

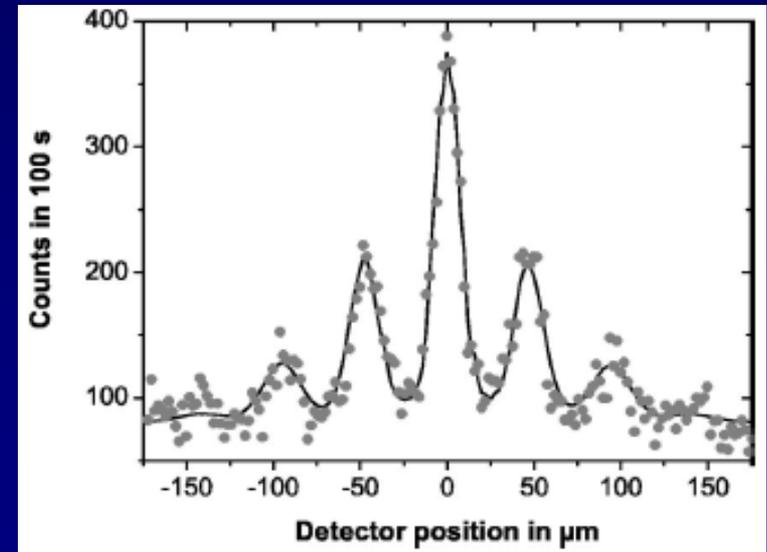
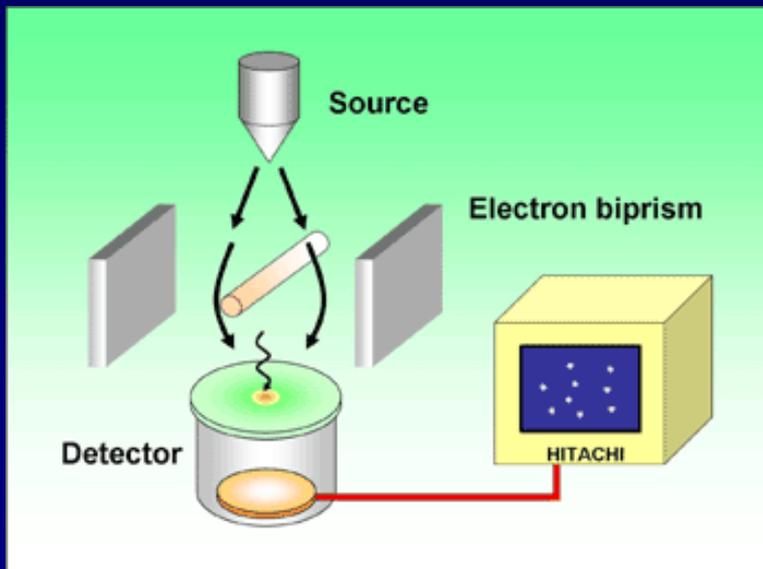
"We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery."

--Richard P. Feynman

# Lecture 8:

## Introduction to Quantum Mechanics

### Matter Waves and the Uncertainty Principle



# This week and next are critical for the course:

Week 3, Lectures 7-9:

Light as Particles

Particles as waves

Probability

Uncertainty Principle

Week 4, Lectures 10-12:

Schrödinger Equation

Particles in infinite wells, finite wells

**Midterm Exam** Monday, Feb. 14.

It will cover lectures 1-10 and some aspects of lectures 11-12.

Practice exams: Old exams are linked from the course web page.

Review Sunday, Feb. 13, 3-5 PM in 141 Loomis

Office hours: Feb. 13 and 14

# Last Time

The important results from last time:

Quantum mechanical entities can exhibit either wave-like or particle-like properties, depending on what one measures.

We saw this phenomenon for photons, and claimed that it is also true for matter (e.g., electrons).

The wave and particle properties are related by these universal equations:

$$E = hf \quad \text{Energy-frequency} \quad (= hc/\lambda \text{ only for photons})$$

$$p = h/\lambda \quad \text{Momentum-wavelength}$$

# Today

Interference, the 2-slit experiment revisited

Only indistinguishable processes can interfere

Wave nature of particles

Proposed by DeBroglie in 1923, to explain atomic structure.

Demonstrated by diffraction from crystals – just like X-rays!

Matter-wave Interference

Double-slit interference pattern, just like photons

Electron microscopy

Heisenberg Uncertainty Principle

An object cannot have both position and momentum simultaneously.

Implications for measurements in QM

Measuring one destroys knowledge of the other.

# Two Slit Interference: Conclusions

Photons (or electrons ...) can produce interference patterns even one at a time !

With one slit closed, the image formed is simply a single-slit pattern.  
We “know” (*i.e.*, we have constrained) which way the particle went.

With both slits open, a particle interferes with itself to produce the observed two-slit interference pattern.

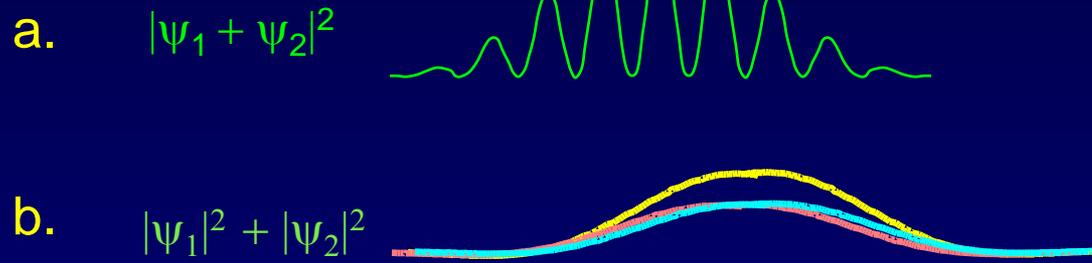
This amazing interference effect reflects, in a fundamental way, the indeterminacy of which slit the particle went through. We can only state the probability that a particle would have gone through a particular slit, if it had been measured.

Confused? You aren't alone! We do not know how to understand quantum behavior in terms of our everyday experience. Nevertheless - as we will see in the next lectures – we know how to use the QM equations and make definite predictions for the probability functions that agree with careful experiments!

The quantum wave,  $\psi$ , is a probability amplitude. The intensity,  $P = |\psi|^2$ , tells us the probability that the object will be found at some position.

# Act 1

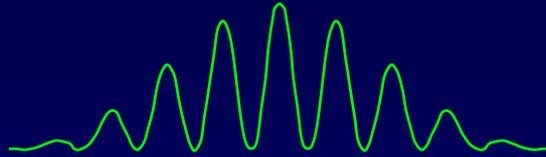
Suppose we measure with the upper slit covered for half the time and the lower slit covered for the other half of the time. What will be the resulting pattern?



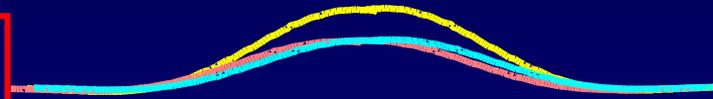
# Solution

Suppose we measure with the upper slit covered for half the time and the lower slit covered for the other half of the time. What will be the resulting pattern?

a.  $|\psi_1 + \psi_2|^2$



b.  $|\psi_1|^2 + |\psi_2|^2$



At any given time, there is only one contributing amplitude ( $\psi_1$  or  $\psi_2$ , but not both). Therefore, for half the time pattern P1 will build up, and for the other half we'll get P2. There is no interference. The result will be the sum of the two single-slit diffraction patterns.

In order for waves to interfere, they must both be present at the same time.

# Interference - What Really Counts

We have seen that the amplitudes from two or more physical paths interfere if nothing else distinguishes the two paths.

Example: (2-slits)

$\psi_{\text{upper}}$  is the amplitude corresponding to a photon traveling through the upper slit and arriving at point  $y$  on the screen.

$\psi_{\text{lower}}$  is the amplitude corresponding to a photon traveling through the lower slit and arriving at point  $y$  on the screen.

If these processes are **distinguishable** (*i.e.*, if there's some way to know which slit the photon went through), **add the probabilities**:

$$P(y) = |\psi_{\text{upper}}|^2 + |\psi_{\text{lower}}|^2$$

If these processes are **indistinguishable**, **add the amplitudes** and take the absolute square to get the probability:

$$P(y) = |\psi_{\text{upper}} + \psi_{\text{lower}}|^2$$

What does “distinguishable” mean in practice?

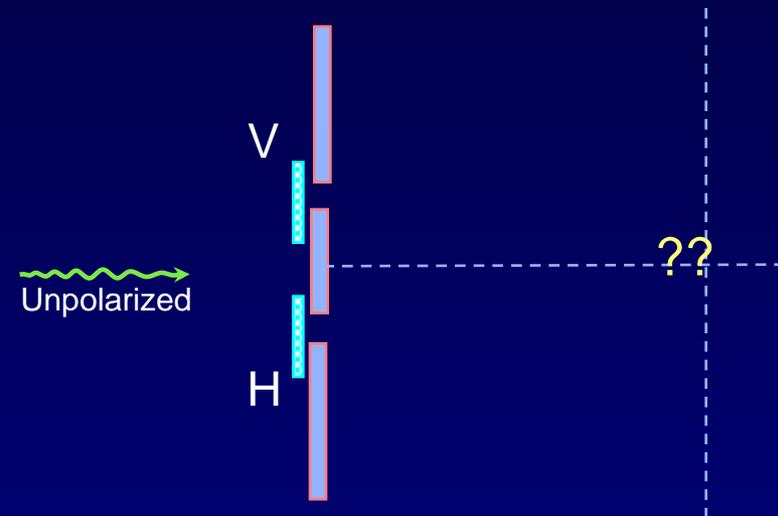
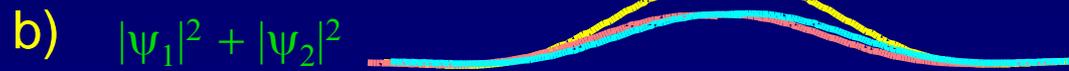
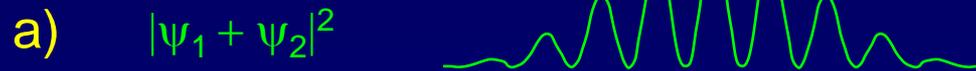
# Act 2

Let's modify the 2-slit experiment a bit. Recall that EM waves can be polarized – electric field in the vertical or horizontal directions.

Send in unpolarized photons.

Cover the upper slit with a vertical polarizer and cover the lower slit with a horizontal polarizer

Now the resulting pattern will be:



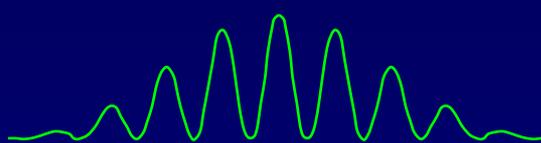
# Solution

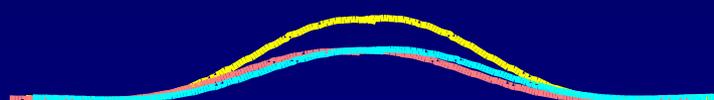
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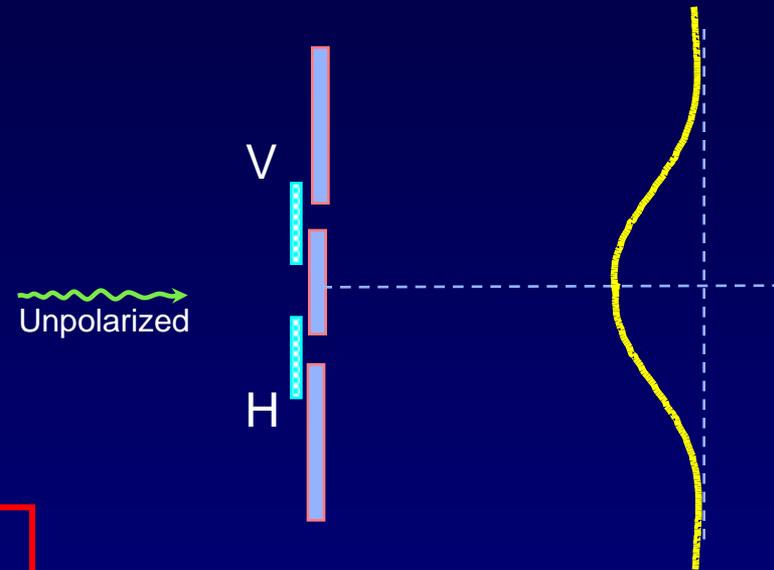
Send in unpolarized photons.

Cover the upper slit with a vertical polarizer and cover the lower slit with a horizontal polarizer

Now the resulting pattern will be:

a)  $|\psi_1 + \psi_2|^2$  

b)  $|\psi_1|^2 + |\psi_2|^2$  



The photon's polarization labels which way it went. Because the two paths are in principle distinguishable there is no interference. Note, that we don't actually need to measure the polarization. The mere possibility that one could measure it destroys the interference.

Bonus Question: How could we recover the interference?

# Matter Waves

We described one of the experiments (the photoelectric effect) which shows that light waves also behave as particles. The wave nature of light is revealed by interference - the particle nature by the fact that light is detected as quanta: “photons”.

Photons of light have energy and momentum given by:

$$E = hf \quad \text{and} \quad p = h/\lambda$$

Prince Louis de Broglie (1923) proposed that particles also behave as waves; i.e., for all particles there is a quantum wave with frequency and wavelength given by the same relation:

$$f = E/h \quad \text{and} \quad \lambda = h/p$$

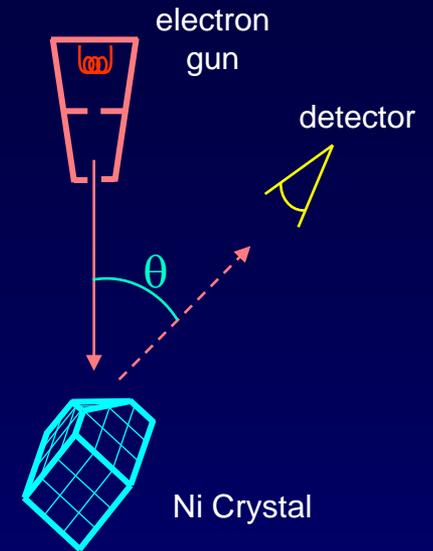
# Matter Waves

Interference demonstrates that matter (electrons) can act like waves. In 1927-8, Davisson & Germer\* showed that, like x-rays, electrons can diffract off crystals !

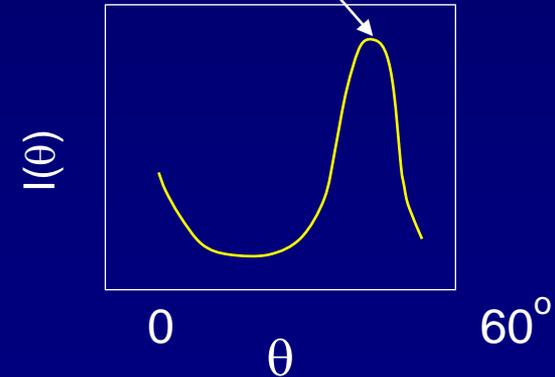
Electrons can act like waves,  
just like photons!

You'll study electron diffraction in discussion.

\*Work done at Bell Labs, Nobel Prize



Interference peak !



# Act 3: Matter Wavelengths

What size wavelengths are we talking about? Consider a photon with energy 3 eV, and therefore momentum  $p = 3 \text{ eV}/c$ .<sup>\*</sup> Its wavelength is:

$$\lambda = \frac{h}{p} = \frac{4.14 \times 10^{-15} \text{ eV} \cdot \text{s}}{3 \text{ eV}} \times c = (1.4 \times 10^{-15} \text{ s}) \times (3 \times 10^8 \text{ m/s}) = 414 \text{ nm}$$

What is the wavelength of an electron with the same momentum?

a)  $\lambda_e < \lambda_p$

b)  $\lambda_e = \lambda_p$

c)  $\lambda_e > \lambda_p$

<sup>\*</sup>It is an unfortunate fact of life that there is no named unit for momentum, so we're stuck with this cumbersome notation.

# Solution

What size wavelengths are we talking about? Consider a photon with energy 3 eV, and therefore momentum  $p = 3 \text{ eV}/c$ . Its wavelength is:

$$\lambda = \frac{h}{p} = \frac{4.14 \times 10^{-15} \text{ eV} \cdot \text{s}}{3 \text{ eV}} \times c = (1.4 \times 10^{-15} \text{ s}) \times (3 \times 10^8 \text{ m/s}) = 414 \text{ nm}$$

What is the wavelength of an electron with the same momentum?

a)  $\lambda_e < \lambda_p$

b)  $\lambda_e = \lambda_p$

c)  $\lambda_e > \lambda_p$

$\lambda = h/p$  for all objects, so equal  $p$  means equal  $\lambda$ .

Note that the kinetic energy of the electron does not equal the energy of a photon with the same momentum (and wavelength):

$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{(6.625 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(414 \times 10^{-9} \text{ m})^2}$$
$$= 1.41 \times 10^{-24} \text{ J} = 8.8 \times 10^{-6} \text{ eV}$$

# Wavelength of an Electron

The DeBroglie wavelength of an electron is inversely related to its momentum:

$$\lambda = h/p$$

$$h = 6.626 \times 10^{-34} \text{ J-sec}$$

Frequently we need to know the relation between the electron's wavelength  $\lambda$  and its kinetic energy  $E$ . Because the electron has  $v \ll c$ ,  $p$  and  $E$  are related through the Physics 211 formula:

$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

Valid for all (non-relativistic) particles

For  $m = m_e$ :  
(electrons)

$$h = 4.14 \times 10^{-15} \text{ eV-sec}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$E_{\text{electron}} = \frac{1.505 \text{ eV} \cdot \text{nm}^2}{\lambda^2}$$

( $E$  in eV;  $\lambda$  in nm)

Don't confuse this with  $E_{\text{photon}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}$  for a photon.

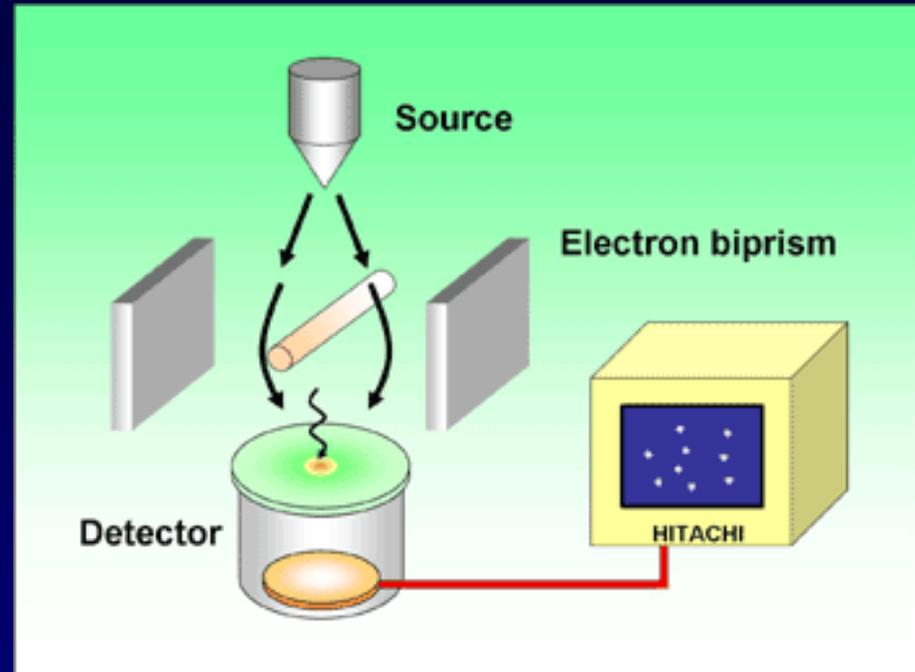
# "Double-slit" Experiment for Electrons

Electrons are accelerated to 50 keV  
→  $\lambda = 0.0055$  nm

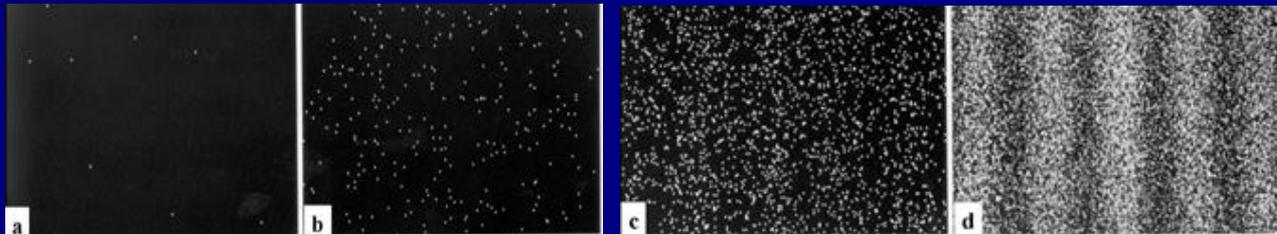
Central wire is positively charged → bends electron paths so they overlap.

A position-sensitive detector records where they appear.

$\ll 1$  electron in system at any time



Video by A. TONOMURA (Hitachi) --pioneered electron holography.  
<http://www.hqrd.hitachi.co.jp/rd/moviee/doubleslite.wmv>



Exposure time: 1 s

10 s

5 min

20 min

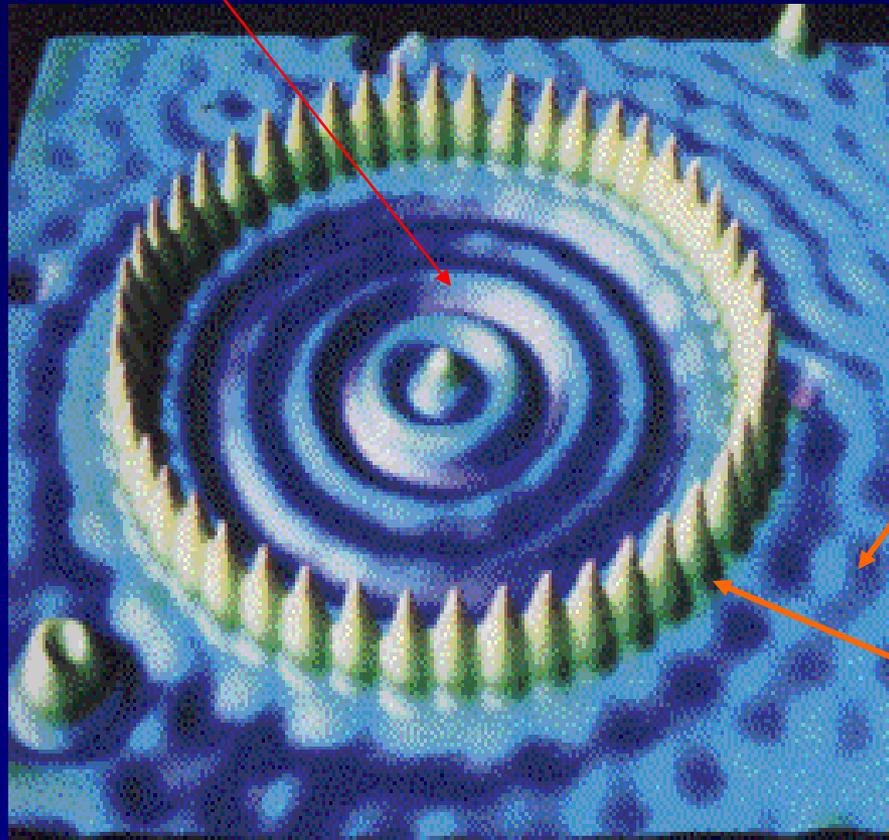
See also this Java simulation: <http://www.quantum-physics.polytechnique.fr/index.html>

# Observation of an electron wave "in a box"

Image taken with a scanning tunneling microscope (more later)

(Note: the color is not real! - it is a representation of the electrical current observed in the experiment)

Real standing waves of electron density in a "quantum corral"



IBM  
Almaden

Cu

Single  
atoms (Fe)

# Summary: Photon & Matter Waves

Everything

$$E = hf$$

$$p = h/\lambda$$

Light ( $v = c$ )

$$E = pc, \text{ so}$$

$$E = hc/\lambda$$

$$E_{\text{photon}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}$$

Slow Matter ( $v \ll c$ )

$$KE = p^2/2m, \text{ so}$$

$$KE = h^2/2m\lambda^2$$

For electrons:

$$KE = \frac{1.505 \text{ eV} \cdot \text{nm}^2}{\lambda^2}$$

# Where do we go from here?

Two approaches pave the way:

## Uncertainty principle (today)

- In quantum mechanics one can only calculate a probability distribution for the result of a measurement.
- The Heisenberg uncertainty principle provides a way to use simple arguments and a simple inequality to draw important conclusions about quantum systems.

## Schrödinger equation (next week)

- This differential equation describes the evolution of the quantum wave function,  $\Psi$ .  $\Psi$  itself has no uncertainty.
- $|\Psi|^2$  will then tell us the probabilities of obtaining various measurement results. That's where the uncertainty enters.

# Heisenberg Uncertainty Principle

All QM objects (we think that includes everything) have wave-like properties. One mathematical property of waves is:

$$\Delta k \cdot \Delta x \geq 1$$

(See the supplementary slide for some discussion)

$$k = 2\pi/\lambda$$

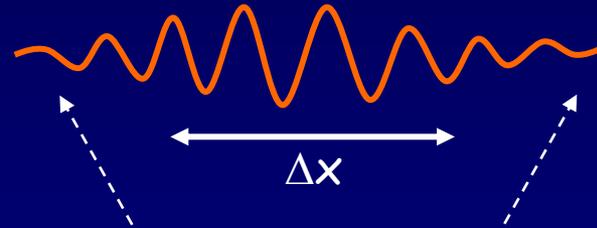
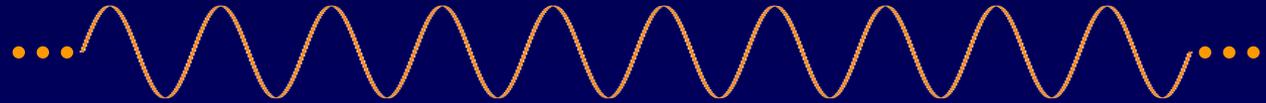
Examples:

- Infinite sine wave:

A definite wavelength must extend forever.

- Finite wave packet:

A wave packet requires a spread\* of wavelengths.



We need a spread of wavelengths in order to get destructive interference.

Using  $p = h/\lambda = \hbar k$ , we have:

$$\hbar (\Delta k \cdot \Delta x \geq 1) \Rightarrow (\hbar \Delta k) \cdot \Delta x \geq \hbar \Rightarrow \Delta p \cdot \Delta x \geq \hbar$$

This relation is known as the Heisenberg Uncertainty Principle.

It limits the accuracy with which we can know the position and momentum of objects.

\* We will not use the statistically correct definition of “spread”, which, in this context, we also call “uncertainty”.

# Supplement: Wave Uncertainty

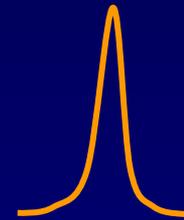
Mathematically, one can produce a localized function by superposing sine waves with a “spread” of wave numbers,  $\Delta k$ :  $\Delta k \cdot \Delta x \geq 1$ .



This is a result of Fourier analysis, which most of you will learn in Math.

It means that making a short wave packet requires a broad spread in wavelengths.

Conversely, a single-wavelength wave would extend forever.



So far, this is just math. The physics comes in when we make the wavelength-momentum connection:  $p = h/\lambda = \hbar k$ .

Example:

How many of you have experienced a close lightning strike (within a couple hundred feet)? If you were paying attention, you may have noticed that the sound, which is a very short pulse, is very weird. That weirdness is a result of the very broad range of frequencies that is needed to construct a very short pulse. One doesn't normally experience such a broad frequency range.

# Why "Uncertainty"?

"Uncertainty" refers to our inability to make definite predictions.

Consider this wave packet:



- Where is the object?
- What is its momentum?

The answer is, We don't know. We can't predict the result of either measurement with an accuracy better than the  $\Delta x$  and  $\Delta p$  given to us by the uncertainty principle.

Each time you look, *you find a local blip* that is in a different place (in fact, it is your looking that *causes* the wavefunction to "collapse"!).

If you look many times, you will find a **probability distribution** that is spread out

But you're **uncertain about where that local blip will be in any one of the times you look** -- it could be anywhere in the spread.

An important point: **You never observe the wave function itself.**

The wave merely gives the probabilities of obtaining the various measurement results. A measurement of position or momentum will always result in a definite result. You can infer the properties of the wave function by repeating the measurements (measure the probabilities), but that's not the same as a direct observation.

# Uncertainty Principle -Implications

The uncertainty principle explains why electrons in atoms don't simply fall into the nucleus: If the electron were confined too close to the nucleus (small  $\Delta x$ ), it would have a large  $\Delta p$ , and therefore a very large average kinetic energy ( $\approx (\Delta p)^2/2m$ ).

The uncertainty principle does not say "everything is uncertain". Rather, it tells us what the limits of uncertainty are when we make measurements of quantum systems.

Some classical features, such as paths, do not exist precisely, because having a definite path requires both a definite position and momentum. One consequence, then, is that electron orbits do not exist. The atom is not a miniature solar system.

Other features can exist precisely. For example, in some circumstances we can measure an electron's energy as accurately as technology allows.

Serious philosophical issues remain open to vigorous debate, e.g., whether all outcomes or only one outcome actually occur.

# Example

The position of an electron in the lowest-energy state of a hydrogen atom; is known to an accuracy of about  $\Delta x = 0.05 \text{ nm}$  (the radius of the atom).

What is the minimum range of momentum measurements? Velocity?

# Solution

The position of an electron in the lowest-energy state of a hydrogen atom; is known to an accuracy of about  $\Delta x = 0.05 \text{ nm}$  (the radius of the atom).

What is the minimum range of momentum measurements? Velocity?

$$\Delta x \Delta p \geq \hbar$$

Heisenberg's Uncertainty Principle

$$\Delta p \geq \hbar / \Delta x$$

$$\hbar = h / 2\pi$$

$$= 2.1 \times 10^{-24} \text{ J} \cdot \text{s/m}$$

$$= 2.1 \times 10^{-24} \text{ kg} \cdot \text{m/s}$$

$$\Delta v = \Delta p / m_e$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$= 2.3 \times 10^6 \text{ m/s}$$