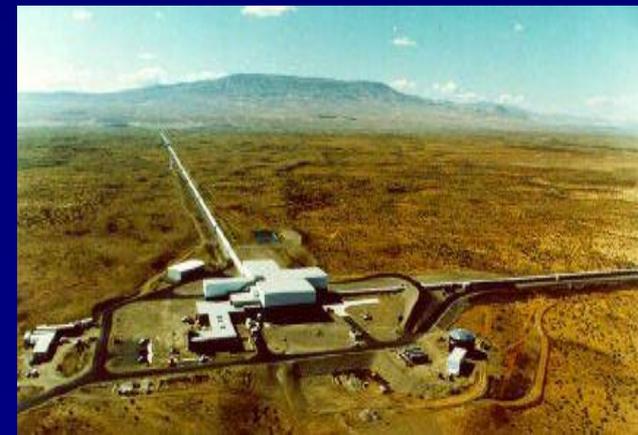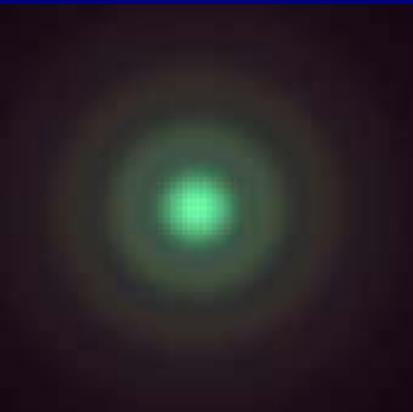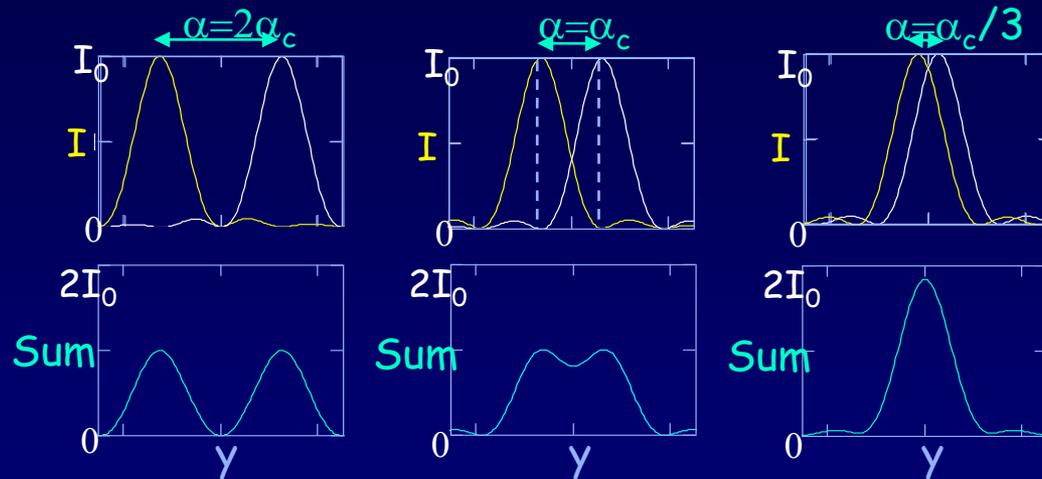# Lecture 5:

Applications of Interference and Diffraction

Today

Circular Diffraction

- Angular resolution (Rayleigh's criterion)
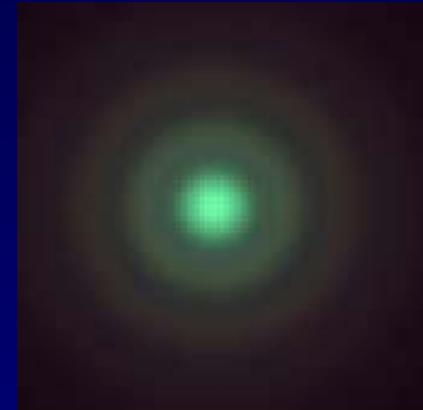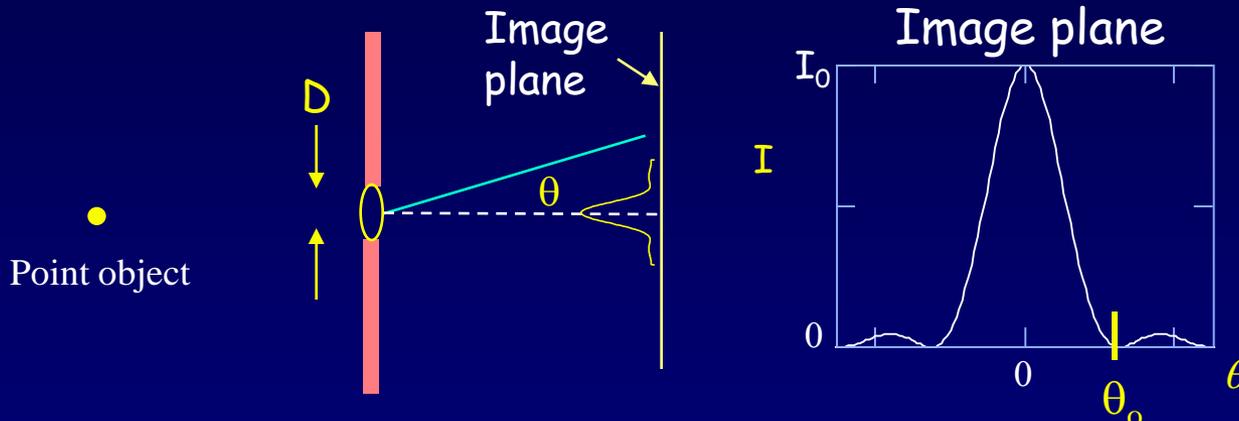- Minimum spot size

Interferometers

- Michelson
- Applications

Diffraction-limited Optics

Diffraction has important implications for optical instruments

Even for perfectly designed optics the image of a point source
will be a little blurry - the circular aperture produces diffraction.

The size of the spot is determined by the diameter, D,
of the aperture, and wavelength, λ, of the incident light.

The "Airy disk".
The central lobe contains
84% of power.

Diffraction by a circular aperture is similar to single-slit
diffraction. But note the difference:

Slit	$\theta_0 \approx \dfrac{\lambda}{a}$

Circular aperture	$\theta_0 \approx 1.22 \dfrac{\lambda}{D}$

Slits and circular apertures

Monochromatic light source at a great distance, or a laser.

Slit, width a
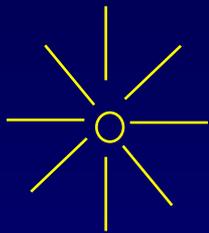
Observation screen:

$$I_0$$
$$I$$
$$0$$
$$-\lambda/a \quad 0 \quad \lambda a \quad \theta$$
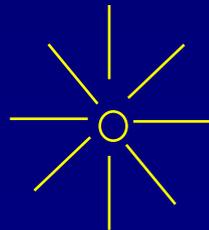
Pinhole, diameter D

Observation screen:

$$I_0$$
$$I$$
$$0$$
$$-1.22\lambda/D \quad 0 \quad 1.22\lambda/D \quad \theta$$

Object at any distance:

Lens, diameter D
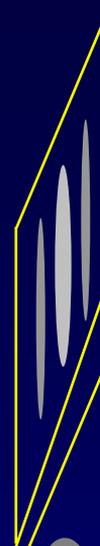
Image Plane:
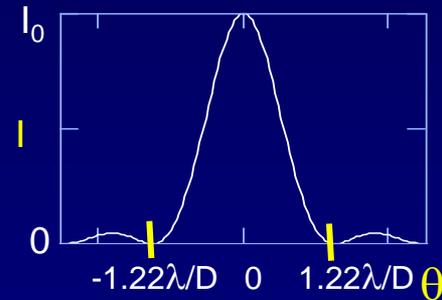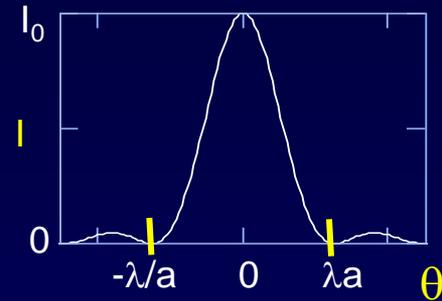
The focusing effect of the lens is independent of the diffraction effect due to the aperture

Exercise: Expansion of a Laser beam

In 1985, a laser beam with a wavelength of λ = 500 nm was fired from the earth and reflected off the space shuttle Discovery, in orbit at a distance of L = 350 km away from the laser.

If the circular aperture of the laser was D = 4.7 cm, what was the beam diameter d at the space shuttle?

Solution

In 1985, a laser beam with a wavelength of $\lambda = 500$ nm was fired from the earth and reflected off the space shuttle Discovery, in orbit at a distance of $L = 350$ km away from the laser.
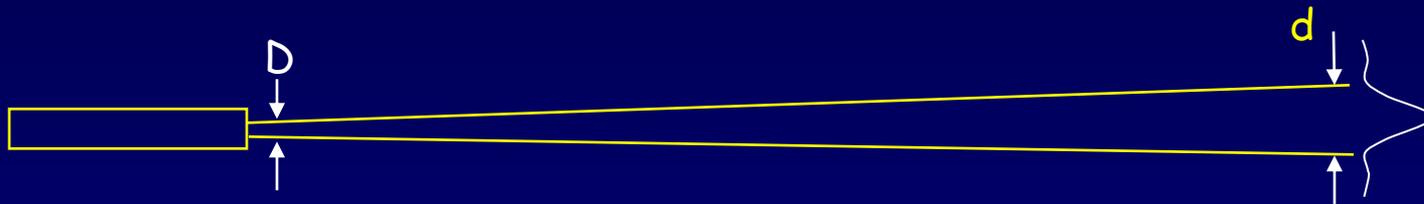
If the circular aperture of the laser was $D = 4.7$ cm, what was the beam diameter d at the space shuttle?

Half-angle-width of diffraction maximum:

$$\theta_o = 1.22\frac{\lambda}{D} = 1.22\frac{500 \times 10^{-9}}{4.7 \times 10^{-2}} = 1.3 \times 10^{-5} \text{ radians}$$

$$d \approx 2\theta_o L = 2(1.3 \times 10^{-5})(350 \times 10^3 \text{m}) = \boxed{9.1 \text{ m}}$$

Act 1

In 1985, a laser beam with a wavelength of $\lambda = 500$ nm was fired from the earth and reflected off the space shuttle Discovery, in orbit at a distance of L = 350 km away from the laser.
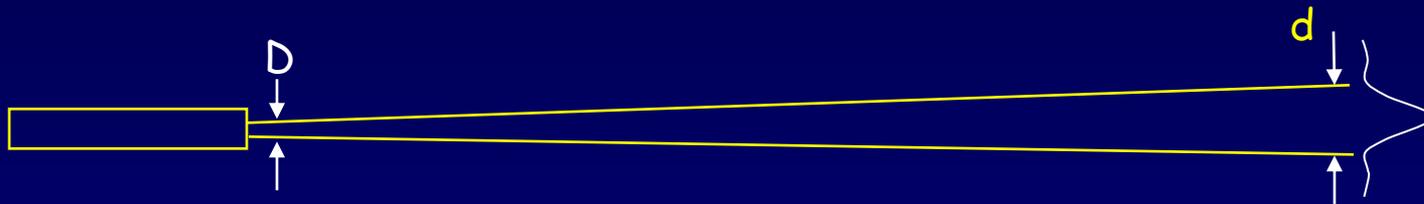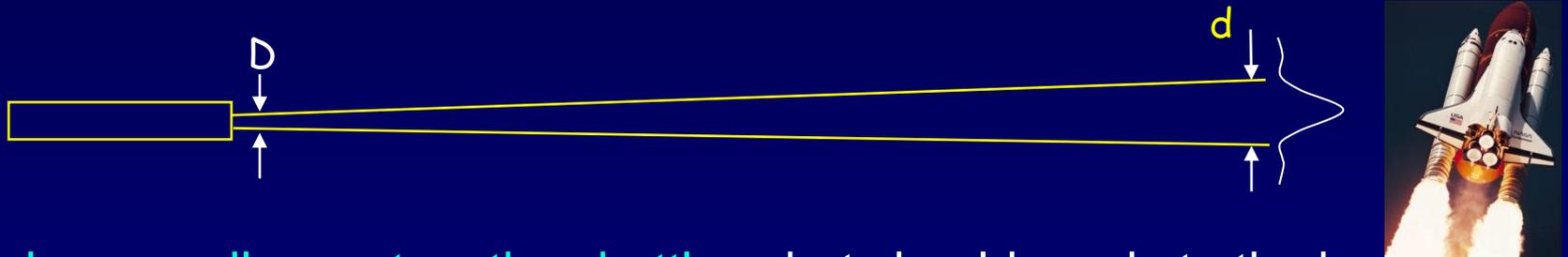

To make a smaller spot on the shuttle, what should we do to the beam diameter at the source?

      a. reduce it   b. increase it   c. cannot be made smaller

Solution

In 1985, a laser beam with a wavelength of λ = 500 nm was fired from the earth and reflected off the space shuttle Discovery, in orbit at a distance of L = 350 km away from the laser.
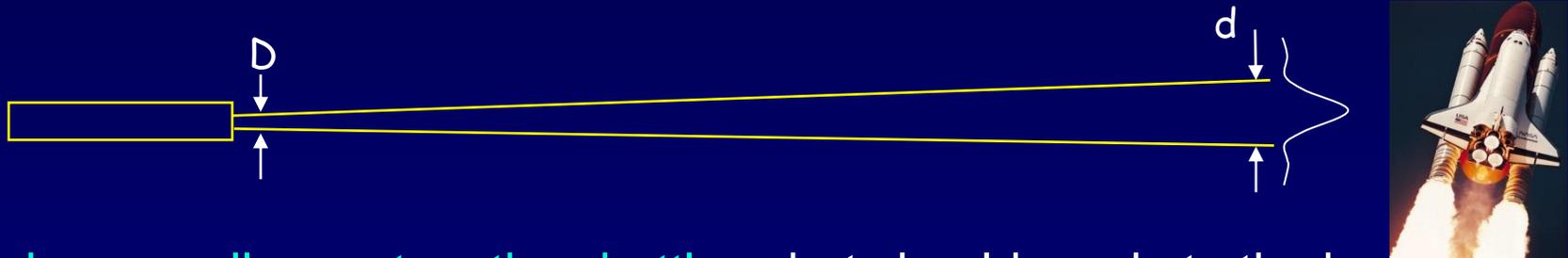
To make a smaller spot on the shuttle, what should we do to the beam diameter at the source?

a. reduce it   b. increase it   c. cannot be made smaller

Counter-intuitive as this is, it is correct – you reduce beam divergence by using a bigger beam. (Note: this will work as long as D < d.)

We'll see that this can be understood as a non-quantum version of the uncertainty principle: $\Delta x \, \Delta p_x > \hbar$.

Exercise: Focusing a laser beam

There are many times you would like to focus a laser beam to as small a spot as possible. However, diffraction limits this.
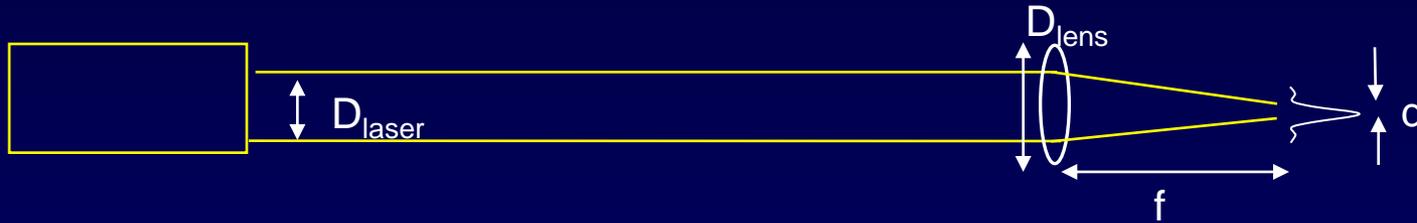
The circular aperture of a laser (λ = 780 nm) has D_{laser} = 5 mm.
What is the spot-size d of the beam
after passing through a perfect lens with focal length f = 5mm
and diameter D_{lens} = 6 mm?

Solution

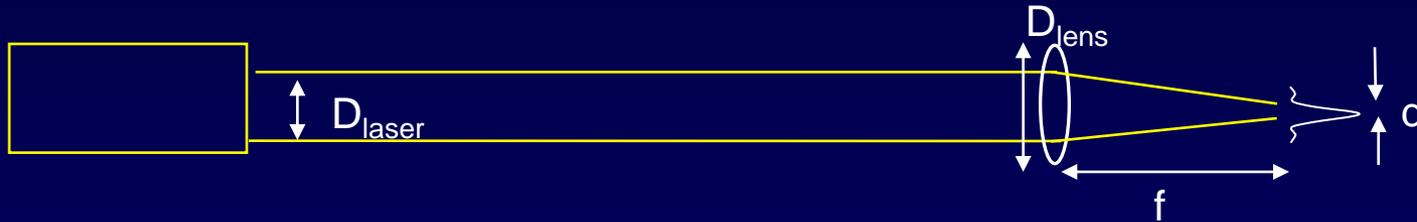There are many times you would like to focus a laser beam to as small a spot as possible. However, diffraction limits this.

The circular aperture of a laser (λ = 780 nm) has D_{laser} = 5 mm.
What is the spot-size d of the beam
after passing through a perfect lens with focal length f = 5mm
and diameter D_{lens} = 6 mm?

The angular spread of the beam is determined by the smaller of D_{laser} and D_{lens}. Here, it's D_{laser}.

$$\theta_o = 1.22\lambda / D_{laser}$$

Light at this angle will intercept the focal plane at d/2 ~ f θ_o.

$$d \approx 2\theta_o f = 2.44\lambda f / D_{laser}$$

$$= 2.44(0.78\mu m)(5mm)/(5mm) = \boxed{1.9\mu m}$$

Act 2

There are many times you would like to focus a laser beam to as small a spot as possible. However, diffraction limits this.

(See lecture 4, exercise 2)
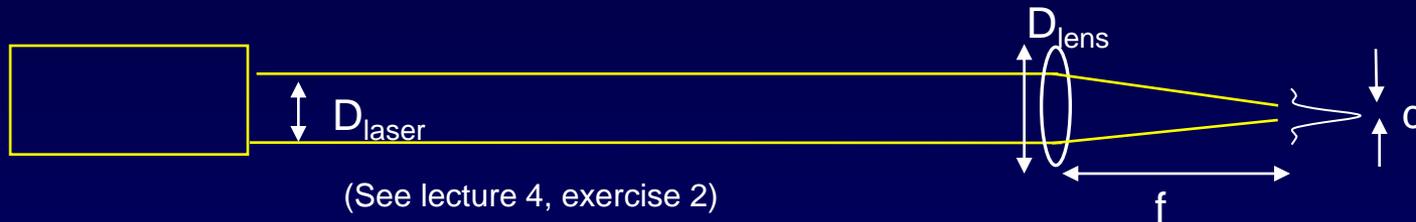
$\lambda = 780$ nm, $D_{laser} = 5$ mm, $f = 5$ mm, $D_{lens} = 6$ mm.
Which of the following will reduce the spot size?

a. increase λ b. decrease λ c. increase D_{lens} d. decrease D_{lens}

Solution

There are many times you would like to focus a laser beam to as small a spot as possible. However, diffraction limits this.
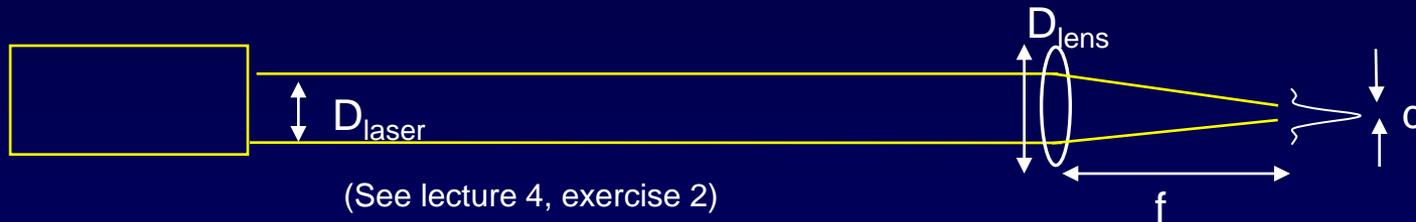

(See lecture 4, exercise 2)

$\lambda = 780$ nm, $D_{laser} = 5$ mm, $f = 5$ mm, $D_{lens} = 6$ mm.
Which of the following will reduce the spot size?

a. increase λ b. decrease λ c. increase D_{lens} d. decrease D_{lens}

The diffraction is already limited by D_{laser}. Increasing D_{lens} doesn't help.

There is a huge industry devoted to developing cheap blue diode lasers ($\lambda \sim 400$ nm) for just this purpose, i.e., to increase DVD capacity.
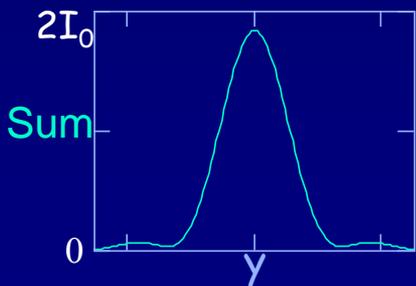
"Blue-Ray" technology!

Angular Resolution

Diffraction also limits our ability to "resolve" (*i.e.*, distinguish) two point sources. Consider two point sources (e.g., stars) with angular separation α viewed through a circular aperture or lens of diameter D.

Two point sources

α

D

α
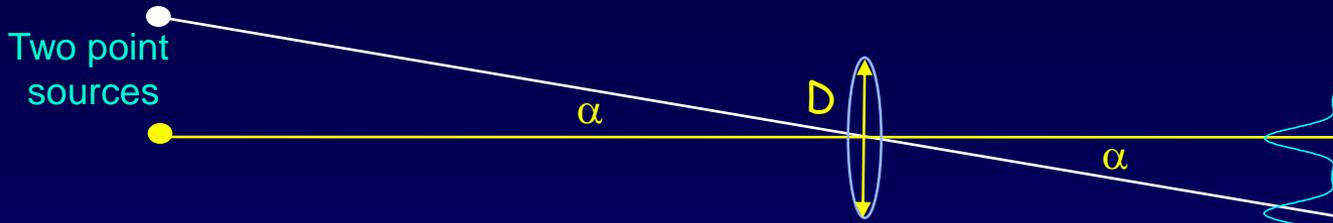
Rayleigh's Criterion defines the images to be resolved if the central maximum of one image falls on or further than the first minimum of the second image.

$$\alpha_c = 1.22 \frac{\lambda}{D}$$

$\alpha = 2\alpha_c$

I_0

I

0

$\alpha = \alpha_c$

I_0

I

0

$\alpha = \alpha_c/3$

I_0

I

0

$2I_0$

Sum

0

y

$2I_0$

Sum

0

y

$2I_0$

Sum

0

y
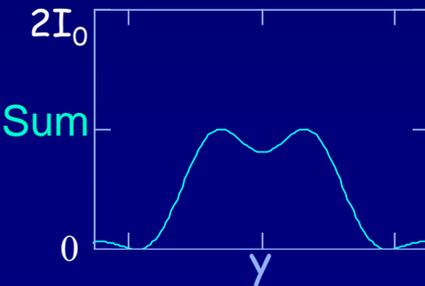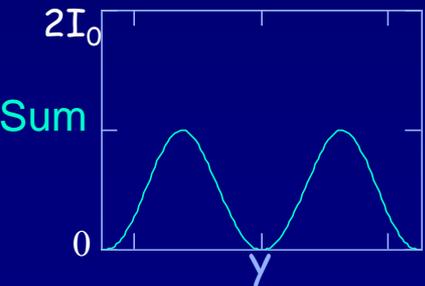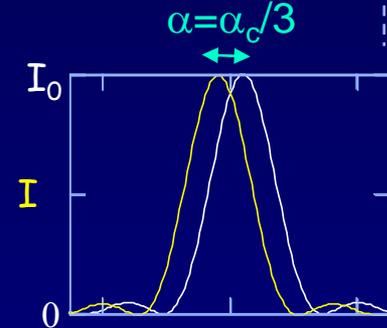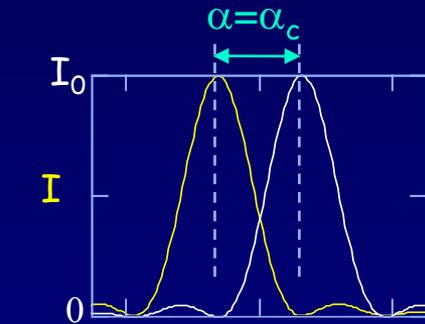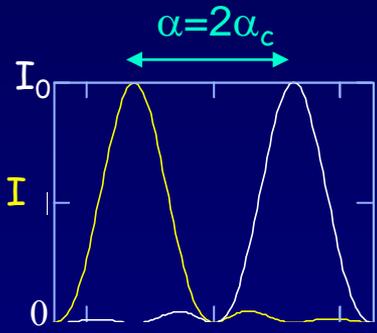
Two images resolvable

'Diffraction limit' of resolution

Two images not resolvable

NOTE:
No interference!!
Why not?

FYI:Coherent and Incoherent Waves

We only observe interference when the sources have a definite (usually constant) phase difference. In this case, the sources are said to be coherent.

Examples of coherent sources:

- Sound waves from speakers driven by electrical signals that have the same frequency and a definite phase.

- Laser light. In a laser, all the atoms emit light with the same frequency and phase. This is a quantum effect that we'll study later in the course.
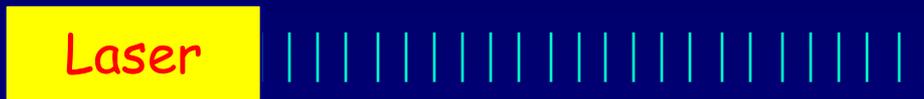
Laser | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

The laser light is also all going the same direction.

Incoherent waves: The phase relation is random.

Waves from two unrelated sources.

- Examples: light from two points on the sun or two atoms on a light bulb filament, or two people singing the same note.

- Incoherent intensities add. The average of constructive and destructive interference is no interference!

Act 3: Resolving Stars

Halley's Comet

1. Assuming diffraction-limited optics, what is the minimum angular separation of two stars that can be resolved by a $D = 5$ m telescope using light of $\lambda = 500$ nm?

  a. 0.1 μrad   b. 1 μrad   c. 10 μrad

2. If the two point sources are not quite resolved at screen 1, will they be resolved at screen 2?

  a. Yes   b. No

screen 1   screen 2

Solution

Halley's Comet

1. Assuming diffraction-limited optics, what is the minimum angular separation of two stars that can be resolved by a D = 5 m telescope using light of λ = 500 nm?

   a. 0.1 μrad   b. 1 μrad   c. 10 μrad

$$\alpha_c = 1.22 \frac{\lambda}{D} \approx 1 \times 10^{-7} = 0.1 \mu rad$$

2. If the two point sources are not quite resolved at screen 1, will they be resolved at screen 2?

   a. Yes   b. No

screen 1   **screen 2**

Solution

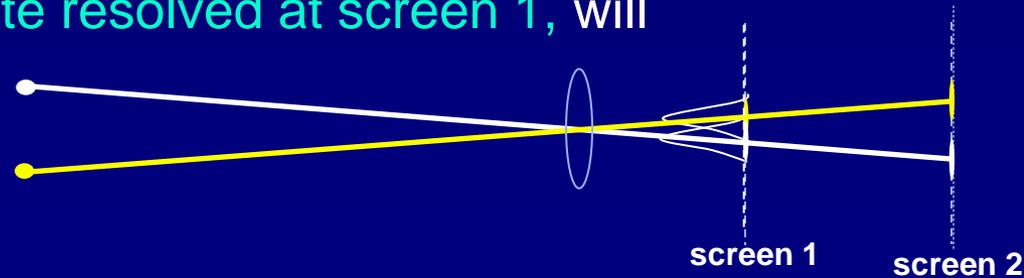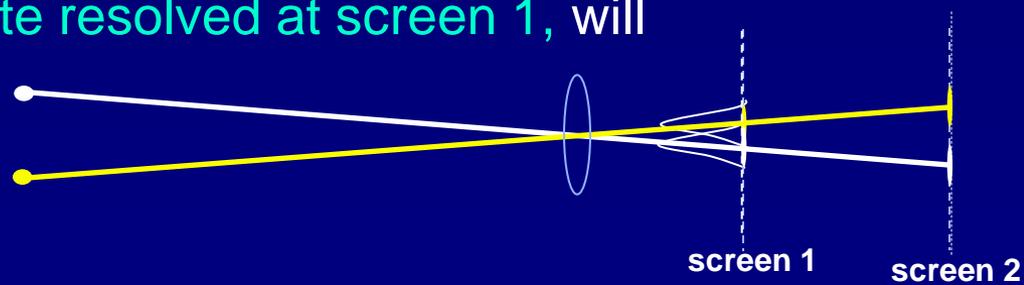**Halley's Comet**

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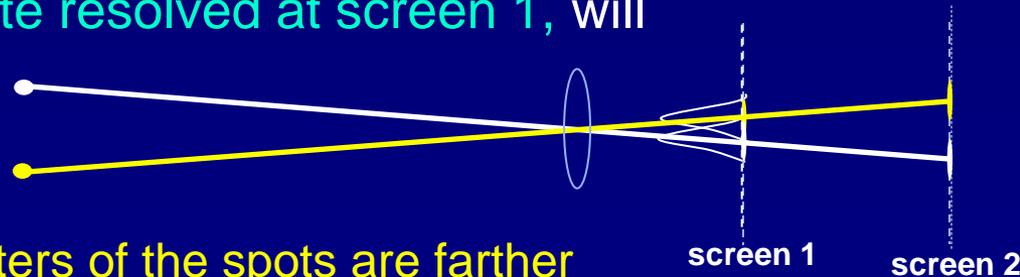
    a. Yes   b. No

screen 1   **screen 2**

α_c only depends on λ and D. The centers of the spots are farther apart, but the spots are also wider by the same amount.

Example: Camera resolution
(Next week's discussion)

Digital cameras look something like this:

Aperture, D = 3 mm

Photosensor

lens

Focal length
f = 10 mm

Photosensor:

5 mm

7 mm
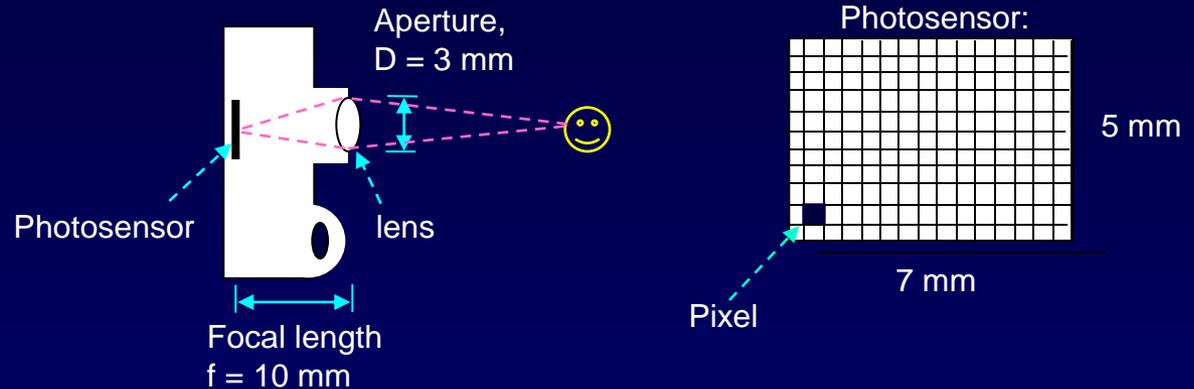
Pixel

If the distance between adjacent pixels is less than the minimum resolvable separation due to diffraction, then diffraction limits the image quality.

d
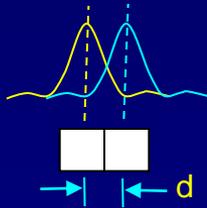
The "f-number" of a lens is defined as f/D. To minimize diffraction, you want a small f-number, i.e., a large aperture*.

*This assumes a 'perfect lens'. In practice, lens aberrations limit the resolution if D is too big.

Optical Interferometers

Interference arises whenever there are two (or more) ways for something to happen, *e.g.*, two slits for the light to get from the source to the screen.

$I = 4I_1\cos^2(\phi/2)$, with $\phi = 2\pi\delta/\lambda$, and path-length difference δ

An interferometer is a device using mirrors and "beam splitters" (half of the light is transmitted, half is reflected) to give two separate paths from source to detector.

Two common types:

Mach-Zehnder:

Michelson :

beam-splitter

mirror

mirror

beam splitter

mirrors

beam splitter
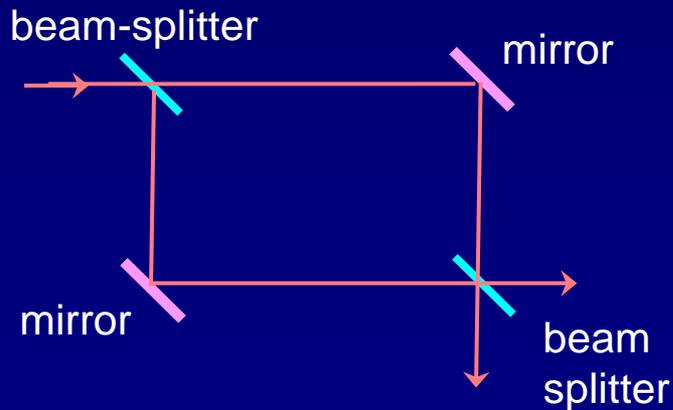
Michelson Interferometer

The Michelson interferometer works by varying the phase difference between the two paths the light can take.

One possibility is to vary the lengths L_1 or L_2.

This makes possible very accurate measurements of displacements.
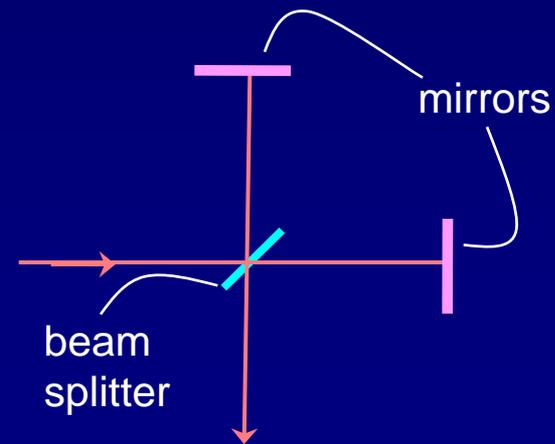
mirror

Total path length L_1

$2I_1$ $2I_1$

Source

$4I_1$

Total path length L_2

$2I_1$

mirror

$2I_1$

beam splitter

I_1 I_1

Path-length difference
$\delta = L_2 - L_1$

Be careful !!

What's important is the <u>total</u> time to get back to the beam splitter...

$$I = 4I_1 \cos^2(\phi/2), \text{ with } \phi = 2\pi\, \delta/\lambda$$
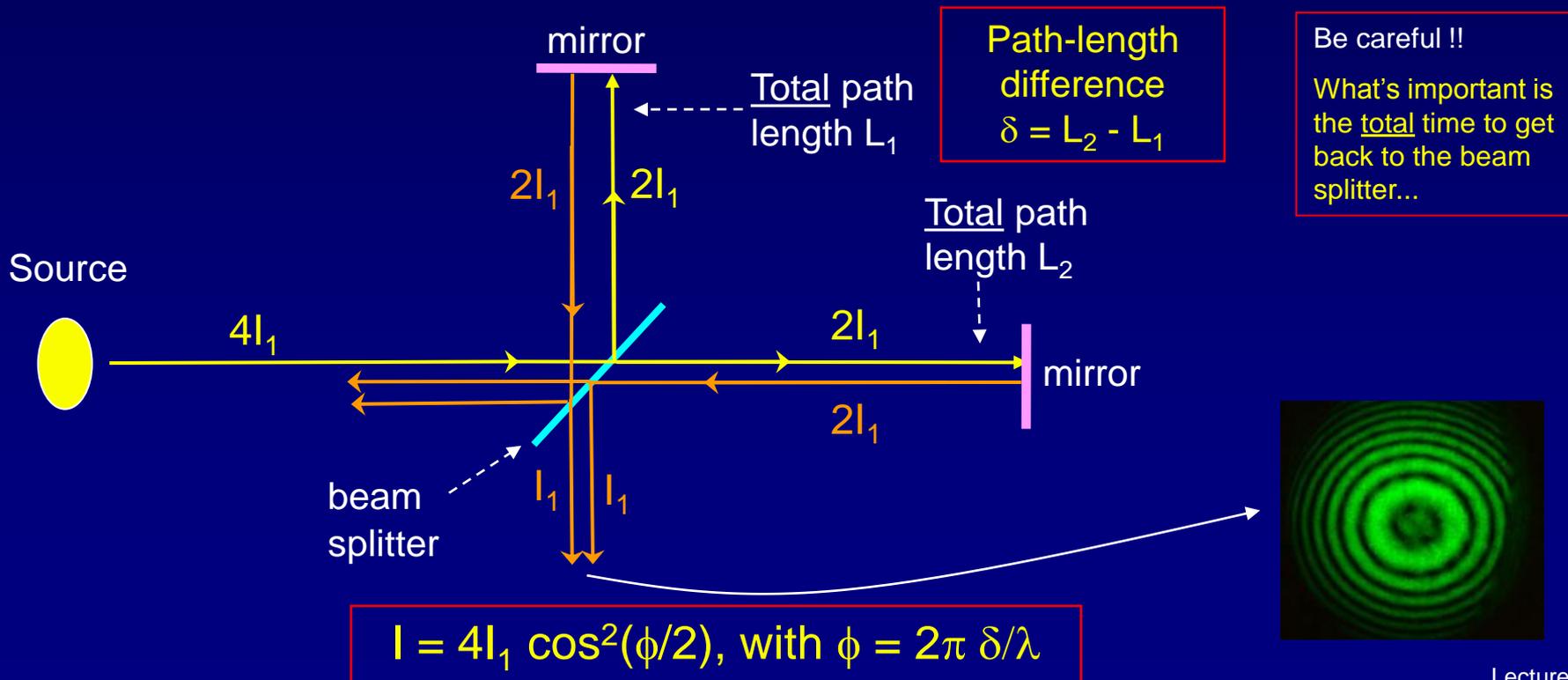
ACT 4

Consider the following Michelson interferometer. Suppose that for the setup shown, all the light (with λ = 500 nm) comes out the bottom port.

1. How much does the top mirror need to be moved so that none of the light comes out the bottom port?

   a. 125 nm   b. 250 nm   c. 500 nm

2. Where does the light then go?
   a. down   b. up   c. left   d. right

Solution

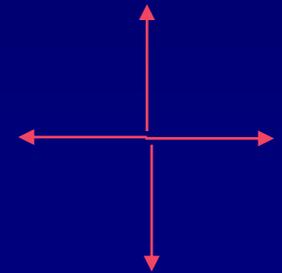Consider the following Michelson interferometer. Suppose that for the setup shown, all the light (with λ = 500 nm) comes out the bottom port.

1. How much does the top mirror need to be moved so that none of the light comes out the bottom port?

   a. 125 nm   b. 250 nm   c. 500 nm

   We need to go from complete constructive to complete destructive interference $\rightarrow$ $\Delta\phi$ = 180˚ $\rightarrow$ δ = $\lambda/2$.

   However…when we move the mirror by d, we change δ by 2d.

   Therefore, d = $\delta/2$ = $\lambda/4$ = 500/4 = 125 nm.
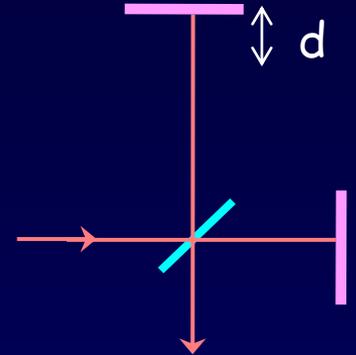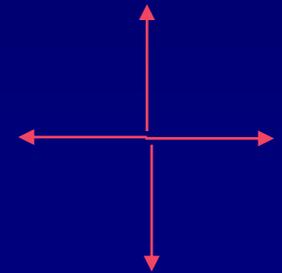
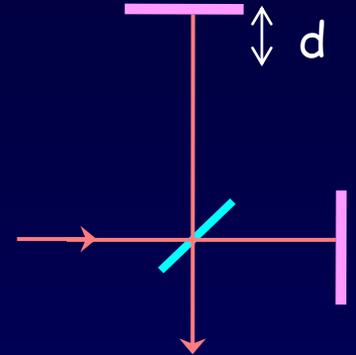2. Where does the light then go?

   a. down   b. up   c. left   d. right

Solution

Consider the following Michelson interferometer. Suppose that for the setup shown, all the light (with $\lambda = 500$ nm) comes out the bottom port.

1. How much does the top mirror need to be moved so that none of the light comes out the bottom port?

  a. 125 nm   b. 250 nm   c. 500 nm

  We need to go from complete constructive to complete destructive interference → $\Delta\phi = 180°$ → $\delta = \lambda/2$.

  However…when we move the mirror by d, we change δ by 2d.

  Therefore, $d = \delta/2 = \lambda/4 = 500/4 = 125$ nm.

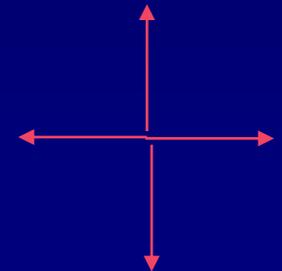2. Where does the light then go?

  a. down   b. up   **c. left**   d. right

  The light goes out the way it came in.
  Energy is conserved --the light can't just disappear!

  The Michelson interferometer is perhaps most famous for disproving the hypothesis that EM waves propagate through an "aether" – this result helped stimulate the Special Theory of Relativity

Michelson Interferometer

Another possibility is to vary the phase by changing the speed of the waves in the two arms.

Recall v=c/n where n = index of refraction.
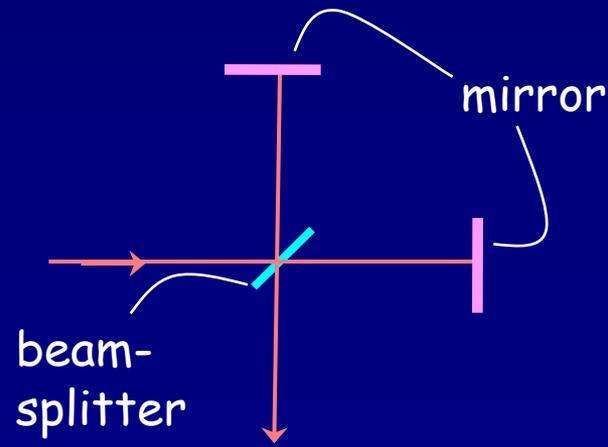
Using $\lambda = v/f$, the number of wavelengths in arm 1 is:

$$N_1 = \frac{L}{\lambda_1} = \frac{n_1 f L}{c}$$ and similarly for arm 2.

(You can think of it as the path being longer by n.)

The phase difference is thus:

$\phi = 2\pi(N_1 - N_2) = 2\pi(fL/c)(n_1 - n_2)$

This makes possible very accurate measurement of changes in the speed of light in the two arms.

mirror

beam-
splitter

FYI: Application
Optical Coherence Tomography

- One "mirror" of the Michelson is replaced by human tissue. The type of tissue controls the amount of reflection and the phase shift.

- By sending in many colors, one can learn about the density, composition, and structure of the tissue.

- Used for medical diagnostics – like a microscope, but you don't have to excise the sample from the body!

- Used to study
  - skin cancer
  - cardiovascular disease (detect bad plaques)
  - glaucoma and macular degeneration (incurable eye disease)
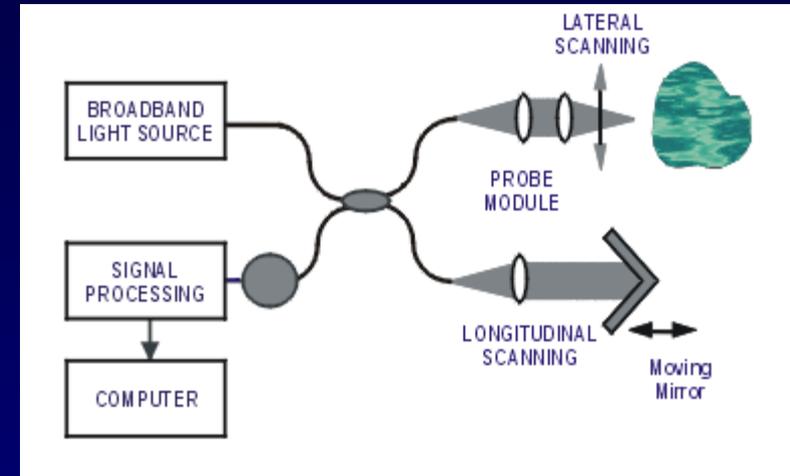
FYI: Gravitational Wave Detection

General relativity predicts that when massive objects accelerate, they produce time-dependent gravitational fields – gravitational waves – that propagate as "warpings" of spacetime at the speed of light. (similar to EM radiation from accelerated charge)

The effect is very tiny: E.g., estimated $\Delta L/L$ of $\sim 10^{-21}$ for in-spiraling binary neutron stars. How to detect this???
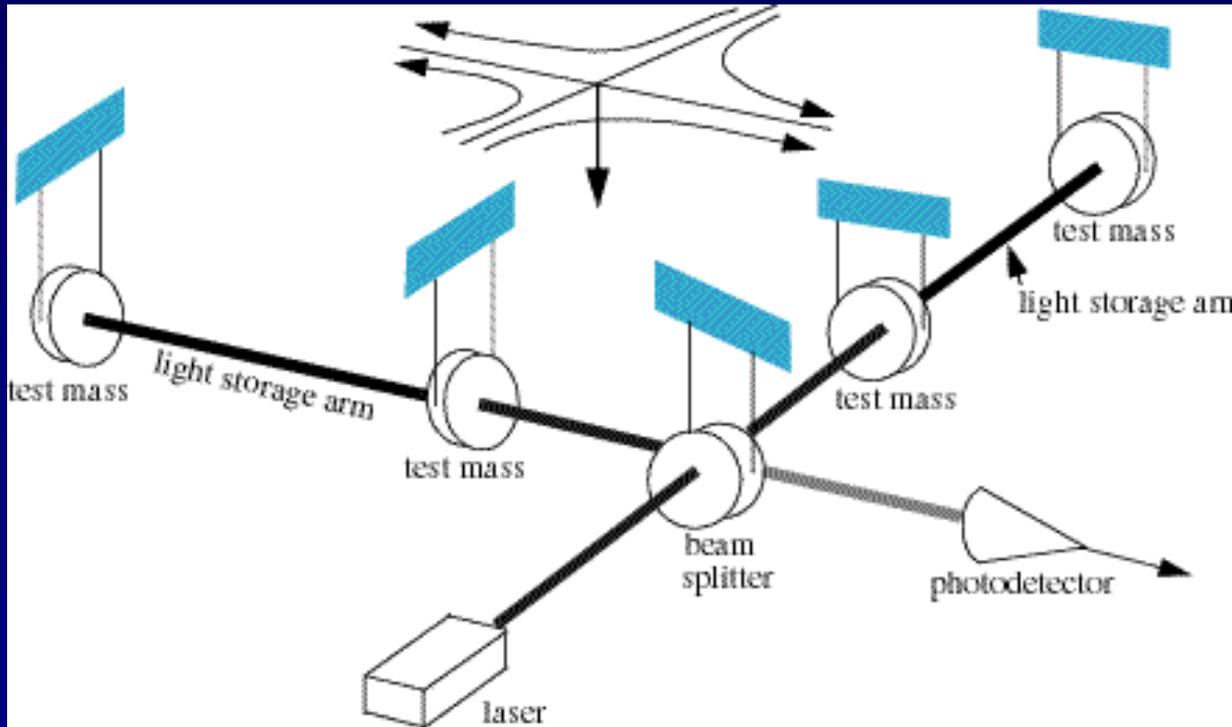
FYI: Application: Gravity Wave Detection

LIGO:

<u>L</u>aser

<u>I</u>nterferometric

<u>G</u>ravitational wave

<u>O</u>bservatory

-World's largest interferometers: 4-km

-2 in Hanford, WA; 1 in Livingston, LO

- >500 scientists

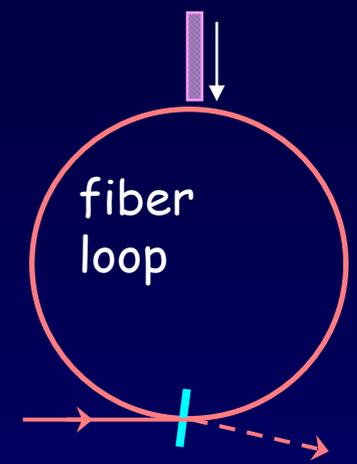-Achieved sensitivity $\Delta L/L \sim 10^{-23}$ → $\Delta L \sim 10^{-20}$ m

-Six data runs completed.

-"Advanced LIGO" should improve sensitivity by another 10x.

FYI: Modern Applications in Navigation

Consider the following "Sagnac" ["sahn-yack"] interferometer. Here the two possible paths are the clockwise and counter-clockwise circuits around the fiber loop.
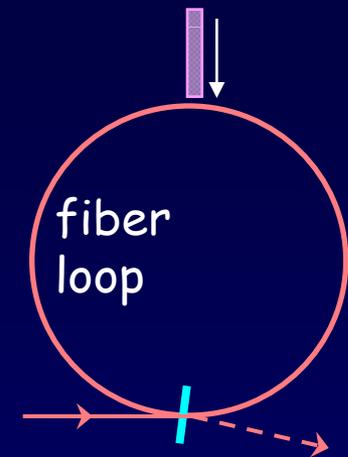
fiber loop

1. If we insert an extra piece of glass as shown, how does the relative path length change?

2. How could we change the relative path-length difference, and thereby change how much light exits the bottom port?

FYI: Modern Applications in Navigation

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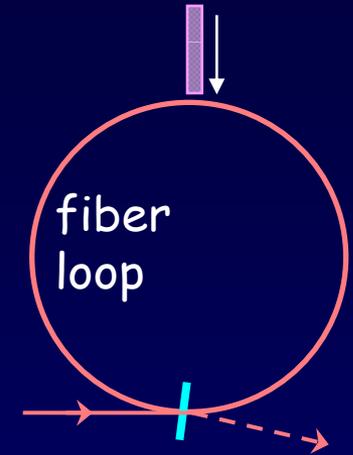
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Rotate the entire interferometer (in the plane of the paper). For example, if we rotate it clockwise, the light making the clockwise circuit will have farther to go (the beamsplitter is "running away"), while the counterclockwise path will be shortened.

It is not difficult to show that

$$\phi \approx \frac{2\pi}{\lambda} \frac{4\left(\pi R^2\right)\omega}{c^2}$$

Monitor output intensity → determine ϕ → rate of rotation ω → "laser ring gyroscope"!

Next Week

Introduction to quantum mechanics

Photoelectric effect

Relation between energy and frequency of a photon

Photon momentum

The key relations of quantum mechanics

Wave-particle duality