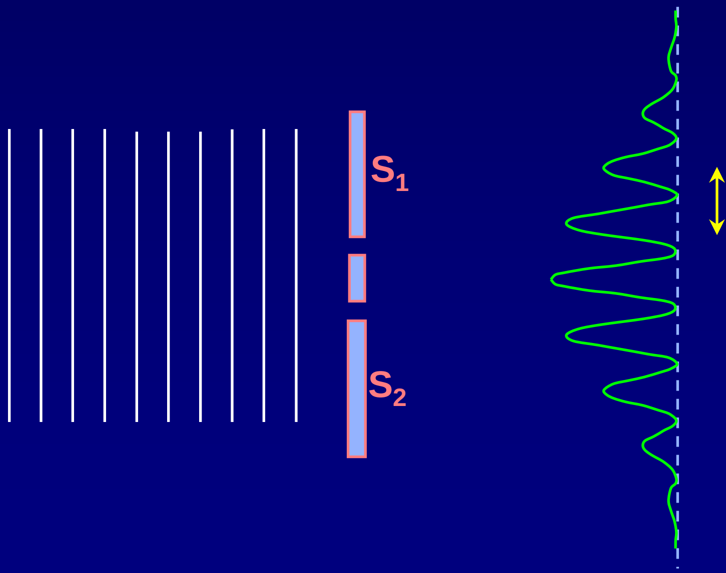
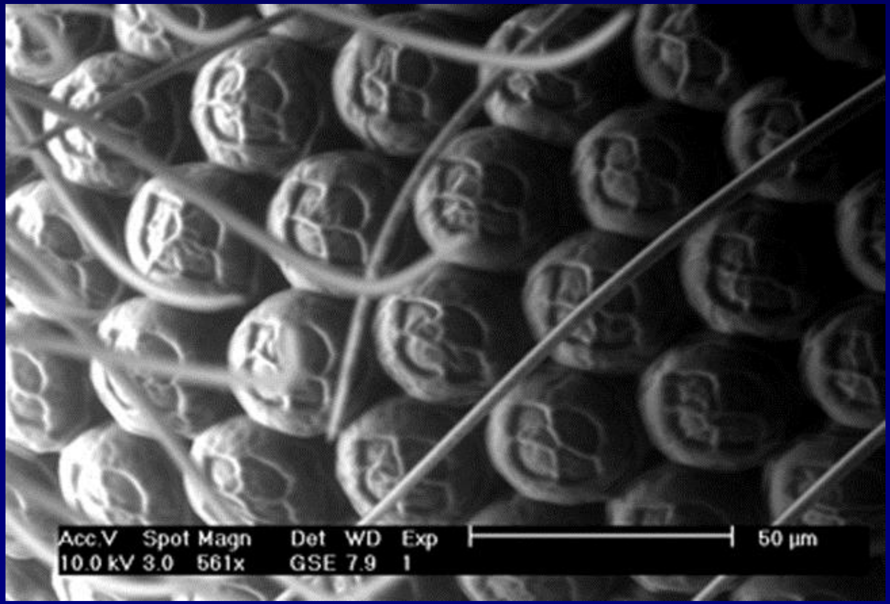

OH ALICE.. YOU'RE THE ONE FOR ME
BUT BOB.. IN A QUANTUM WORLD HOW CAN WE BE SURE?
Physics World

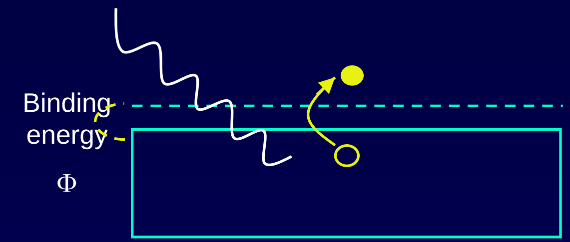
Lecture 9:
Introduction to QM:
Review and Examples





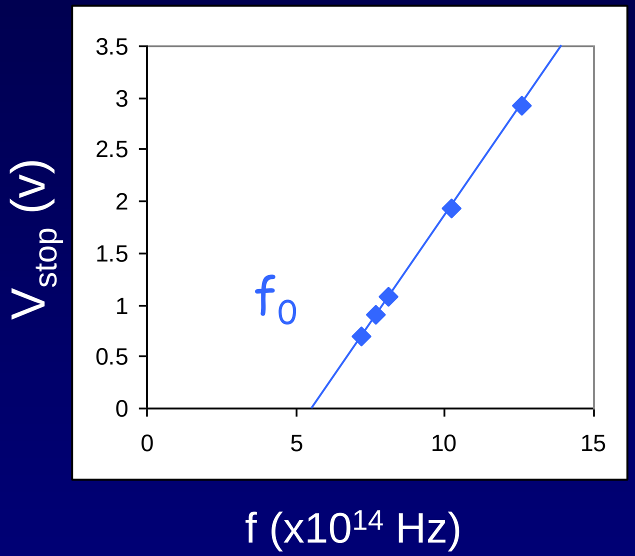
Photoelectric Effect

$$KE_{max} = e \cdot V_{stop} = hf - \Phi$$



The work function:

- Φ is the *minimum* energy needed to strip an electron from the metal.
- Φ is defined as positive.
- Not all electrons will leave with the maximum kinetic energy (due to losses).



Conclusions:

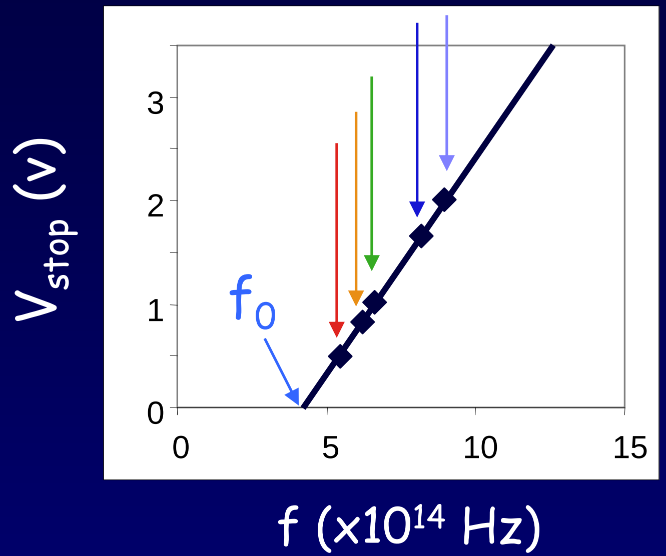
- Light arrives in "packets" of energy (photons).
- $E_{photon} = hf$
- Increasing the intensity increases # photons, not the photon energy. Each photon ejects (at most) one electron from the metal.

Recall: For EM waves, frequency and wavelength are related by $f = c/\lambda$.
Therefore: $E_{photon} = hc/\lambda = 1240\ \text{eV-nm}/\lambda$

Act 1

1. If Planck's constant were somewhat larger, but Φ remained the same, how would the graph change?

   a. Increased slope
   b. Increased f_o
   c. Both a and b



2. If Φ increased, but h remained the same, how would the graph change?

   a. Increased slope
   b. Increased f_o
   c. Both a and b

Act 1 - Solution

1. If Planck's constant were somewhat larger, but Φ remained the same, how would the graph change?

   a. Increased slope
   b. Increased f_o
   c. Both a and b

   From $KE_{max} = e \cdot V_{stop} = h(f - f_0) = hf - \Phi$ we can see that the slope increases, and f_0 decreases.

2. If Φ increased, but h remained the same, how would the graph change?

   a. Increased slope
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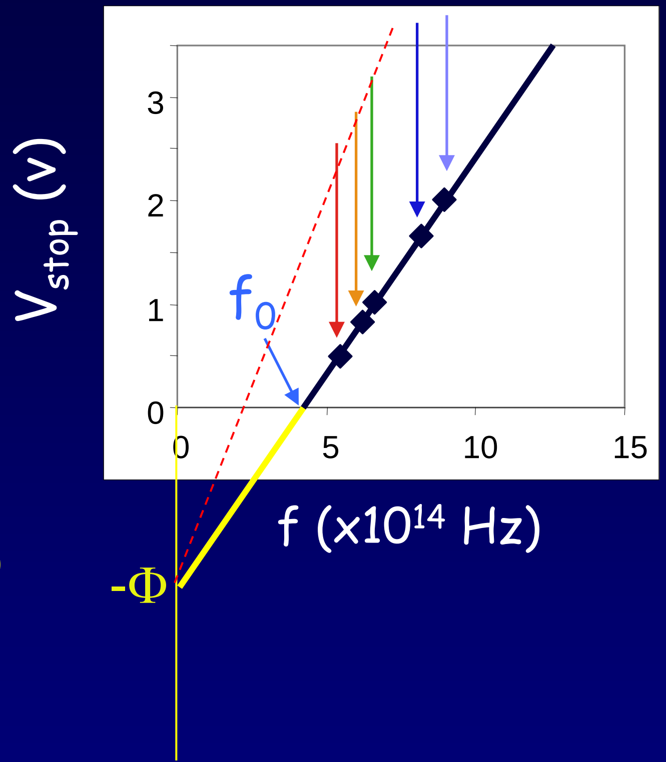

Act 1 - Solution

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2. If Φ increased, but h remained the same, how would the graph change?

   a. Increased slope
   b. Increased f_o
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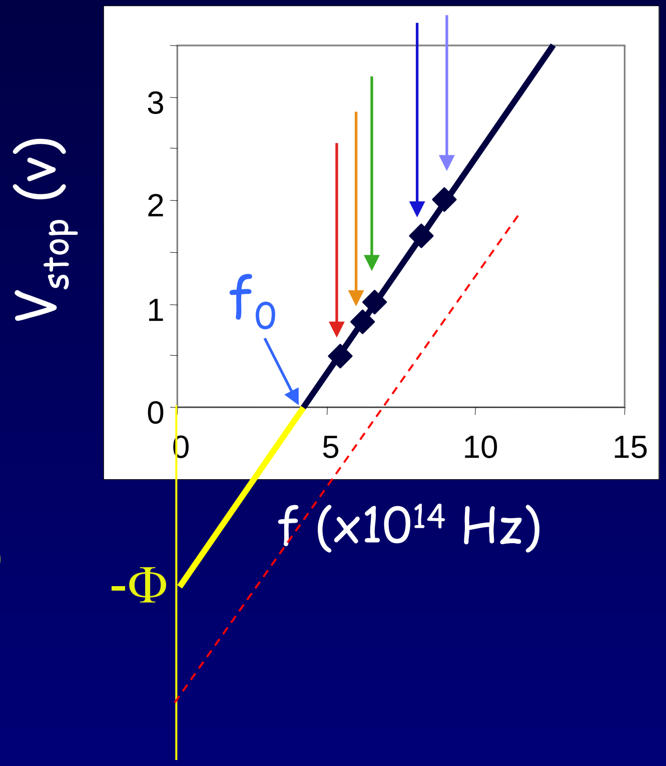
   The y intercept moves down, so the slope is unchanged and f_0 increases.



Summary: Photon & Matter Waves

Everything

$E = hf$

$p = h/\lambda$

Light ($v = c$)

$E = pc$, so

$E = hc/\lambda$

$$E_{photon} = \frac{1240\ \text{eV}\cdot\text{nm}}{\lambda}$$

Slow Matter ($v << c$)

$KE = p^2/2m$, so

$KE = h^2/2m\lambda^2$

For electrons:

$$KE = \frac{1.505\ \text{eV}\cdot\text{nm}^2}{\lambda^2}$$

Act 2: Counting photons

Which emits more photons, a 1-mW cell phone (f = 830 MHz → λ = 0.36 m) or a 1-mW laser (λ = 635 nm)?

a) Laser emits more
b) They emit the same number
c) Cell phone emits more

Act 2: Solution

Which emits more photons, a 1-mW cell phone (f = 830 MHz → λ = 0.36 m) or a 1-mW laser (λ = 635 nm)?

a) Laser emits more
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c) Cell phone emits more

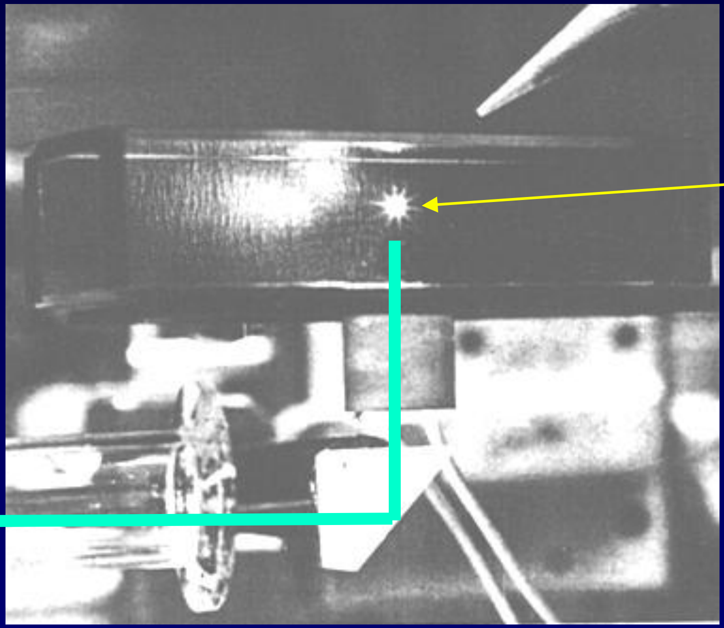
Because the cell frequency is much less than the optical frequency, each cell-phone photon has much less energy. Therefore, you need many more of them to get the same total energy.

$$\text{Rate} \propto \lambda \quad \therefore \quad \frac{Rate_{cell}}{Rate_{laser}} = \frac{\lambda_{cell}}{\lambda_{laser}} = \frac{0.36\ \text{m}}{635\times 10^{-9}\ \text{m}} = 5.7\times 10^{5}$$

Cell phones actually emit ~1W → ~10^{24} photons/sec

Exercise: Optical "Levitation"

What laser power is required to suspend a glass bead weighing 0.01 g?



Glass bead 'floating' on a laser beam!

(AT&T Bell Labs)

Laser

$P = \Delta E/\Delta t = ?$

Assume the bead absorbs all the incident light.

Answer: 30 kW

Exercise: Optical "Levitation"

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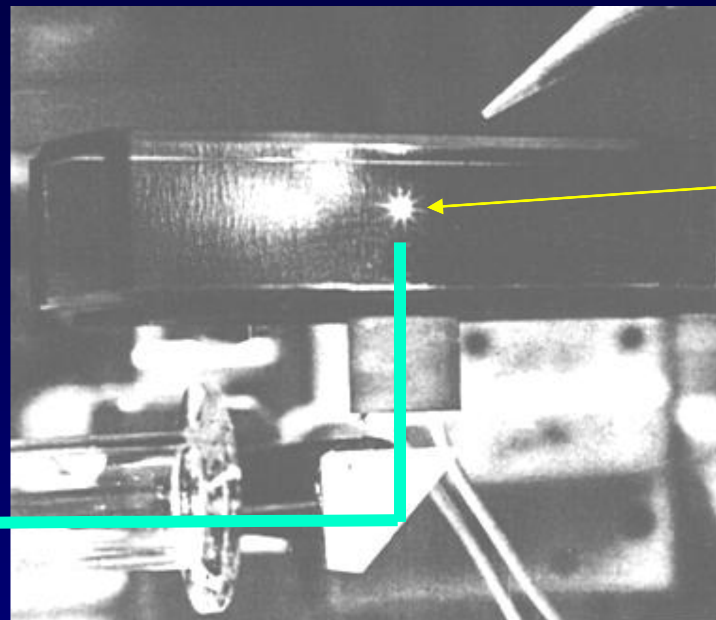


Glass bead 'floating' on a laser beam!

(AT&T Bell Labs)

Assume the bead absorbs all the incident light.

Laser

$P = \Delta E/\Delta t = ?$

$$F = mg = \frac{\Delta p}{\Delta t} = (\text{momentum for each absorbed photon}) \times (\text{\# photons/sec})$$

$$\therefore \text{\#photons/s} = \frac{F}{p} = \frac{mg}{h/\lambda}$$

$$Power = (\text{energy/photon}) \times (\text{\# photons/sec}) = \frac{hc}{\lambda}\frac{mg}{h/\lambda} = mg\,c$$

$$= \left(10^{-5}kg\right)\left(9.8m/s^2\right)\left(3\times10^8 m/s\right) = 30kW$$

Measurement

How does what we measure determine whether we observe wave or particle properties?

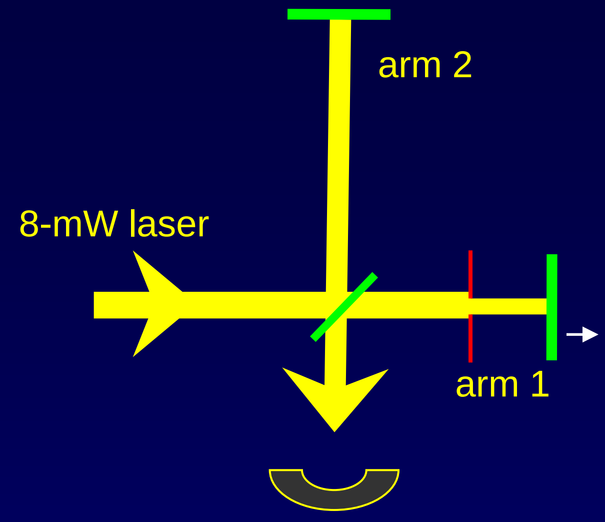
Waves have wavelength, λ, and frequency, f. So, if we measure momentum (wavelength) or energy (frequency), we have observed the wave properties of our object.

Particles have position (and trajectories). If we measure position (*e.g.*, which slit it went through) we have observed a particle property. That's why the "which slit" measurement destroys the interference pattern.

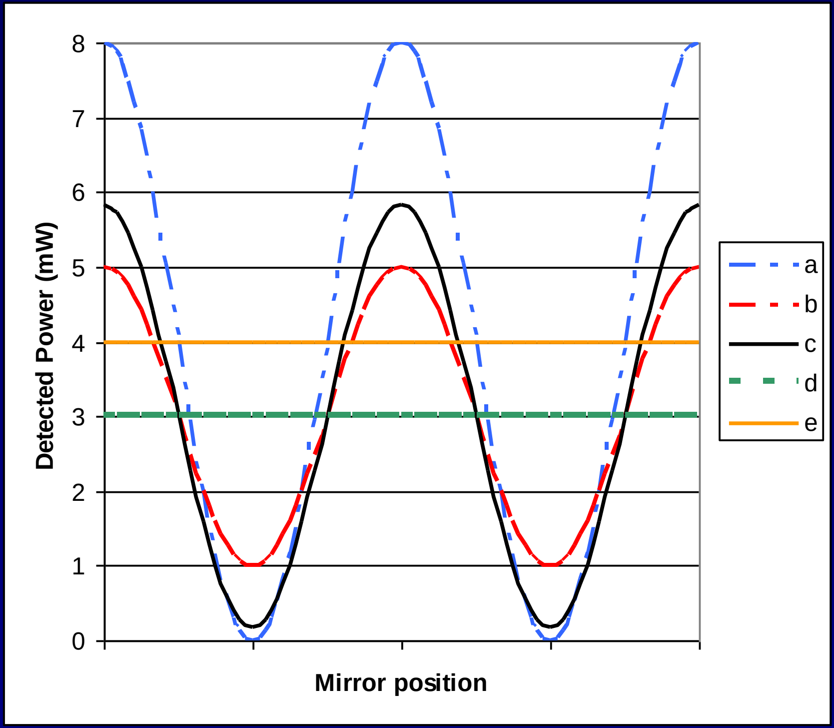
Note that particle and wave properties are incompatible. One can't simultaneously measure both wavelength and position. This is the basis of Heisenberg's "uncertainty principle". (more later)

ACT 3

We can use our rules for quantum mechanical interference to understand classical interference too! Consider a Michelson interferometer, into which is directed an 8-mW laser with a 1-cm beam diameter.



We now put an iris in arm 1, centered on the beam, that reduces its diameter to only 0.71 cm, so that the power coming to the detector just from that arm is only 1 mW (and still 2 mW from the other path, whose beam is still 1 cm in diameter).
As we move the arm 1 mirror outward, which of the following curves might describe the power measured on the detector?
(Hint: what's required for interference.)



a. dash dot curve (varies from 0 to 8 mW)
b. red curve (varies from 1 to 5 mW)
c. solid curve (varies from 0.17 to 5.83 mW, with an average of 3 mW)
d. dashed curve (constant at 3 mW)
e. orange curve (constant at 4 mW)

Solution

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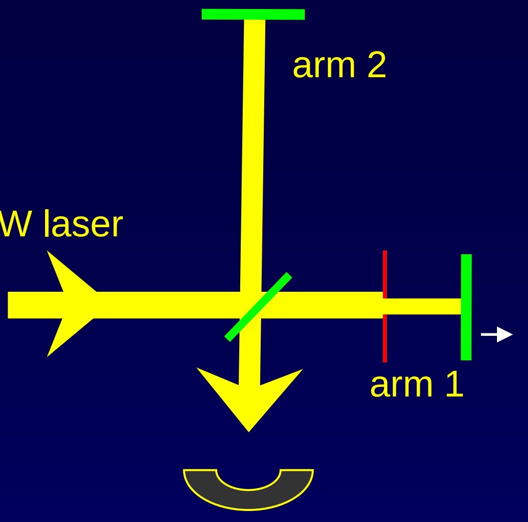

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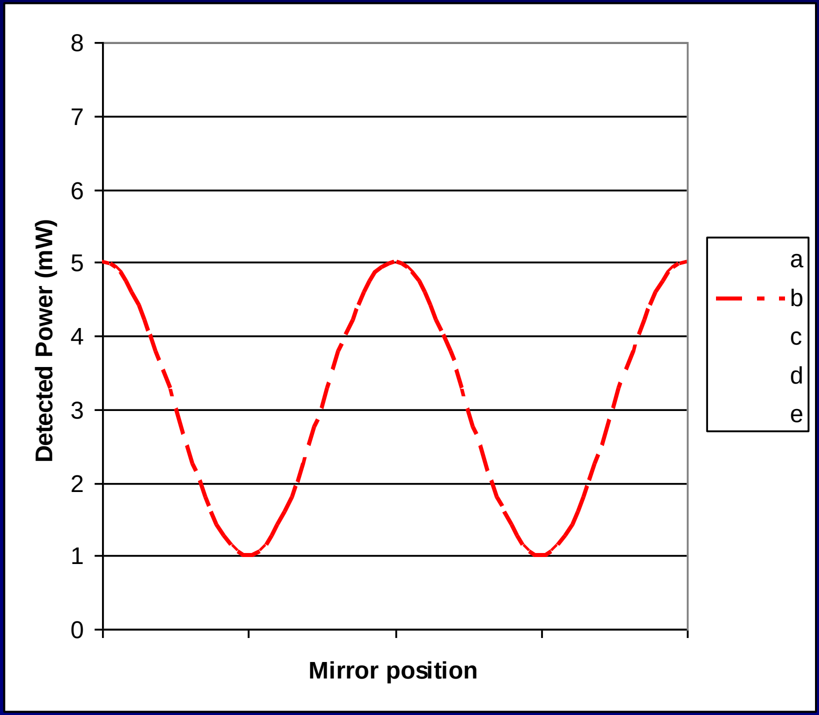
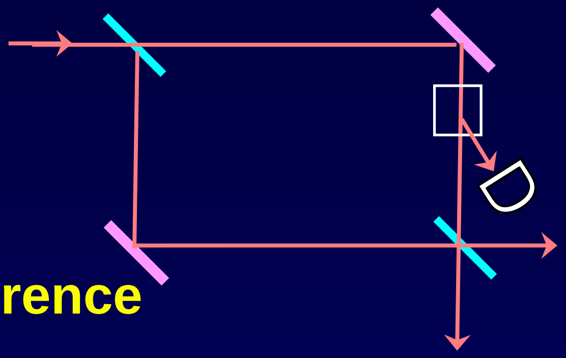
b. red curve (varies from 1 to 5 mW)



Interference can only occur if the contributing processes are indistinguishable. In this problem, that's only the case for photons inside the 0.71-cm diameter disk, which could have come from either arm. Inside that disk, we have perfect interference (0 → 4 mW). But the detector also sees the non-interfering 1 mW from the outer ring from arm 2. This adds as a background.

FYI: More Quantum Weirdness



Consider the following interferometer:

- **photons are sent in one at a time**
- **the experimenter can choose to**
  - **leave both paths open, so that there is interference**
  - **activate switch in the upper path, deflecting that light to a counter**
- **What does it mean?**
  - **Switch OFF → interference → wave-like behavior**
  - **Switch ON → detector "click" or "no click" and no interference → particle-like behavior (trajectory is identified)**

- **What is observed? What kind of behavior you observe depends on what kind of measurement you make. Weird.**

Principle of Complementarity: You can't get perfect particle-like and wave-like behavior in the same setup.

- **It gets worse! In the "*delayed choice*" version of the experiment that was done, the switch could be turned ON and OFF *after* the photon already passed the first beam splitter! The results depended only on the state of the switch when the photon amplitude passed through it!**

Wavelengths of Various "Particles"

Calculate the wavelength of

a. an electron that has been accelerated from rest across a 3-Volt potential difference ($m_e = 9.11\times10^{-31}$ kg).

b. Ditto for a proton ($m_p = 1.67\times10^{-27}$ kg).

c. a major league fastball ($m_{baseball} = 0.15$ kg, $v = 50$ m/s).

Solution

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c. a major league fastball ($m_{baseball} = 0.15$ kg, v = 50 m/s).

a. $E = e\cdot V = 4.8\times10^{-19}$ J

$p = \sqrt{(2m_e E)} = 9.35\times10^{-25}$ kg m/s

$\lambda = h/p = 7.1\times10^{-10}$ m = 0.71 nm

Physics 212
Physics 211
Physics 214

b. $p = \sqrt{(2m_p E)} = 4.00\times10^{-23}$ kg m/s

$\lambda = h/p = 1.7\times10^{-11}$ m

E is the same.
Mass is bigger $\Rightarrow$ λ is smaller.

c. $p = mv = 7.5$ kg m/s

$\lambda = h/p = 8.8\times10^{-35}$ m

SI units were designed to be convenient for macroscopic objects.

QM wave effects are negligible in the motion of macroscopic objects. 10^{-35} m is many orders of magnitude smaller than any distance that has ever been measured (10^{-19} m, at Fermilab).

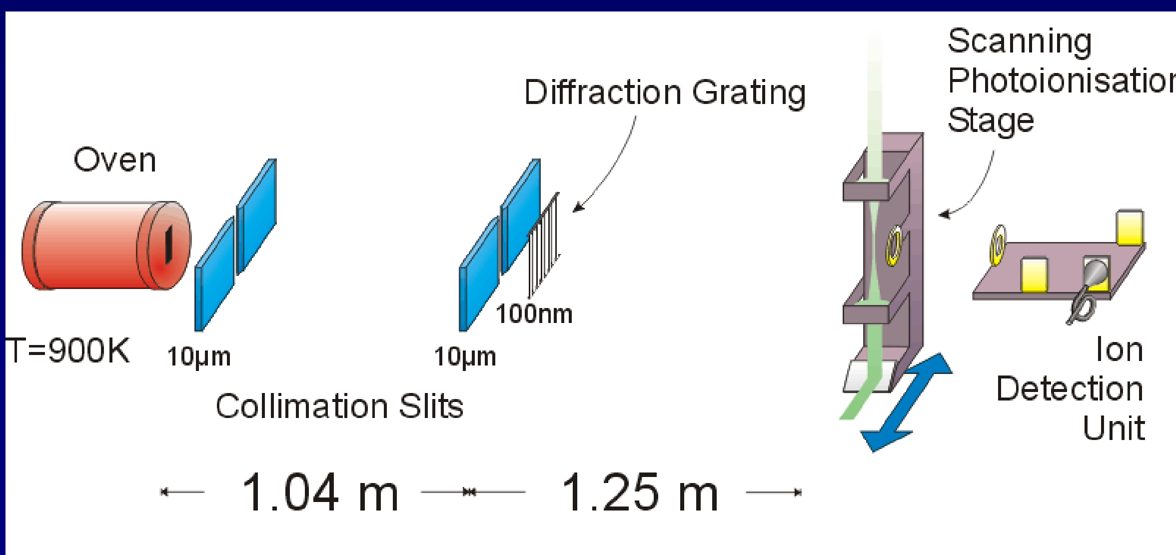
Interference of larger particles

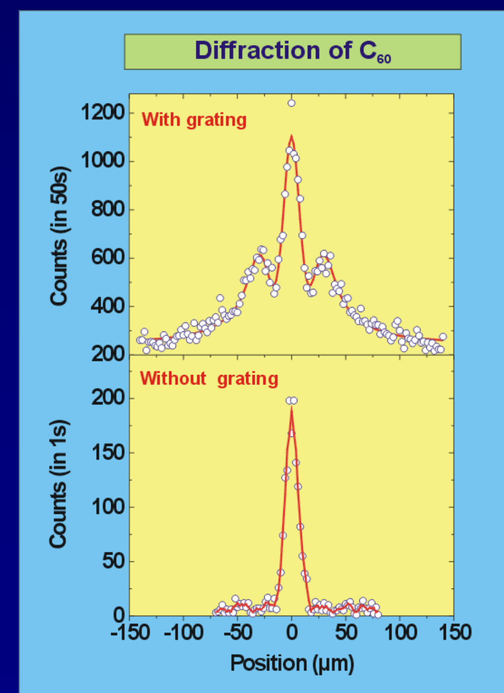
- Matter-wave interference has now been demonstrated with electrons, neutrons, atoms, small molecules, BIG molecules, & biological molecules
- Recent Example: Interference of C_{60}, a.k.a. "fullerenes", "buckyballs"

Mass = (60 C)(12 g/mole) = 1.2×10^{-24} kg

$$\frac{\langle p^2 \rangle}{2m} = K.E. \approx \frac{3}{2}kT \quad \Rightarrow \quad \langle p \rangle = \sqrt{3kTm} = 2.1\times10^{-22}\,kg\,m/s$$

$\lambda = h/p = 2.9$ pm (c.f. C_{60} is ~ 1 nm across!)

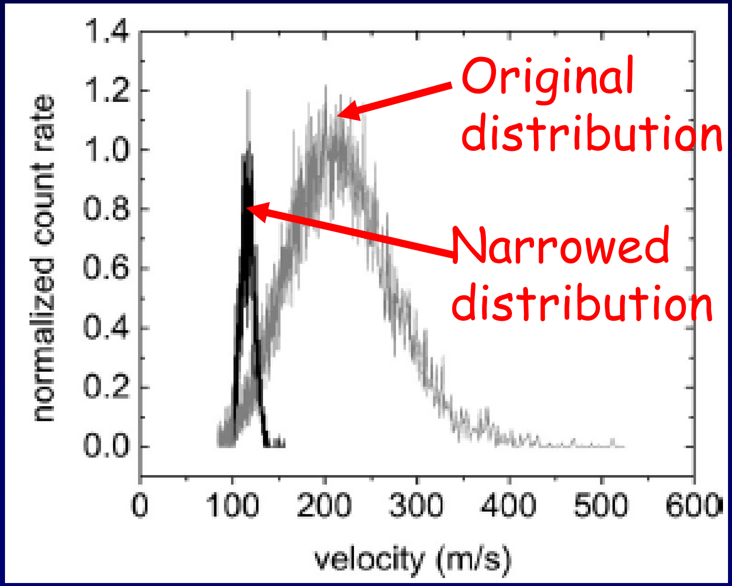


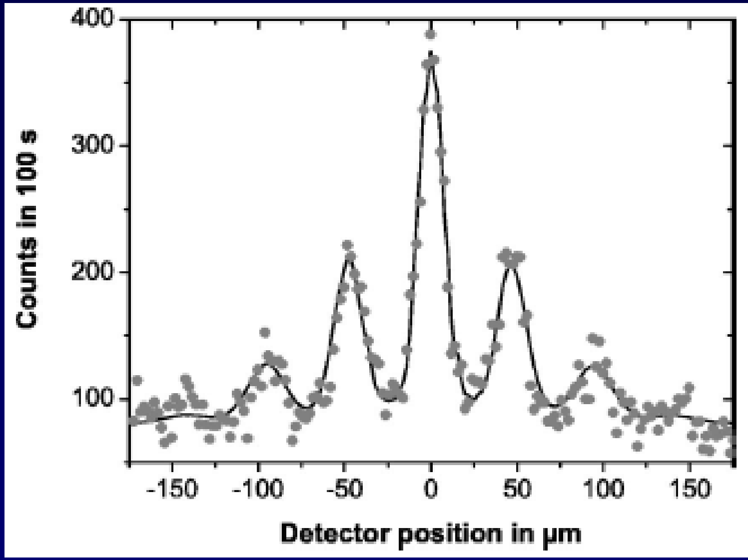


[A. Zeilinger (U. Vienna), 1999]

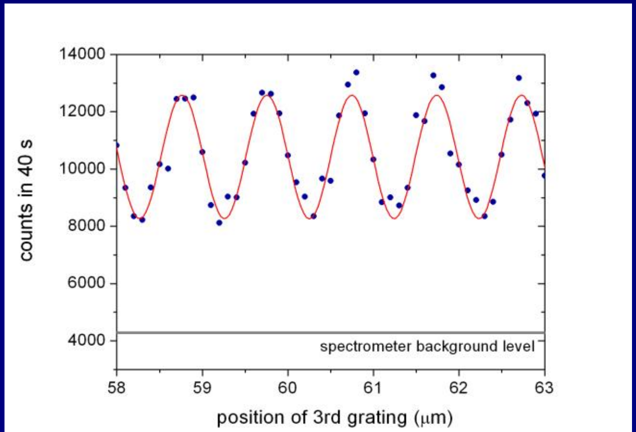
FYI: More on Interference of larger particles

- Using a velocity selector, they could make the atoms more monochromatic → improved interference:





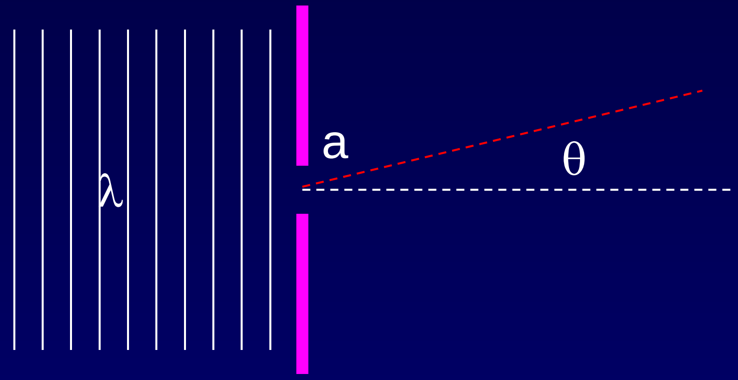
- In 2003 interference was observed with porphyrin, a bio. molecule:



Now they're trying to do something like this with a virus!

Diffraction and the Uncertainty Principle

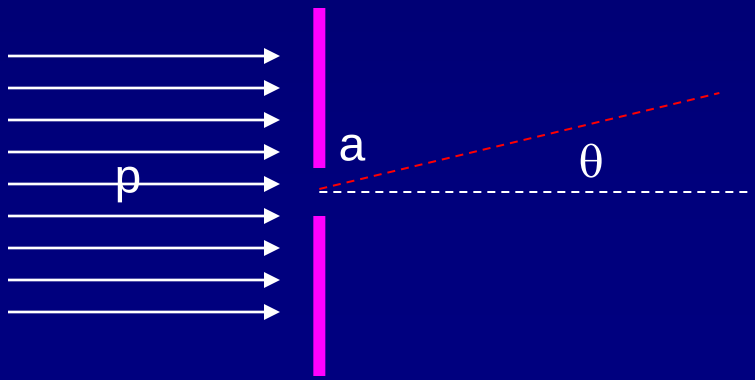
Remember single-slit diffraction:



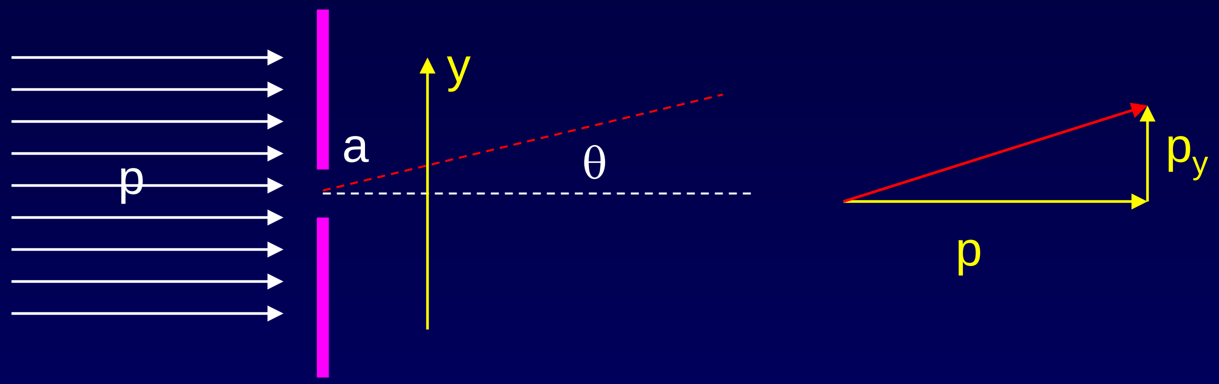
Wavelength: λ
Slit width: a
Diffraction angle: $\theta = \lambda/a$
angle to first zero

Let's analyze this problem using the uncertainty principle.

Suppose a beam of electrons of momentum p approaches a slit of width a. How big is the angular spread of motion after it passes through the slit?



Solution



Consider the momentum uncertainty in the y-direction.

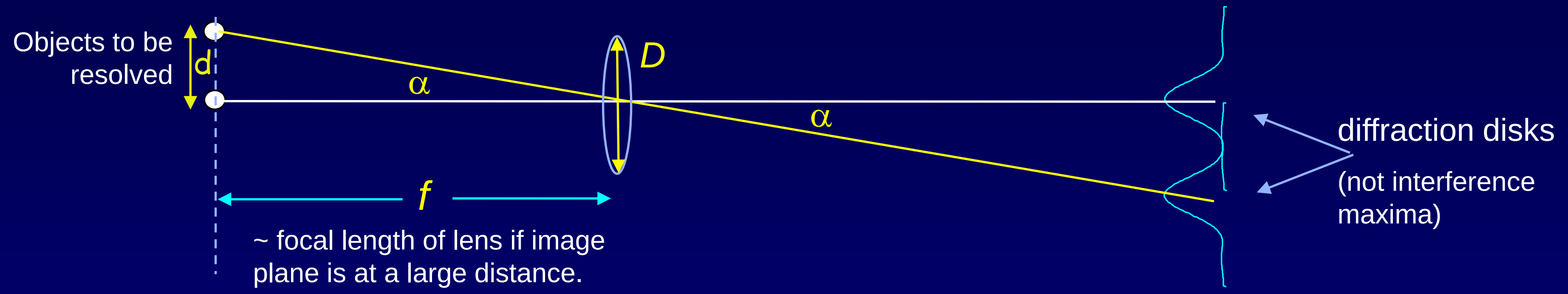
- Before the slit, the y-position is not known, so the uncertainty of p_y can be zero. We know that $p_y = 0$.
- Just after the slit, the y-position has an uncertainty of about a/2.
  Therefore p_y must have an uncertainty $\Delta p_y \geq 2\hbar/a$.
  This corresponds to a change of direction by an angle, $\theta = \Delta p_y / p = 2\hbar/ap$.
  Using $p = h/\lambda$, we have $\theta = \lambda/(\pi a)$.

This is almost the diffraction answer: $\theta = \lambda/a$. The extra factor of π is due to our somewhat sloppy treatment of the uncertainty.

The important point is that the uncertainty principle results because matter behaves as a wave.

Application of Matter Waves:
Electron Microscopy

The ability to resolve tiny objects improves as the wavelength decreases.
Consider the microscope:



Rayleigh's criterion: $$\alpha_c = 1.22\frac{\lambda}{D}$$

The minimum d for which we can still resolve two objects is α_c times the focal length: $$d_{min} \approx f\alpha_c = 1.22\lambda\frac{f}{D}$$

the "f-number"

The objective lens of a good optical microscope has f/D ≅ 2,
so with λ ~ 500 nm the microscope has a resolution of d_{min} ~ 1 μm.

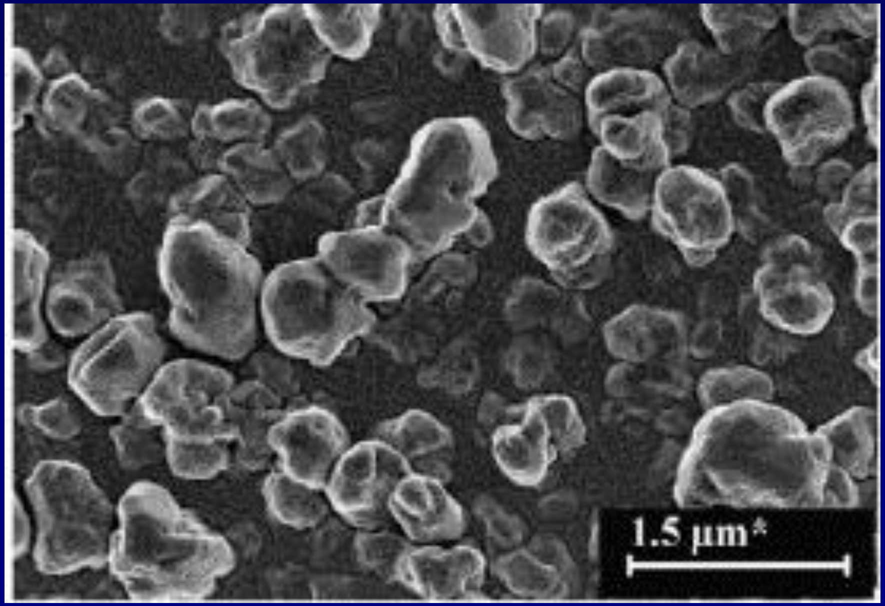
We can do much better with matter waves because electrons with energies of a few keV have wavelengths much less than 1 nm.

The instrument is known as an "electron microscope".

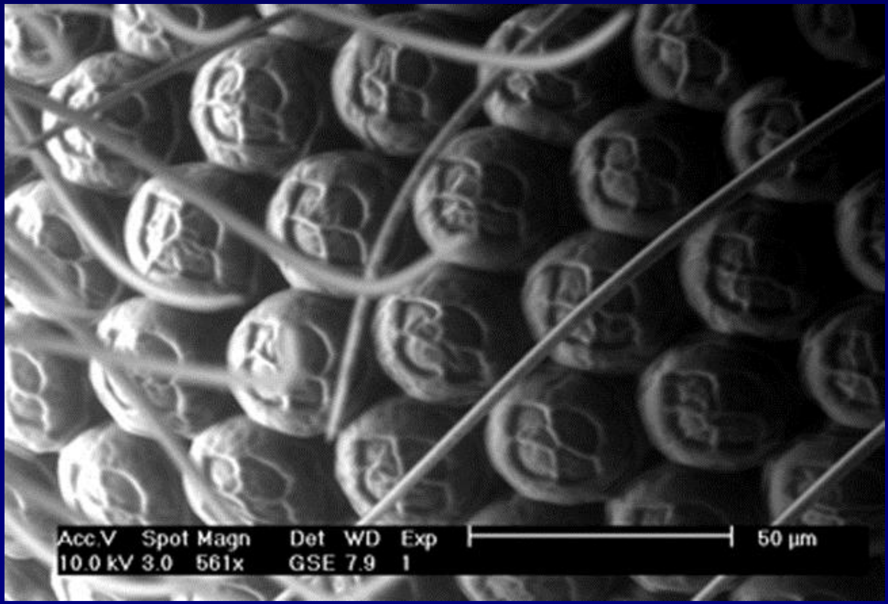
Application of Matter Waves:
Electron Microscopy

Scientists and engineers - such as those here at the Materials Research Lab and the Beckman Institute - use "electron microscopy" to study nanometer-scale structures in materials and biological systems

Cu-In alloy



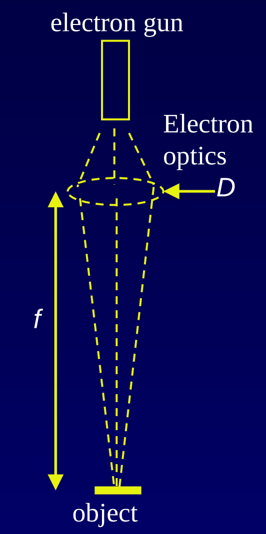
Compound eye of a fly



Imaging technology at Beckman: http://www.itg.uiuc.edu/

Example: Imaging a Virus

You wish to observe a virus with a diameter of 20 nm, much too small to observe with an optical microscope. Calculate the voltage required to produce an electron wavelength suitable for studying this virus with a resolution of d_{min} = 2 nm. The "f-number" for an electron microscope is quite large: $f/D \approx 100$.



Hint: First find λ required to achieve d_{min}.
Then find E of an electron from λ.

Solution

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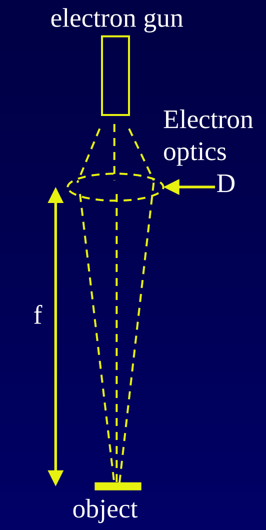

Hint: First find λ required to achieve d_{min}.
Then find E of an electron from λ.

$$d_{min} \approx 1.22 \frac{\lambda}{D} f$$

$$\lambda \approx d_{min}\left(\frac{D}{1.22f}\right) = 2nm\left(\frac{D}{1.22f}\right) = 0.0164\ nm$$

$$E = \frac{h^2}{2m\lambda^2} = \frac{1.505\ eV \cdot nm^2}{(0.0164\ nm)^2} = 5.6\ keV$$

Note:
1.22λ/D is the diffraction angle, θ
f is the lever arm,
So, θf is the spot size.

To accelerate an electron to an energy of 5.6 keV requires 5.6 kilovolts .
(The convenience of electron-volt units)

Hydrogen Atom Example

A hydrogen atom is about 0.1 nm in diameter. Suppose we wanted to measure the position of its electron with an accuracy of, say, 0.01 nm by scattering a photon off it.

How much energy would be transferred to the electron if the photon lost most of its energy in the scattering?

Solution

A hydrogen atom is about 0.1 nm in diameter. Suppose we wanted to measure the position of its electron with an accuracy of 0.01 nm by scattering a photon off it.

How much energy would be transferred to the electron if the photon lost most of its energy in the scattering?

Solution:

The photon would have to have a wavelength of about 0.01 nm.

The energy of the photon is:

$$E_{photon} = pc = \frac{hc}{\lambda} = \frac{1240\ eV \cdot nm}{10^{-11}\ m} = 1.24 \times 10^{5}\ eV$$

Since the binding energy of the electron is only about 13 eV, this measurement would disrupt the atom.