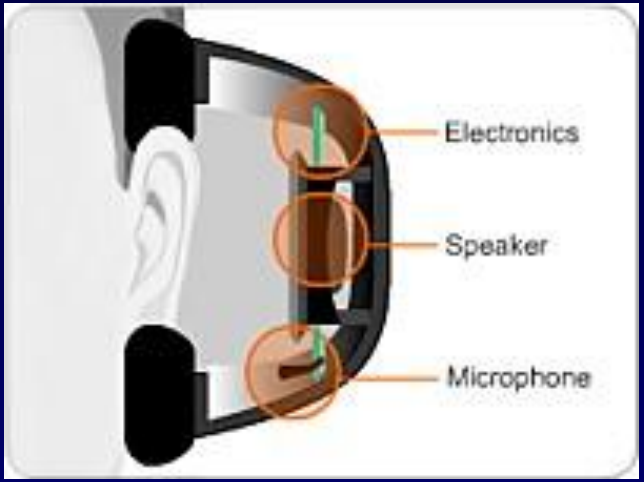
$$y_1 + y_2 \qquad (\phi/2) \qquad (kx - \omega t + \phi/2)$$

$$y = 2A_1\cos(\phi/2)\ \cos(kx - \omega t + \phi/2)$$

Amplitude | Oscillation

ACT 1:
Noise-cancelling Headphones

Noise-canceling headphones work using interference. A microphone on the earpiece monitors the instantaneous amplitude of the external sound wave, and a speaker on the inside of the earpiece produces a sound wave to cancel it.



1. What must be the phase of the signal from the speaker relative to the external noise?

   a. 0 b. 90° c. π d. -180° e. 2π

2. What must be the intensity I_s of the signal from the speaker relative to the external noise I_n?

   a. $I_s = I_n$ b. $I_s < I_n$ c. $I_s > I_n$

Solution

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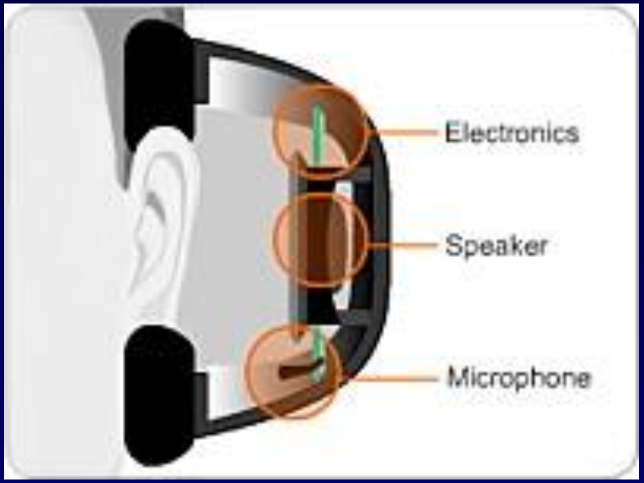

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a. 0　　b. 90˚　　c. π　　d. -180˚　　e. 2π

Destructive interference occurs when the waves are ±180˚ out of phase.

180º = π radians!

2. What must be the intensity I_s of the signal from the speaker relative to the external noise I_n?

a. $I_s = I_n$　　b. $I_s < I_n$　　c. $I_s > I_n$

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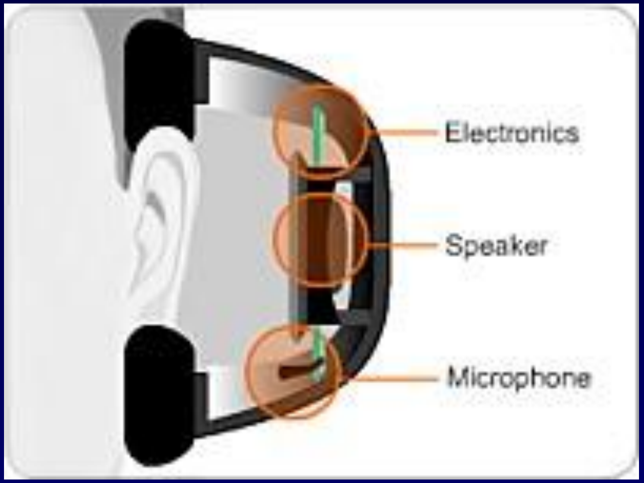

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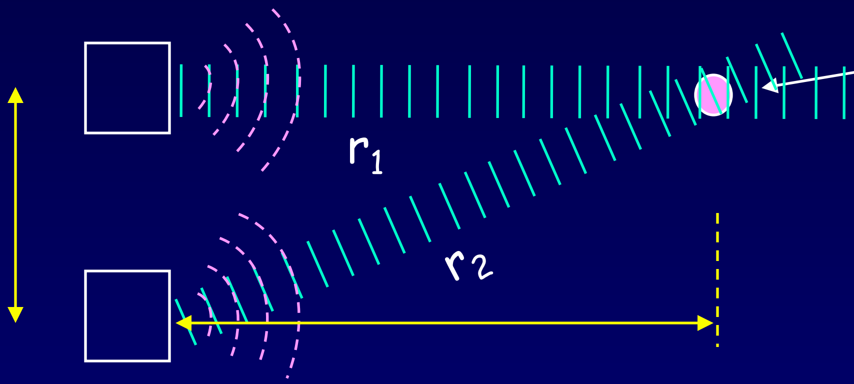
2. What must be the intensity I_s of the signal from the speaker relative to the external noise I_n?

   a. $I_s = I_n$ b. $I_s < I_n$ c. $I_s > I_n$

   We want $A = A_s - A_n = 0$.
   Note that I is never negative.

Interference Exercise

The relative phase of two waves also depends on the relative distances to the sources:



The two waves at this point are "out of phase". Their phase difference ϕ depends on the path difference $\delta \equiv r_2 - r_1$.

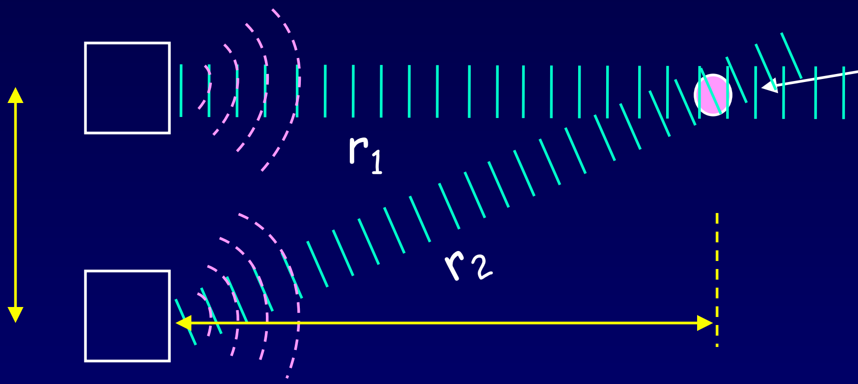
Path difference δ	Phase difference ϕ	$A = 2A_1\cos(\phi/2)$	I
0			
$\lambda/4$			
$\lambda/2$			
λ			

Each fraction of a wavelength of path difference gives that fraction of 360º (or 2π) of phase difference:

$$\phi = 2\pi\frac{\delta}{\lambda} = 360^\circ\frac{\delta}{\lambda}$$

Solution

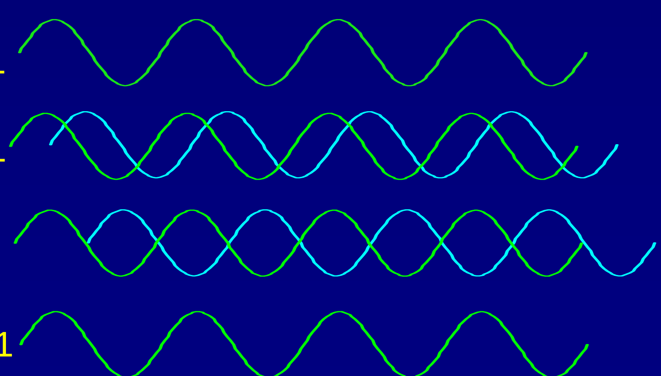
The relative phase of two waves also depends on the relative distances to the sources:



The two waves at this point are "out of phase". Their phase difference ϕ depends on the path difference $\delta \equiv r_2 - r_1$.

Reminder: A can be negative.
"Amplitude" is the absolute value.

Path difference δ	Phase difference ϕ	$A = 2A_1\cos(\phi/2)$	I
0	0	$2A_1$	$4I_1$
$\lambda/4$	$\pi/2$	$\sqrt{2}A_1$	$2I_1$
$\lambda/2$	π	0	0
λ	2π	$2A_1$	$4I_1$

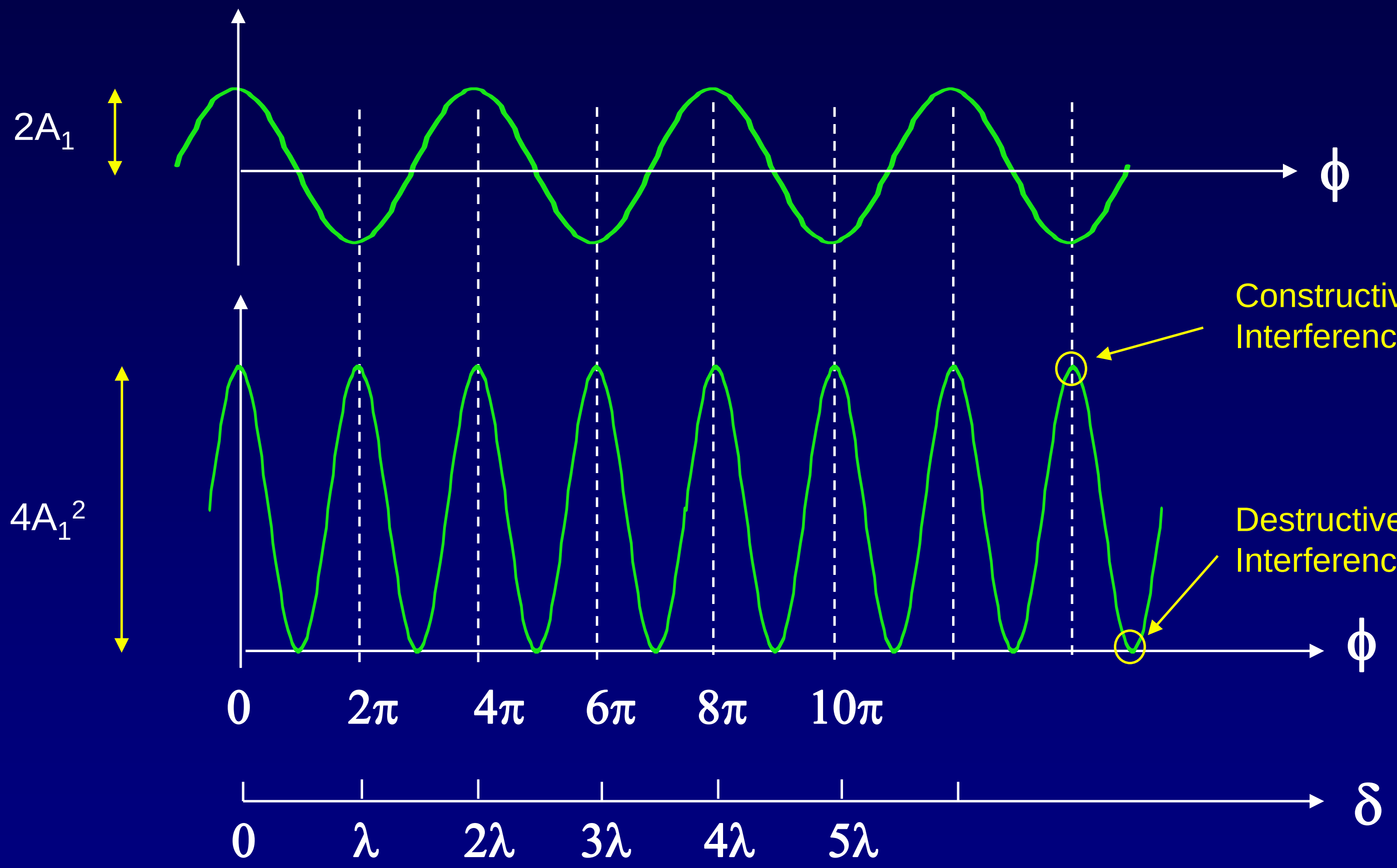


Each fraction of a wavelength of path difference gives that fraction of 360º (or 2π) of phase difference:

$$\phi = 2\pi\frac{\delta}{\lambda} = 360°\frac{\delta}{\lambda}$$

Amplitude vs. Intensity for 2 Interfering Waves

Plot $2A_1\cos(\phi/2)$ and $4A_1^2\cos^2(\phi/2)$ as a function of ϕ.

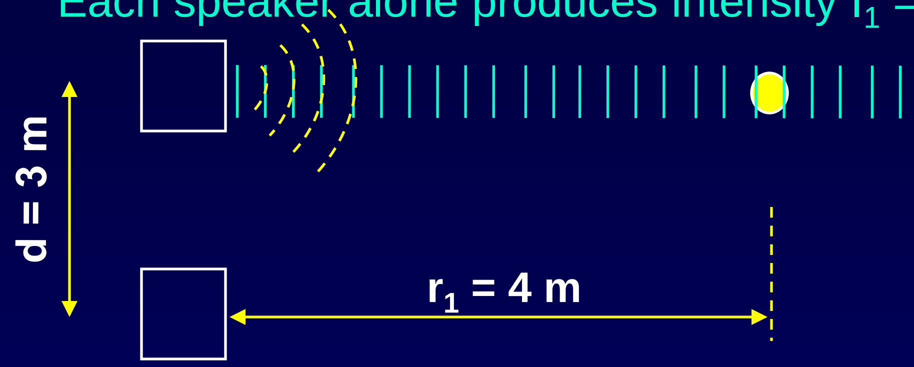


Q: What is the spatial average intensity?

A: $I_{av} = 4I_1 * 0.5 = 2I_1$ Does this make sense?

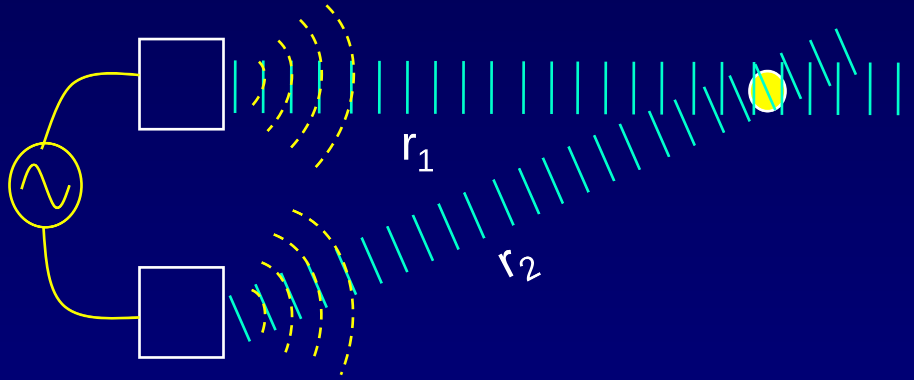
Example: Path-Length Dependent Phase

Each speaker alone produces intensity I_1 = 1W/m^2 at the listener, and f = 300 Hz.



Sound velocity: v = 330 m/s

Drive speakers in phase. Compute the intensity I at the listener in this case:



Procedure:

1) Compute path-length difference: δ =
2) Compute wavelength: λ =
3) Compute phase difference: ϕ =
4) Write formula for resultant amplitude: A =
5) Compute the resultant intensity: $I = A^2$ =

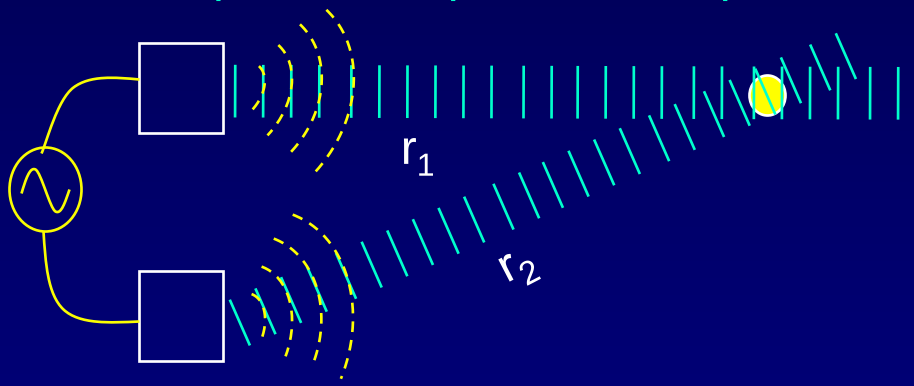
Nice demo on web: www.falstad.com/interference

Solution

Each speaker alone produces intensity I_1 = 1W/m^2 at the listener, and f = 300 Hz.

Sound velocity: v = 330 m/s

Drive speakers in phase. Compute the intensity I at the listener in this case:



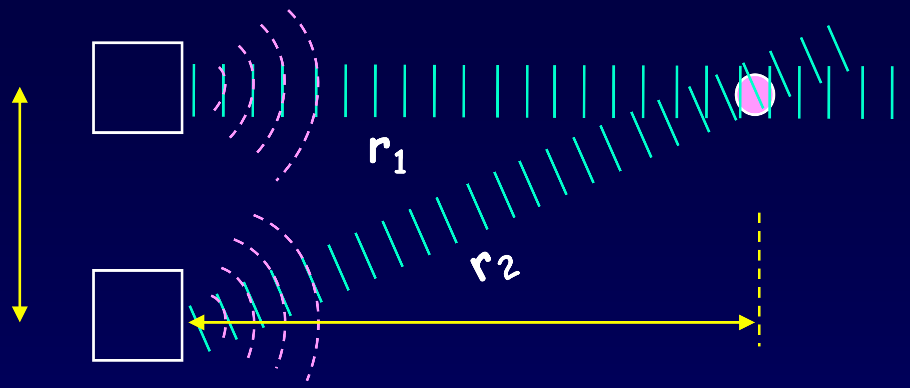
Procedure:

1) Compute path-length difference: δ = 5 m - 4 m = 1 m
2) Compute wavelength: λ = v/f = 330 m/s / 300 Hz = 1.1 m
3) Compute phase difference: ϕ = 2π(1 m / 1.1 m) = 5.71 rad = 327°
4) Write formula for resultant amplitude: A = $2A_1\cos(\phi/2)$ = 2*1*cos(2.86) = -1.92
5) Compute the resultant intensity: I = A^2 = 3.69 W/m^2

The - sign is not significant.
We care about |A|.

Nice demo on web: www.falstad.com/interference

Act 1: Speaker interference

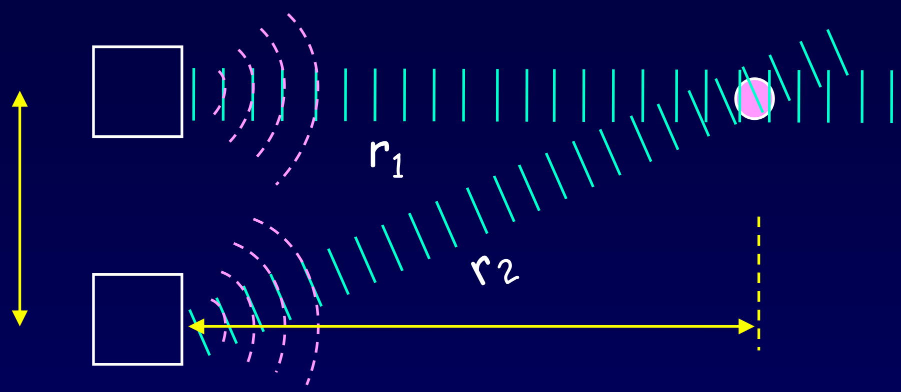


What happens to the intensity at the listener if we decrease the frequency f by a small amount?

a. decrease   b. stay the same   c. increase

Hint: How does intensity vary with ϕ when $\phi = 327°$?

Solution



What happens to the intensity at the listener if we decrease the frequency f by a small amount?

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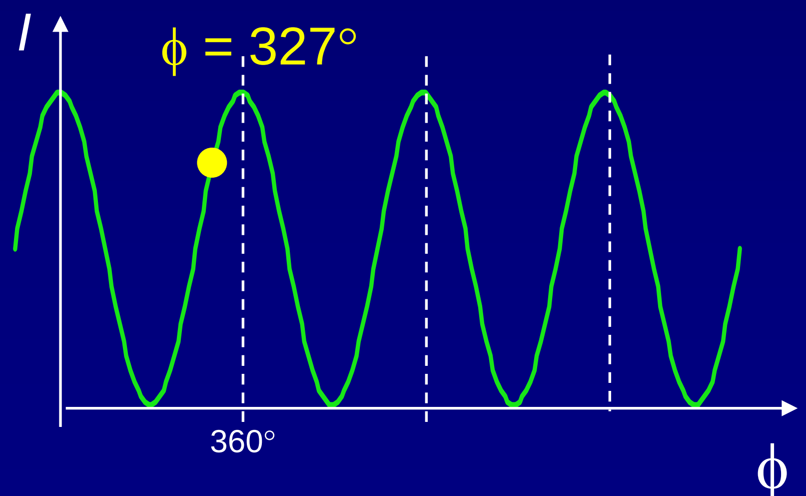
Hint: How does intensity vary with ϕ when $\phi = 327°$?

Draw the graph of $I(\phi)$:



f decreases:

→ λ increases

→ δ/λ decreases

→ ϕ decreases

→ I decreases

Summary

Interference of coherent waves

Resultant intensity of two equal-intensity waves of the same wavelength at the same point in space:

$$I = 4\, I_1 \cos^2(\phi/2)$$

In order to calculate I, we need to know ϕ.

For unequal intensities, the maximum and minimum intensities are

$$I_{max} = |A_1 + A_2|^2$$
$$I_{min} = |A_1 - A_2|^2$$

The phase difference between the two waves may be due to a difference in their source phases or in the path difference to the observer, or both. The difference due to path difference is:

$$\phi = 2\pi(\delta/\lambda)$$

where $\delta = r_2 - r_1$

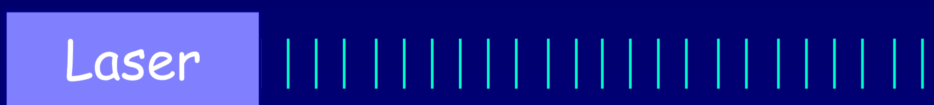
Note: The phase difference can also be due to an index of refraction, because that will change the wavelength.

FYI:Coherent and Incoherent Waves

We only observe interference when the sources have a definite (usually constant) phase difference. In this case, the sources are said to be coherent.

Examples of coherent sources:

- Sound waves from speakers driven by electrical signals that have the same frequency and a definite phase.
- Laser light. In a laser, all the atoms emit light with the same frequency and phase. This is a quantum effect that we'll study later in the course.



The laser light is also all going the same direction.

Incoherent waves: The phase relation is random.

Waves from two unrelated sources.

- Examples: light from two points on the sun or two atoms on a light bulb filament, or two people singing the same note.
- Incoherent intensities add. The average of constructive and destructive interference is no interference!

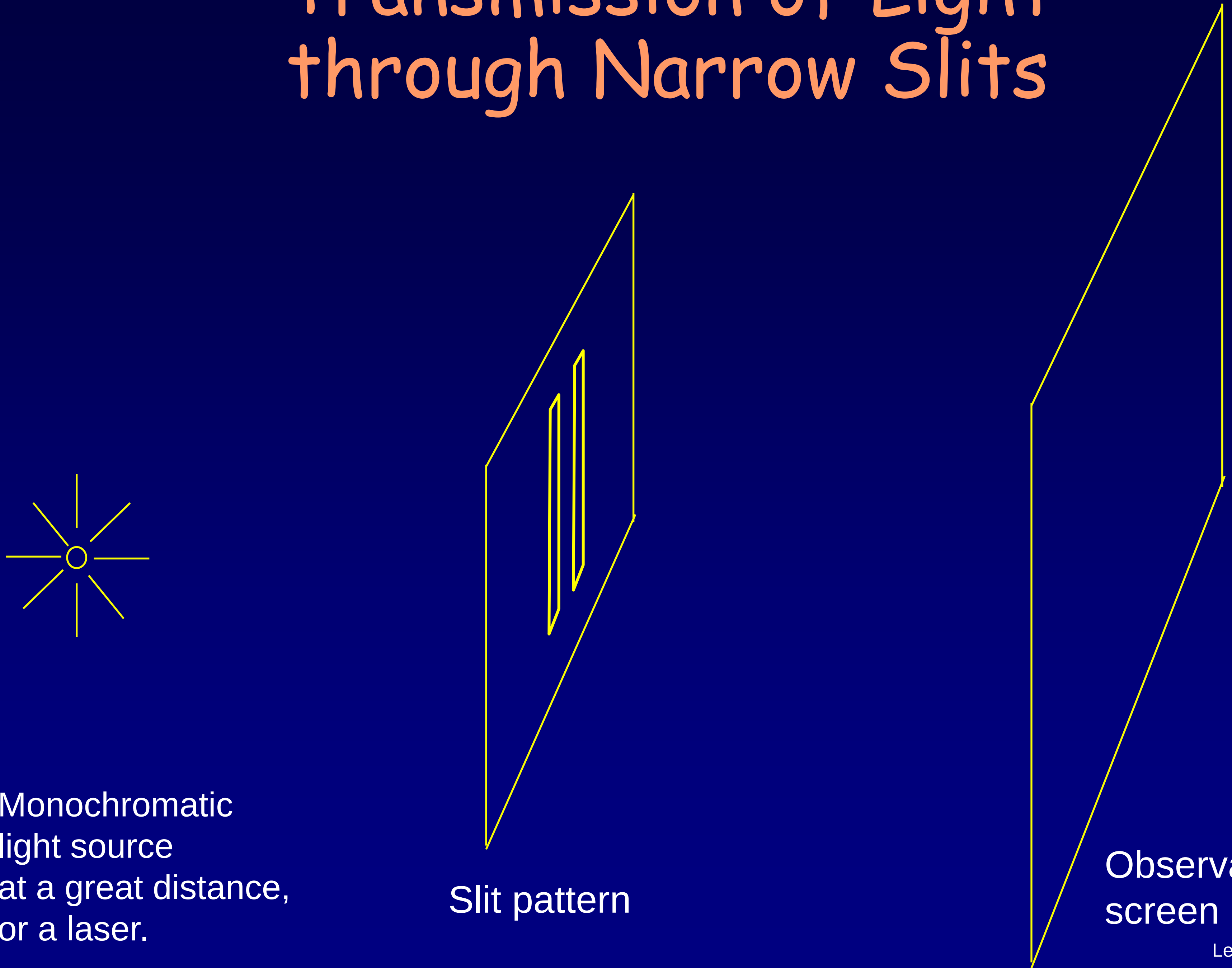
Light - Particle or Wave?

Diffraction of light played an important historical role.

- 1818: French Academy held a science competition
- Fresnel proposed the diffraction of light.
- One judge, Poisson, knew light was made of particles, and thought Fresnel's ideas ridiculous; he argued that if Fresnel ideas were correct, one would see a bright spot in the middle of the shadow of a disk.
- Another judge, Arago, decided to actually do the experiment…
(our lecture demo)

- **Conclusion:**
Light *must* be a wave, since particles don't diffract!

Transmission of Light through Narrow Slits

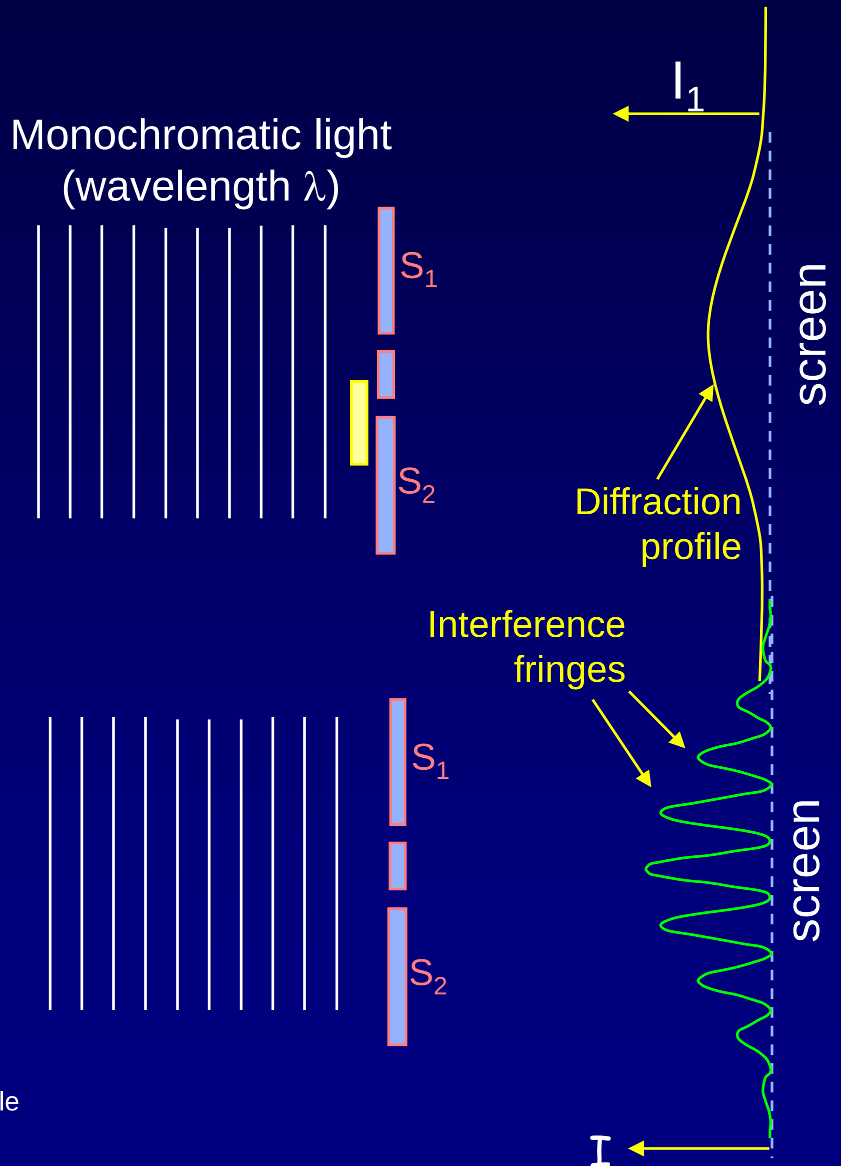


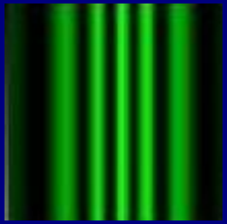
Double-slit interference

Light (wavelength λ) is incident on a two-slit (two narrow, rectangular openings) apparatus:

If either one of the slits is closed, a spread-out image of the open slit will appear on the screen. (The image is spread due to diffraction. We will discuss diffraction in more detail later.)

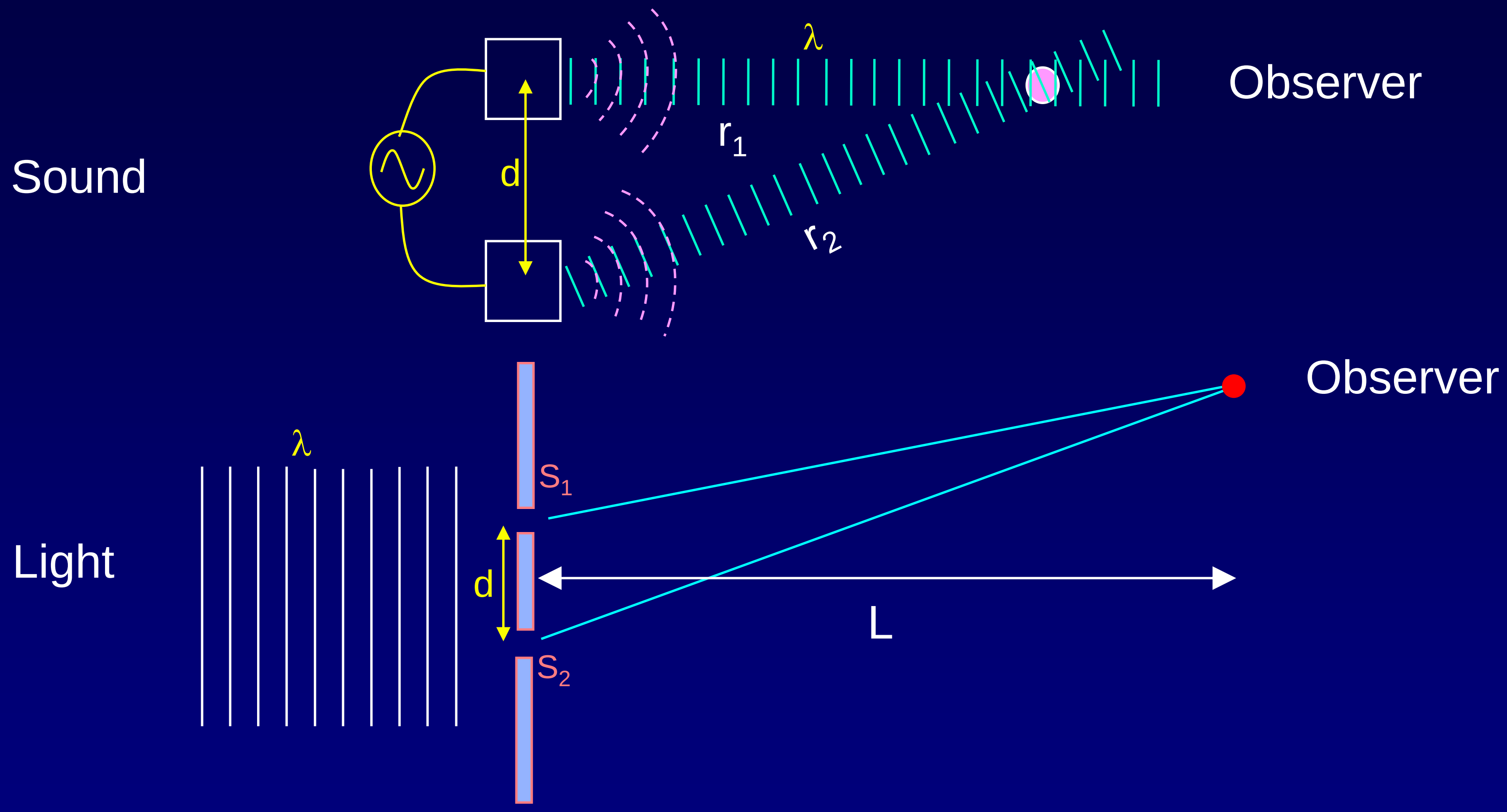
If both slits are open, we see interference “fringes” (light and dark bands), corresponding to constructive and destructive interference of the wave passing through the two slits.





Note: In the laser demo, there is little vertical spread, because the laser spot is small in that direction.

Sound and Light Waves Interfere the Same Way

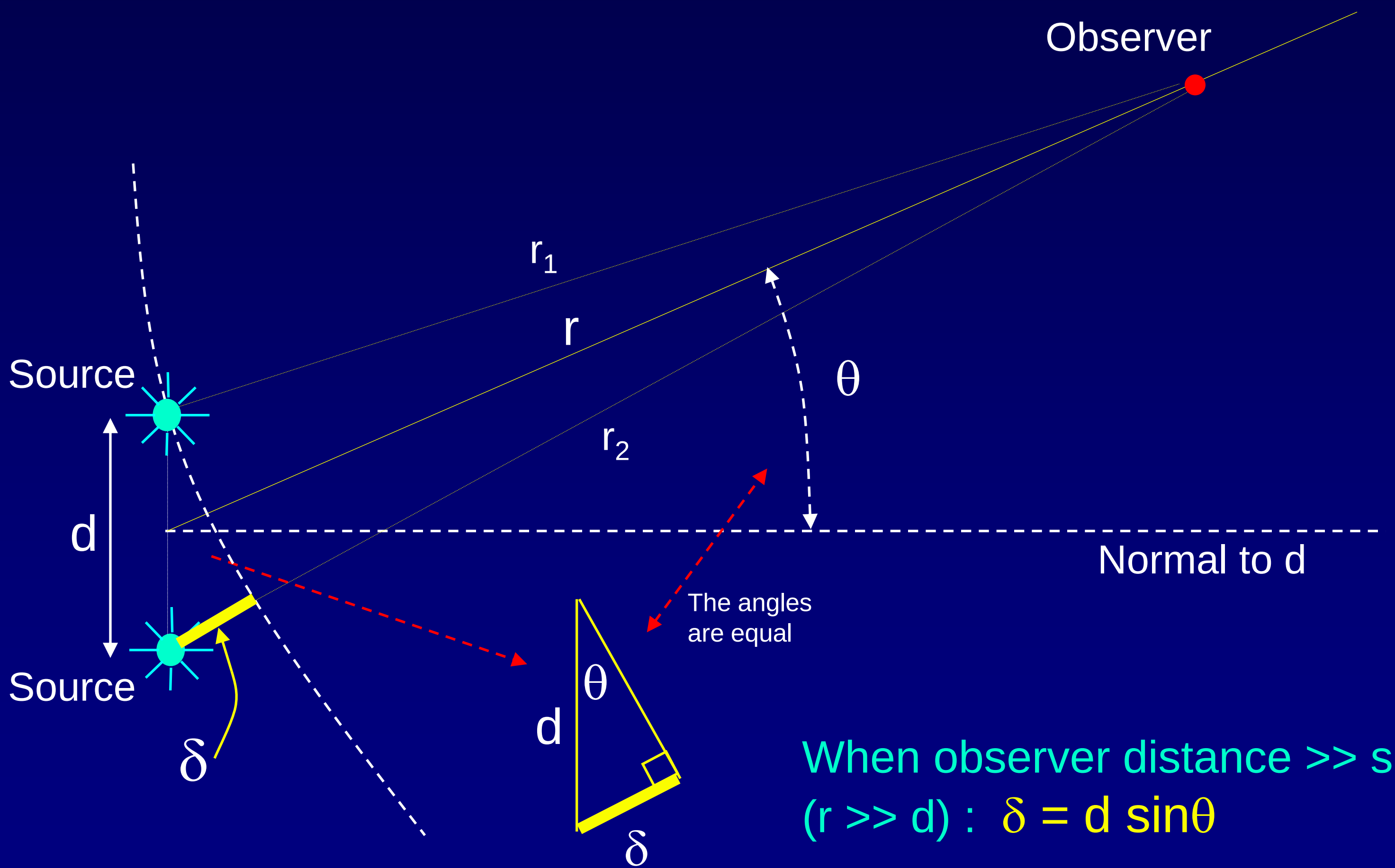


In both cases, $I = 4\,I_1 \cos^2(\phi/2)$ with $\phi = 2\pi(\delta/\lambda)$, $\delta = r_2 - r_1$

However, for light, the distance L is generally much greater than the wavelength λ and the slit spacing d: $L \gg \lambda$, $L \gg d$.

Simple formula for the path difference, δ , when the observer is far from sources.

Assume 2 sources radiating in phase:



When observer distance >> slit spacing

(r >> d) : $\delta = d \sin\theta$

Therefore: $\phi = 2\pi(\delta/\lambda) = 2\pi(d \sin\theta / \lambda)$

Two-Slit Interference

Constructive interference:

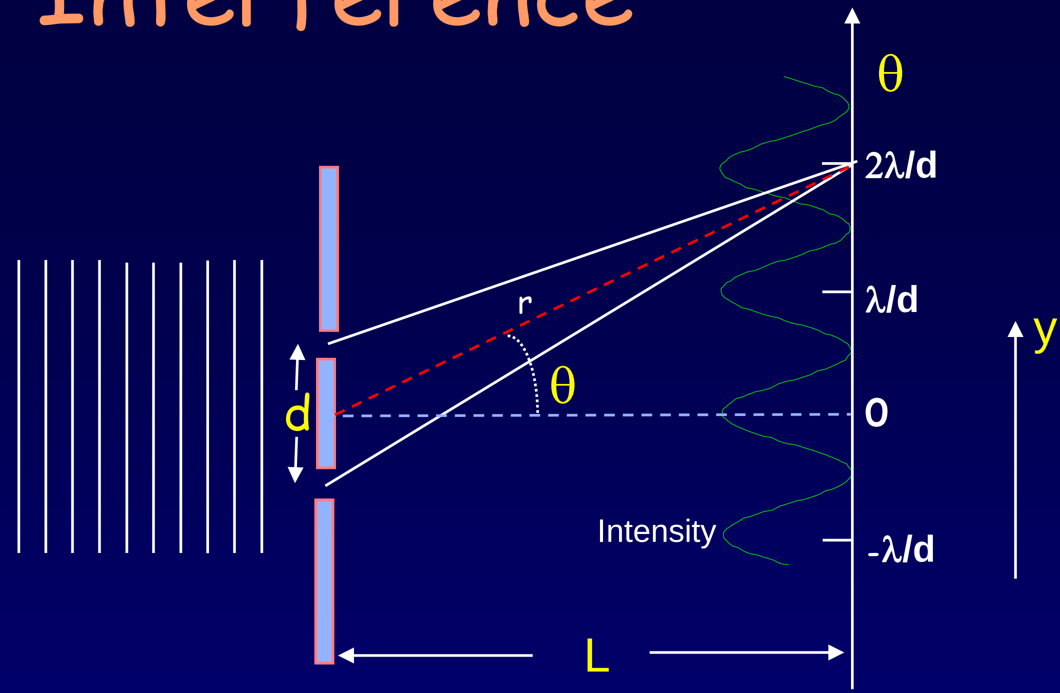
$\delta = d\sin\theta = m\lambda$

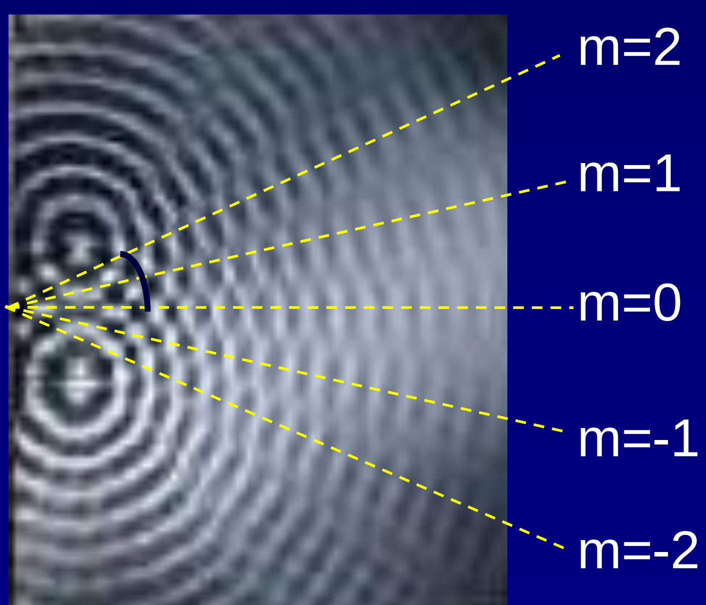
$m = 0, \pm1, \pm2, \ldots$

Destructive Interference:

$\delta = d\sin\theta = (m+½)\lambda$

$m = 0, \pm1, \pm2, \ldots$





Lines of constructive interference

Usually we care about the linear displacement y of the pattern (because our screens are flat):

$y = L\tan\theta$

Two-Slit Interference, small angles:

Often, $d \gg \lambda$, so that θ is small.

Then we can use the small angle approximation to simplify our results:
For small angles: ($\theta \ll 1$ radian):
$\sin\theta \approx \theta \approx \tan\theta$ (only in radians!)

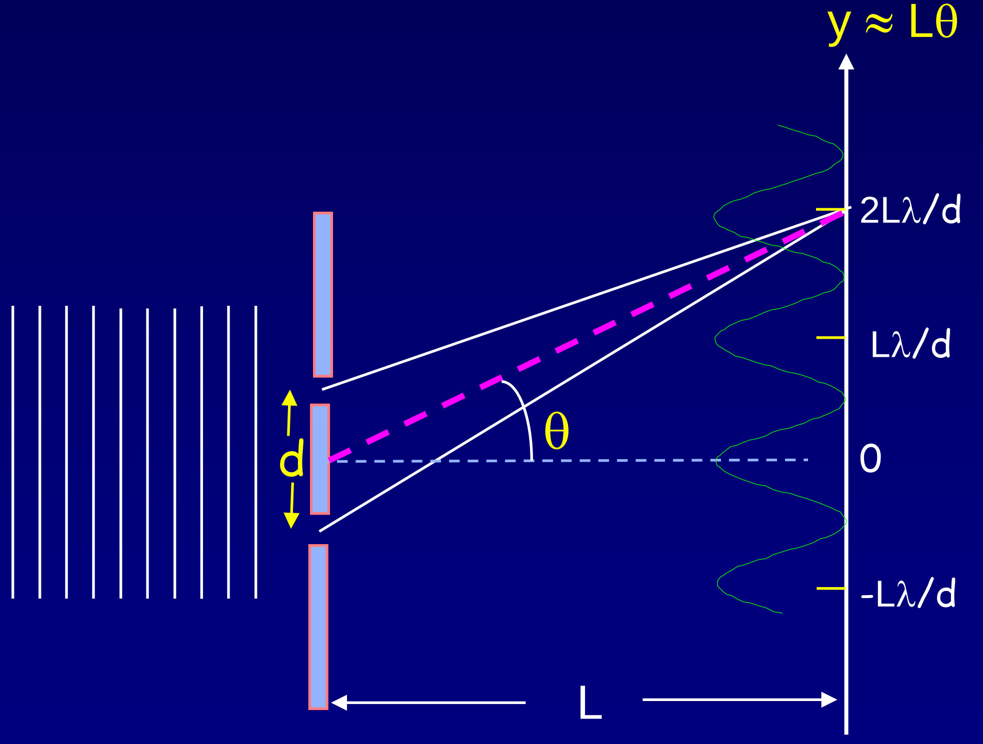
$y = L\tan\theta \approx L\theta$

Constructive interference:

$\theta \approx m(\lambda/d)$
$y \approx m(\lambda/d)L$
$m = 0, \pm 1, \pm 2, \ldots$

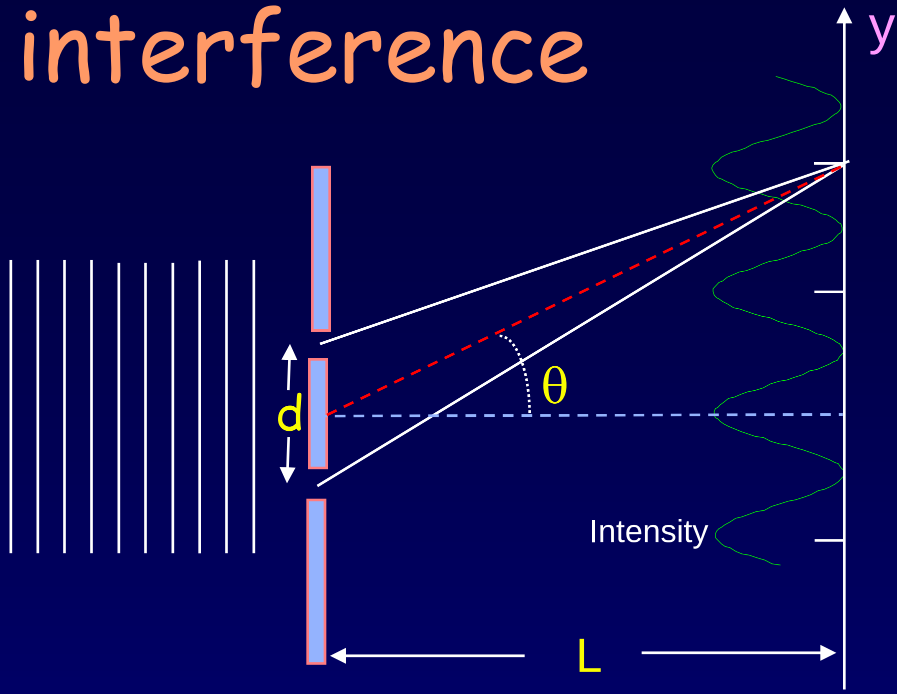
Destructive interference:

$\theta \approx (m+½)(\lambda/d)$
$y \approx (m+½)(\lambda/d)L$
$m = 0, \pm 1, \pm 2, \ldots$



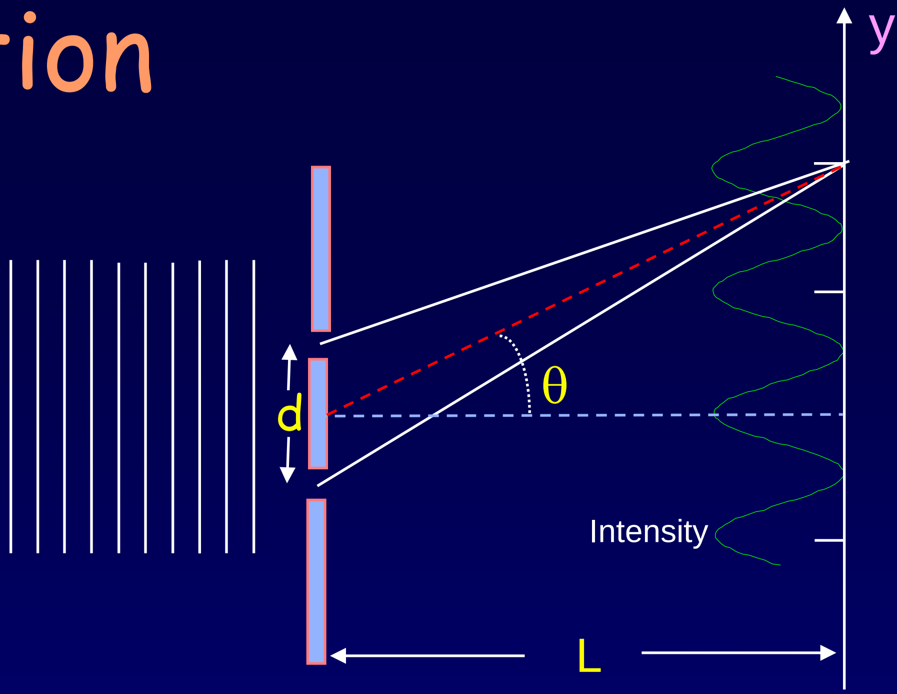
Example: 2-slit interference

A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.



1. What is the spacing Δy between adjacent fringe maxima (*i.e.*, $\Delta m = 1$) on a screen 2m away?

Solution



A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.

1. What is the spacing Δy between adjacent fringe maxima (*i.e.*, $\Delta m = 1$) on a screen 2m away?

First: Can we use the small angle approximation?

$d = 125\ \mu m$; $\lambda = 0.633\ \mu m$ → $d >> \lambda$ → θ is small.

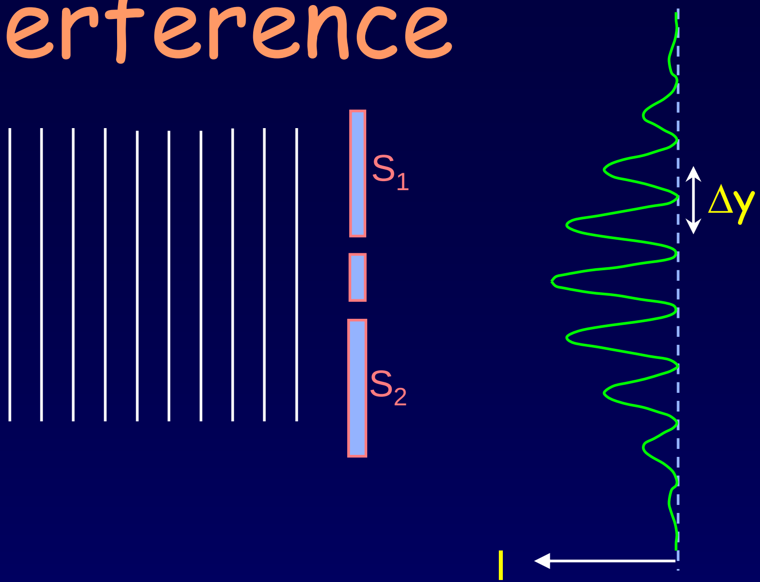
$d \sin\theta_m = m\lambda \sim d\theta_m$ → $\theta_1 \approx (\lambda/d) = (0.633/125) = 0.00451$ rad $= 0.29°$ (small!)

$\Delta y \approx L(\theta_2 - \theta_1) \approx L(2 - 1)(\lambda/d) = L\lambda/d = (2\ m)(0.633\ \mu m)/125\ mm = 0.01\ m \sim$ **1 cm**

Could have also used 1 – 0 (or 6 – 5).

Act 2: 2-slit interference

A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.



1. If we increase the spacing between the slits, what will happen to Δy?

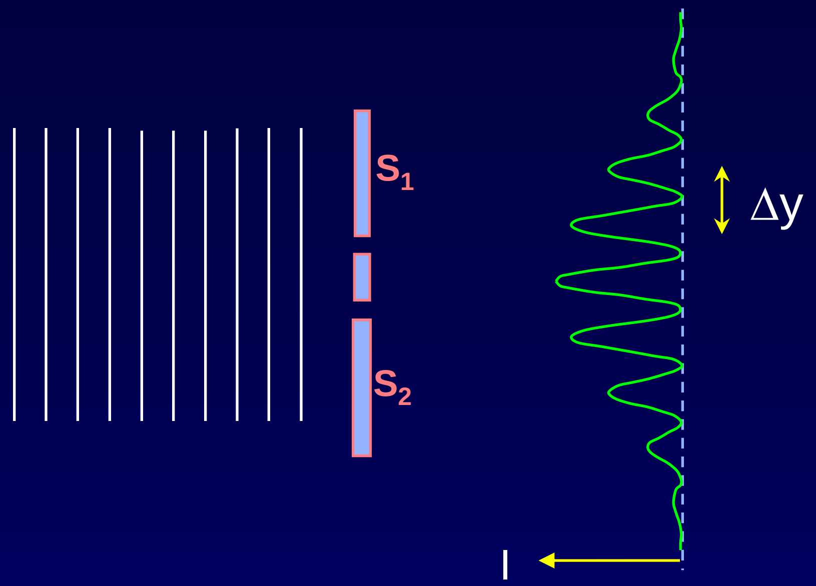
   a. decrease　　b. stay the same　　c. increase

2. If we instead use a green laser (smaller λ), Δy will?

   a. decrease　　b. stay the same　　c. increase

Solution

A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.



1. If we increase the spacing between the slits, what will happen to Δy?

   a. decrease   b. stay the same   c. increase

   $\Delta y \propto 1/d$, so it decreases. This is a general phenomenon: the more spread out the sources are, the narrower the interference pattern is.

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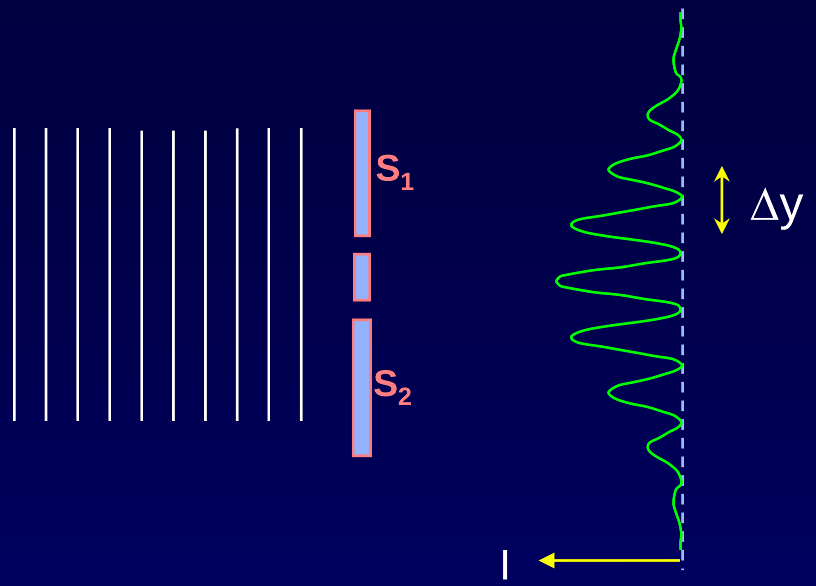

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2. If we instead use a green laser (smaller λ), Δy will?

   a. decrease   b. stay the same   c. increase

   $\Delta y \propto \lambda$, so it decreases.

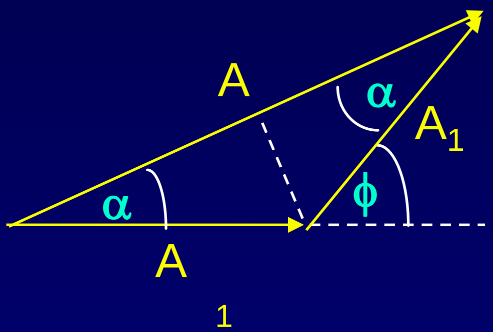
"Beam me up Scotty –
It ate my phasor!"

Phasors

- See the supplementary slide.
- See text: 35.3, 36.3, 36.4.
- See Physics 212 lecture 20.
- Phasors make it easier to solve other problems later.

Lets find the resultant amplitude of two waves using phasors.

Suppose the amplitudes are the same. Represent each wave by a vector with magnitude (A_1) and direction (ϕ). One wave has $\phi = 0$.

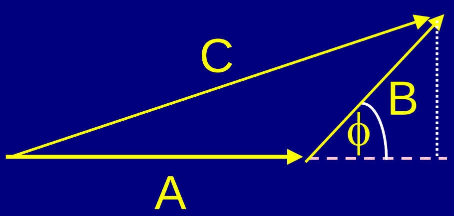


Isosceles triangle: $\alpha = \phi/2$. So, $A = 2A_1 \cos\left(\frac{\phi}{2}\right)$

This is identical to our previous result !

More generally, if the phasors have different amplitudes A and B:

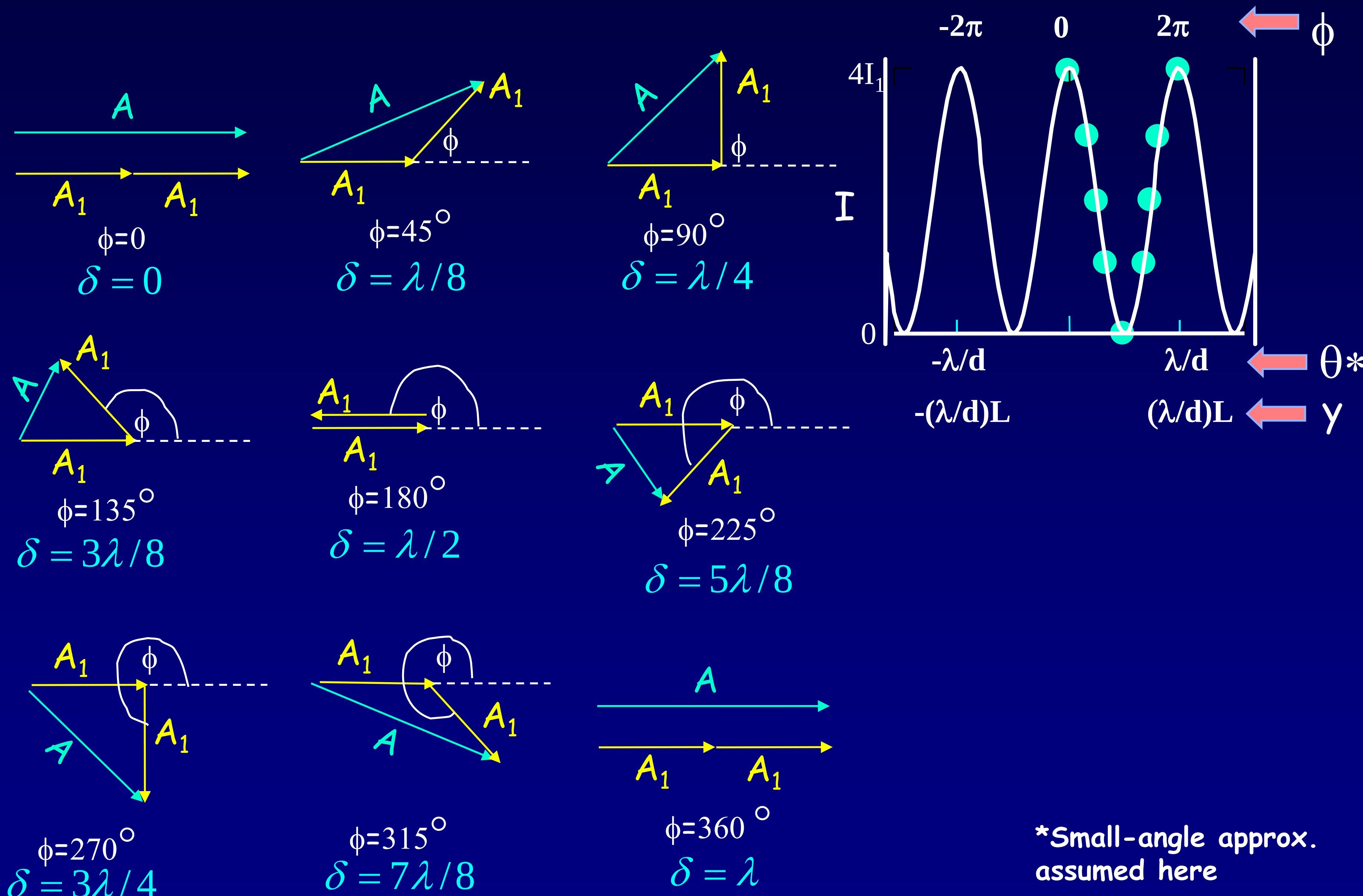
$$C^2 = A^2 + B^2 + 2AB \cos \phi$$



Here ϕ is the *external* angle.

Phasors for 2-Slits

- **Plot the phasor diagram for different ϕ:**



*Small-angle approx. assumed here

You should work through "Lect 3" slides

Phasors, review, examples, examples, examples

Next week

Diffraction from a single slit

Multiple-slit interference

Diffraction and Spectroscopy

Text – Ch. 36 + added material

Applications – resolution of telescopes and microscopes, interferometers, crystallography, etc...