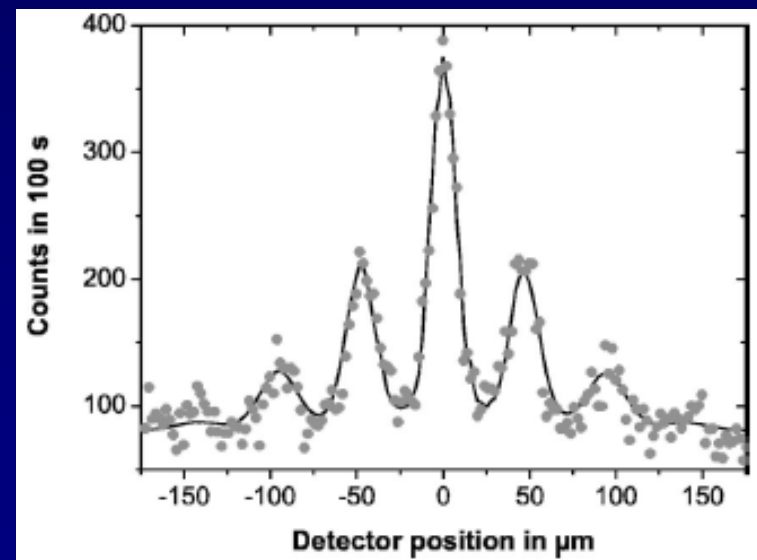
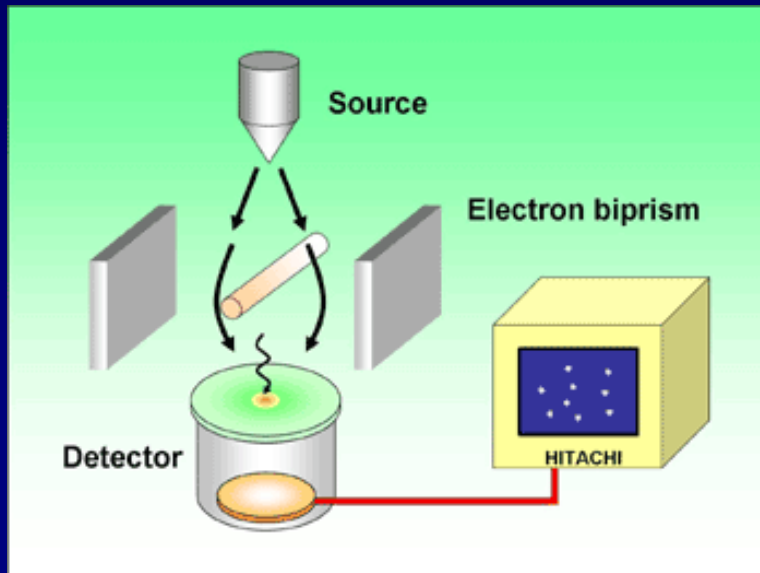


"We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery."

--Richard P. Feynman

Lecture 8: Wave-Particle Duality



This week and next week are critical for the course:

Week 3, Lectures 7-9:

Light as Particles

Particles as waves

Probability

Uncertainty Principle

Week 4, Lectures 10-12:

Schrödinger Equation

Particles in infinite wells, finite wells

Simple Harmonic Oscillator

Midterm Exam Monday, week 5

It will cover lectures 1-12 (except Simple Harmonic Oscillators)

Practice exams: Old exams are linked from the course web page.

Review: Sunday before Midterm

Office hours: Sunday and Monday

Last Time

The important results from last time:

Light, which we think of as waves, really consists of particles – photons –
with $E = hf$ Energy-frequency ($= hc/\lambda$ only for photons)
 $p = h/\lambda$ Momentum-wavelength

Today we will see that these universal equations apply to *all* particles.

In fact, quantum mechanical entities can exhibit either wave-like or particle-like properties, depending on what one measures.

Today

Interference, the 2-slit experiment revisited

Only indistinguishable processes can interfere

Wave nature of particles

Proposed by DeBroglie in 1923, to explain atomic structure.

Demonstrated by diffraction from crystals – just like X-rays!

Matter-wave Interference

Double-slit interference pattern, just like photons

Wave-Particle "Duality"

Light sometimes exhibits wave-like properties (interference), and sometimes exhibits particle-like properties (trajectories).

We will soon see that matter particles (electrons, protons, etc.) also display both particle-like and wave-like properties!

An important question:

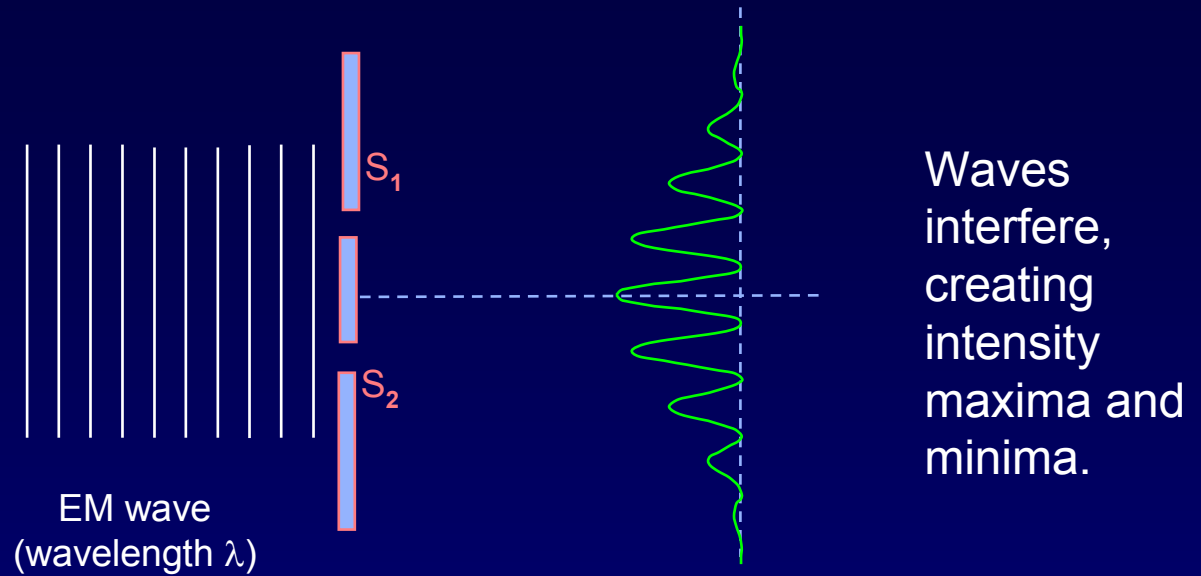
When should we expect to observe wave-like properties, and when should we expect particle-like properties?

To help answer this question, let's reconsider the 2-slit experiment.

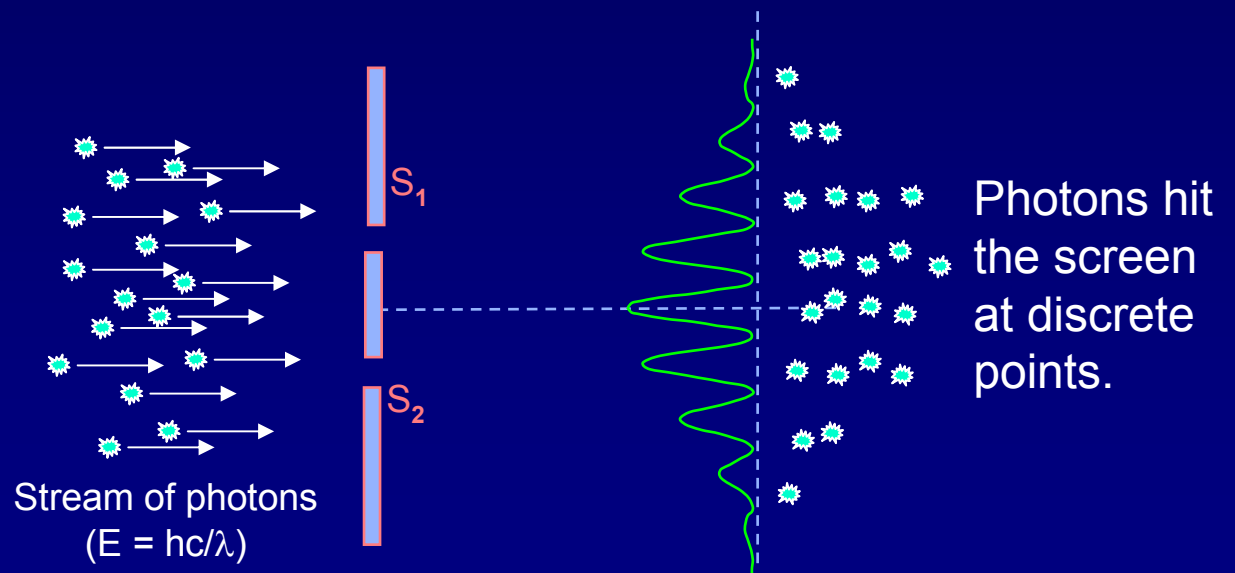
2-slits Revisited (1)

Recall 2-slit interference:

We analyzed it this way
(Wave view):



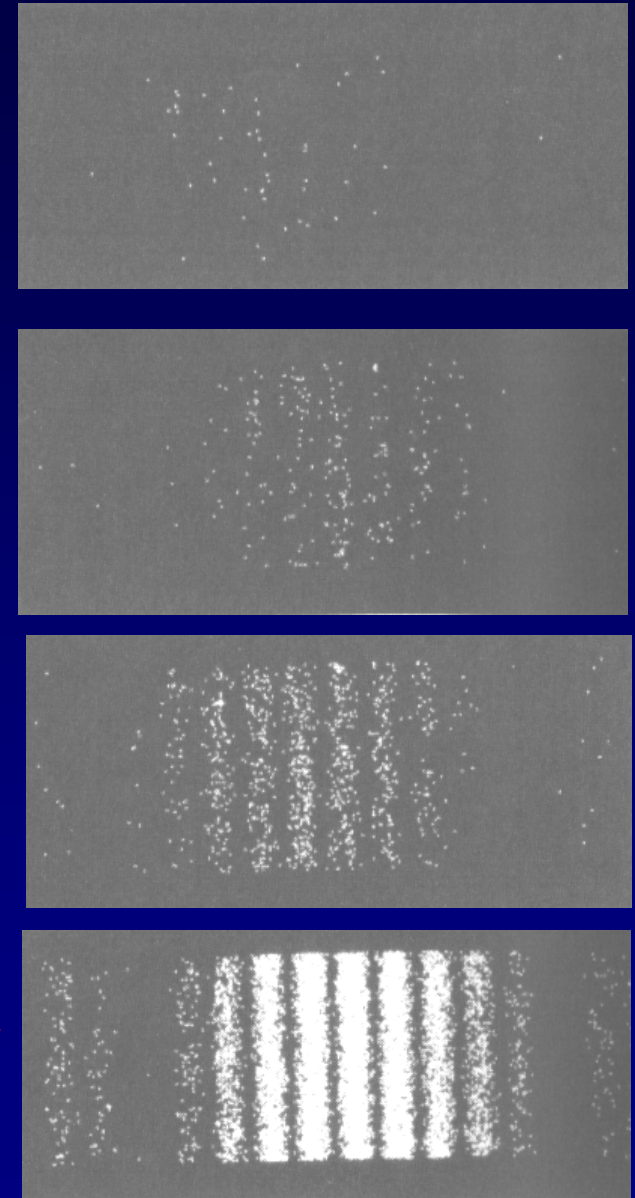
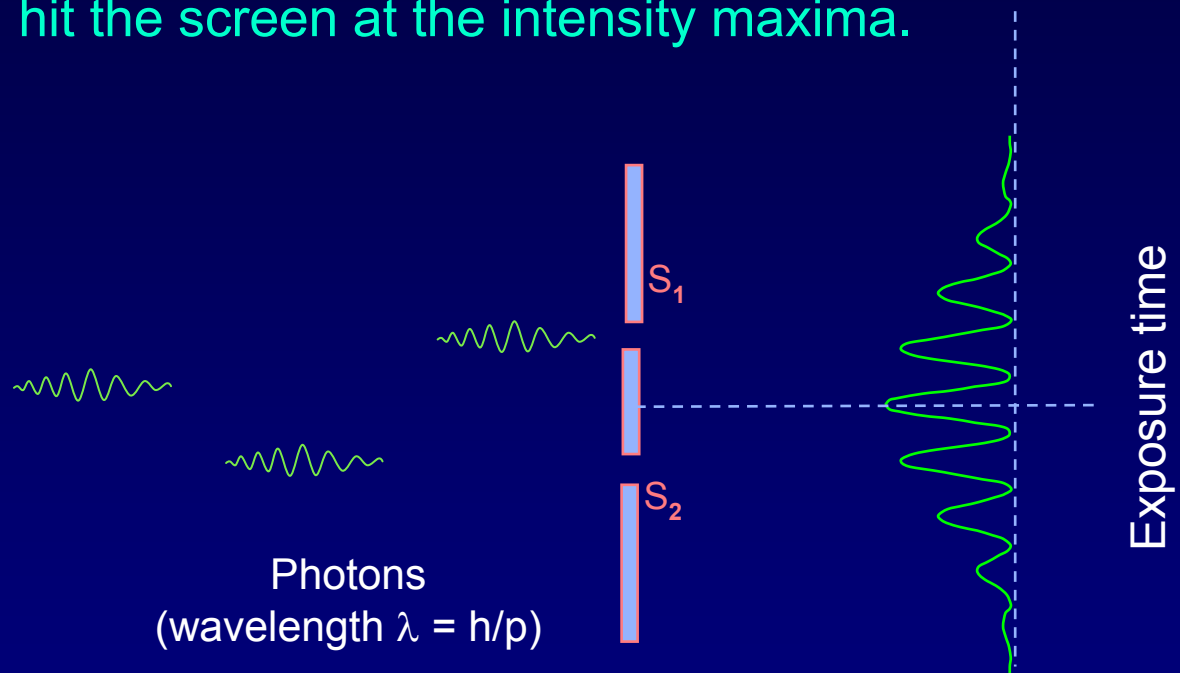
Can we also analyze it this way?
(Particle view):



How can particles yield an interference pattern?

2-slits Revisited (2)

It's just like the formation of a photographic image. More photons hit the screen at the intensity maxima.



The big question ...

What determines where an individual photon hits the screen?

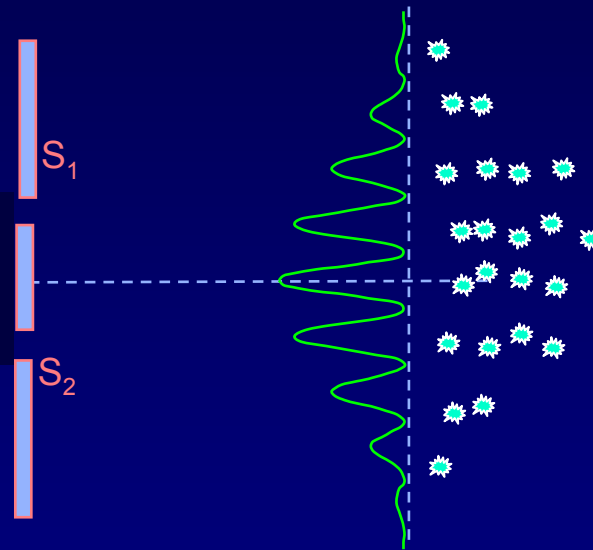
2-slits Revisited (3)

The quantum answer:

The intensity of the wave pattern describes the probability of arrival of quanta. The wave itself is a “probability amplitude”, usually written as ψ .

Light consists of quantum “entities”.

(neither waves nor particles)



One observes a random arrival of photons.

Randomness is intrinsic to QM.

Quantum mechanical entities are neither particles nor waves separately, but both simultaneously. Which properties you observe depends on what you measure.

Very large number of quanta \Rightarrow classical wave pattern

2-slits Revisited (4)

Hold on! This is kind of weird!

How do we get an interference pattern from single “particles” going through the slits one at a time?

Q: Doesn't the photon have to go through either slit 1 or slit 2?

A: No! Not unless we actually measure which slit !

The experimental situation:

- With only one slit open: You get arrival pattern P_1 or P_2 (see next slide).
- With both slits open:
 - If something ‘measures’ which slit the photon goes through, there is no interference: $P_{\text{tot}} = P_1 + P_2$.
 - If nothing ‘measures’ which slit the light goes through, P_{tot} shows interference, as if the photon goes through both slits!

Each individual photon exhibits wave behavior!
QM waves are **not** a collective phenomenon.

2-slits Revisited (5)

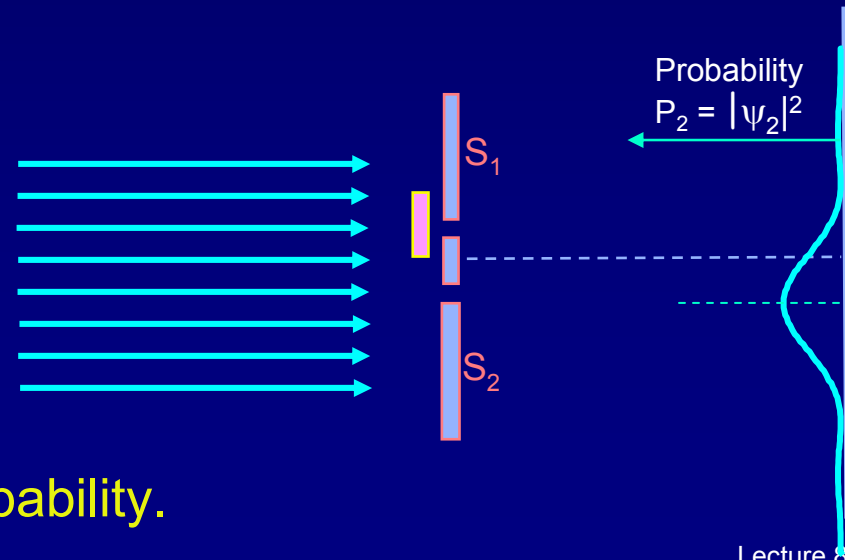
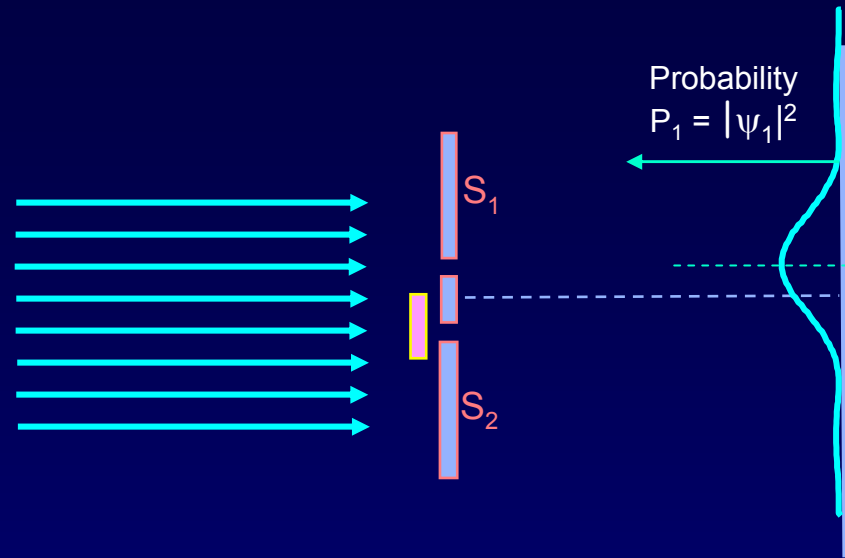
First, cover slit 2; *i.e.*, only light that goes through slit 1 is transmitted. What do we see on the screen?

We get a single-slit diffraction pattern.

Probability amplitude = ψ_1
Probability density = $|\psi_1|^2 = P_1$

Similar results when slit 1 is covered. $|\psi_2|^2 = P_2$

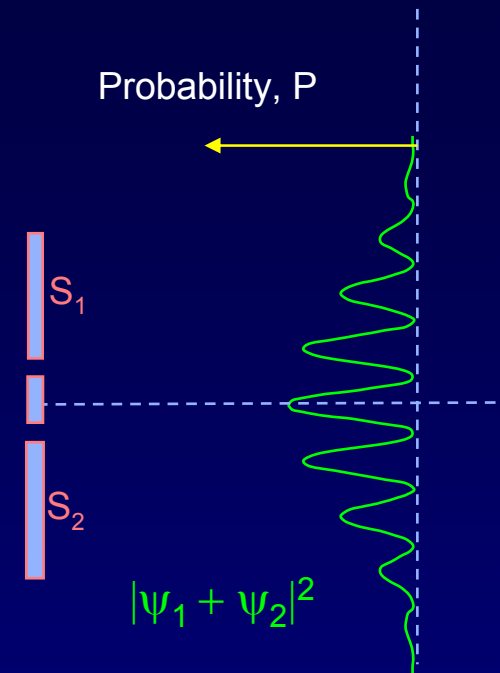
Changing the wave changes the probability.



2-slits Revisited (6)

Now, open both slits. We see interference!

The probability amplitude is now $\psi_1 + \psi_2$, because you don't know which slit the photon went through.



$$\begin{aligned} P_{\text{tot}} &= \text{Probability density} \\ &= |\psi_1 + \psi_2|^2 \\ &= |\psi_1|^2 + |\psi_2|^2 + \text{interference term} \end{aligned}$$

$$P \neq P_1 + P_2$$

Add amplitudes
not intensities.

The interference term will depend on phase differences, just like the wave calculations we did before.

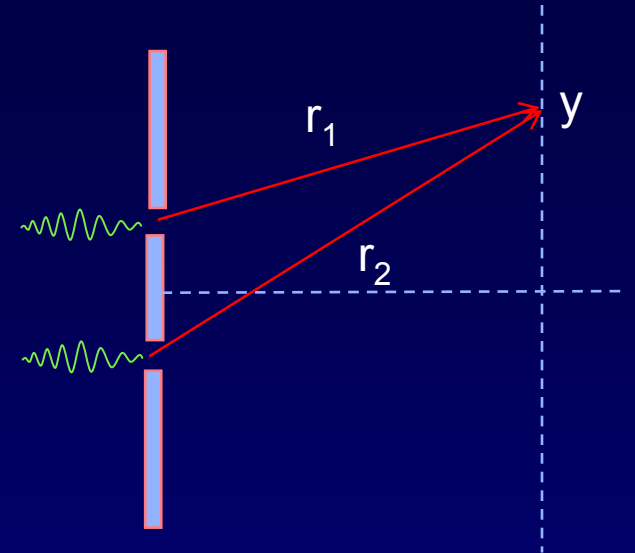
FYI: Two-Slit Experiment, More Carefully

ψ_1 : amplitude to pass through upper slit, and travel to y

$$\psi_1 \sim e^{ikr_1}$$

ψ_2 : amplitude to pass through lower slit, and travel to y

$$\psi_2 \sim e^{ikr_2}$$



Assume that the only difference between ψ_1 and ψ_2 is a result of the difference between r_1 and r_2 .

$$P = |\psi_1 + \psi_2|^2 \sim |e^{ikr_1} + e^{ikr_2}|^2 = (e^{+ikr_1} + e^{+ikr_2})(e^{-ikr_1} + e^{-ikr_2})$$

$$= e^{+ikr_1}e^{-ikr_1} + e^{+ikr_2}e^{-ikr_2} + e^{+ikr_1}e^{-ikr_2} + e^{-ikr_1}e^{+ikr_2}$$

$$= 1 + 1 + e^{+i\phi} + e^{-i\phi}$$

$$= 2 + 2\cos(\phi)$$

$$\phi = 2\pi \frac{r_1 - r_2}{\lambda} = kr_1 - kr_2$$

Two Slit Interference: Conclusions

Photons (or electrons ...) can produce interference patterns even one at a time !

With one slit closed, the image formed is simply a single-slit pattern.
We “know” (*i.e.*, we have constrained) which way the particle went.

With both slits open, a particle interferes with itself to produce the observed two-slit interference pattern.

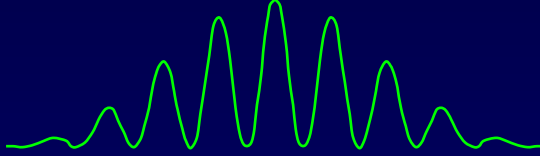
This amazing interference effect reflects, in a fundamental way, the indeterminacy of which slit the particle went through. We can only state the probability that a particle would have gone through a particular slit, if it had been measured.

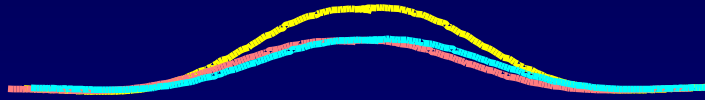
Confused? You aren't alone! We do not know how to understand quantum behavior in terms of our everyday experience. Nevertheless - as we will see in the next lectures – we know how to use the QM equations and make definite predictions for the probability functions that agree with careful experiments!

The quantum wave, ψ , is a probability amplitude. The intensity, $P = |\psi|^2$, tells us the probability that the object will be found at some position.

Act 1

Suppose we measure with the upper slit covered for half the time and the lower slit covered for the other half of the time. What will be the resulting pattern?

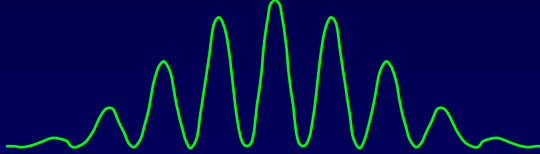
a. $|\psi_1 + \psi_2|^2$ 

b. $|\psi_1|^2 + |\psi_2|^2$ 

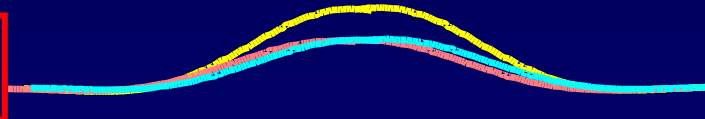
Solution

Suppose we measure with the upper slit covered for half the time and the lower slit covered for the other half of the time. What will be the resulting pattern?

a. $|\psi_1 + \psi_2|^2$



b. $|\psi_1|^2 + |\psi_2|^2$



At any given time, there is only one contributing amplitude (ψ_1 or ψ_2 , but not both). Therefore, for half the time pattern P1 will build up, and for the other half we'll get P2. There is no interference. The result will be the sum of the two single-slit diffraction patterns.

In order for waves to interfere, they must both be present at the same time.

Interference - What Really Counts

We have seen that the amplitudes from two or more physical paths interfere if nothing else distinguishes the two paths.

Example: (2-slits)

ψ_{upper} is the amplitude corresponding to a photon traveling through the upper slit and arriving at point y on the screen.

ψ_{lower} is the amplitude corresponding to a photon traveling through the lower slit and arriving at point y on the screen.

If these processes are **distinguishable** (*i.e.*, if there's some way to know which slit the photon went through), **add the probabilities**:

$$P(y) = |\psi_{\text{upper}}|^2 + |\psi_{\text{lower}}|^2$$

If these processes are **indistinguishable**, **add the amplitudes and take the absolute square** to get the probability:

$$P(y) = |\psi_{\text{upper}} + \psi_{\text{lower}}|^2$$

What does “distinguishable” mean in practice?

Act 2

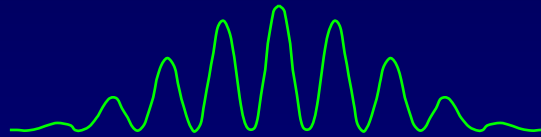
Let's modify the 2-slit experiment a bit. Recall that EM waves can be polarized – electric field in the vertical or horizontal directions.

Send in unpolarized photons.

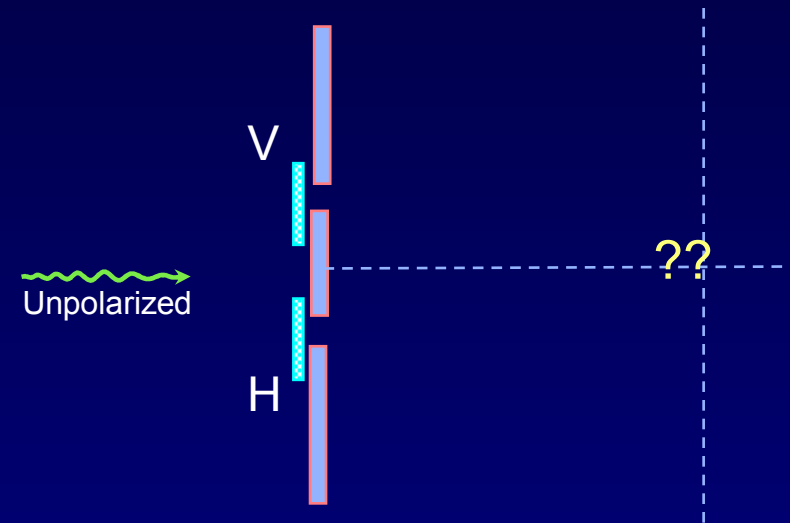
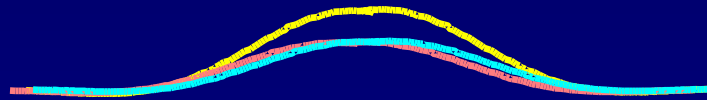
Cover the upper slit with a vertical polarizer and cover the lower slit with a horizontal polarizer

Now the resulting pattern will be:

a) $|\psi_1 + \psi_2|^2$



b) $|\psi_1|^2 + |\psi_2|^2$



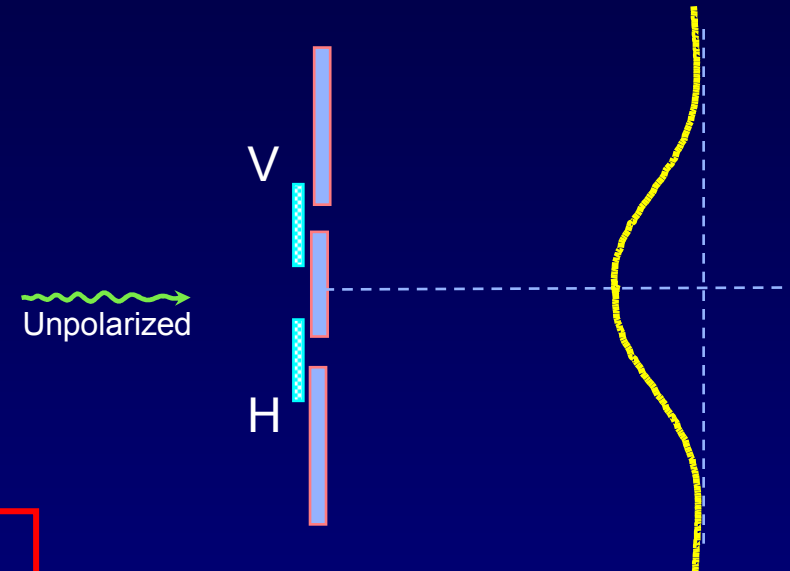
Solution

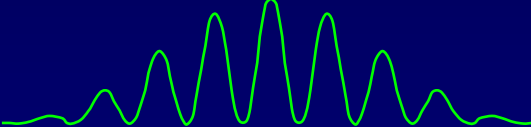
Let's modify the 2-slit experiment a bit. Recall that EM waves can be polarized – electric field in the vertical or horizontal directions.

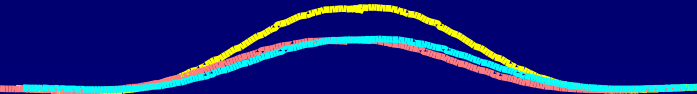
Send in unpolarized photons.

Cover the upper slit with a vertical polarizer and cover the lower slit with a horizontal polarizer

Now the resulting pattern will be:



a) $|\psi_1 + \psi_2|^2$ 

b) $|\psi_1|^2 + |\psi_2|^2$ 

The photon's polarization labels which way it went.
Because the two paths are in principle distinguishable there is no interference.

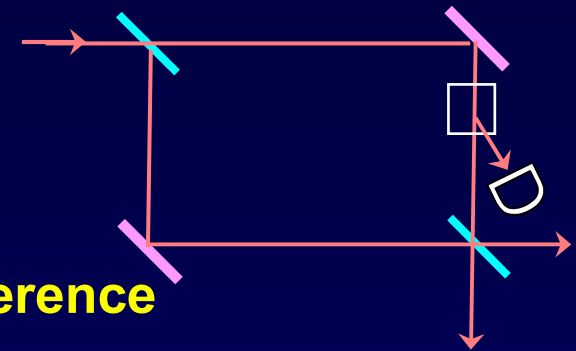
Note, that we don't actually need to measure the polarization.
The mere possibility that one could measure it destroys the interference.

Bonus Question: How could we recover the interference?

FYI: More Quantum Weirdness

Consider the following interferometer:

- photons are sent in one at a time
- the experimenter can choose to
 - leave both paths open, so that there is interference
 - activate switch in the upper path, deflecting that light to a counter
- What does it mean?
 - Switch OFF → interference → wave-like behavior
 - Switch ON → detector “click” or “no click” and no interference → particle-like behavior (trajectory is identified)
- **What is observed?** What kind of behavior you observe depends on what kind of measurement you make. Weird.



Principle of Complementarity: You can't get perfect particle-like and wave-like behavior in the same setup.

• **It gets worse!** In the “*delayed choice*” version of the experiment that was done, the switch could be turned ON and OFF *after* the photon already passed the first beam splitter! The results depended only on the state of the switch when the photon amplitude passed through it!

Matter Waves

We described one of the experiments (the photoelectric effect) which shows that light waves also behave as particles. The wave nature of light is revealed by interference - the particle nature by the fact that light is detected as quanta: “photons”.

Photons of light have energy and momentum given by:

$$E = hf \quad \text{and} \quad p = h/\lambda$$

Prince Louis de Broglie (1923) proposed that particles also behave as waves; i.e., for all particles there is a quantum wave with frequency and wavelength given by the same relation:

$$f = E/h \quad \text{and} \quad \lambda = h/p$$

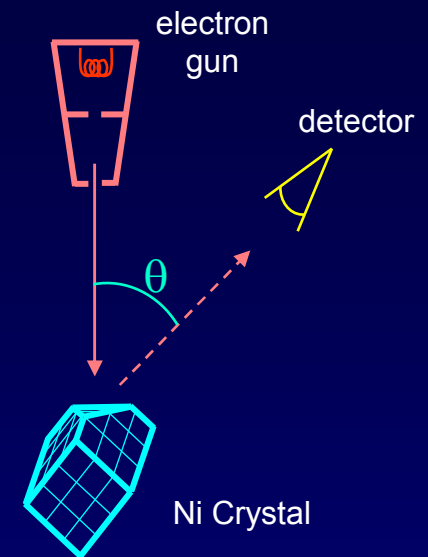
Matter Waves

Interference demonstrates that matter (electrons) can act like waves. In 1927-8, Davisson & Germer* showed that, like x-rays, electrons can diffract off crystals !

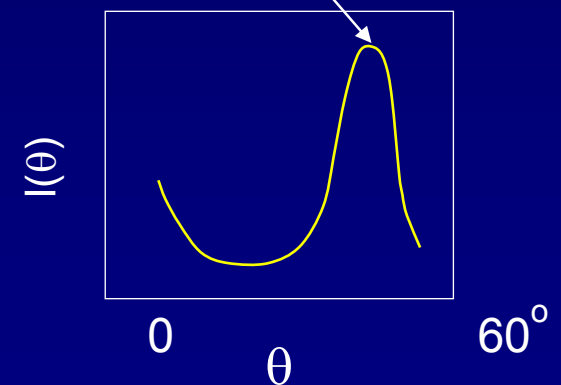
Electrons can act like waves,
just like photons!

You'll study electron diffraction in discussion.

*Work done at Bell Labs, Nobel Prize



Interference peak !



Act 3: Matter Wavelengths

What size wavelengths are we talking about? Consider a photon with energy 3 eV, and therefore momentum $p = 3 \text{ eV}/c$.^{*} Its wavelength is:

$$\lambda = \frac{h}{p} = \frac{4.14 \times 10^{-15} \text{ eV} \cdot \text{s}}{3 \text{ eV}} \times c = (1.4 \times 10^{-15} \text{ s}) \times (3 \times 10^8 \text{ m/s}) = 414 \text{ nm}$$

What is the wavelength of an electron with the same momentum?

a) $\lambda_e < \lambda_p$

b) $\lambda_e = \lambda_p$

c) $\lambda_e > \lambda_p$

^{*}It is an unfortunate fact of life that there is no named unit for momentum, so we're stuck with this cumbersome notation.

Solution

What size wavelengths are we talking about? Consider a photon with energy 3 eV, and therefore momentum $p = 3 \text{ eV}/c$. Its wavelength is:

$$\lambda = \frac{h}{p} = \frac{4.14 \times 10^{-15} \text{ eV} \cdot \text{s}}{3 \text{ eV}} \times c = (1.4 \times 10^{-15} \text{ s}) \times (3 \times 10^8 \text{ m/s}) = 414 \text{ nm}$$

What is the wavelength of an electron with the same momentum?

a) $\lambda_e < \lambda_p$

b) $\lambda_e = \lambda_p$

c) $\lambda_e > \lambda_p$

$\lambda = h/p$ for all objects, so equal p means equal λ .

Note that the kinetic energy of the electron does not equal the energy of a photon with the same momentum (and wavelength):

$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{(6.625 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(414 \times 10^{-9} \text{ m})^2}$$
$$= 1.41 \times 10^{-24} \text{ J} = 8.8 \times 10^{-6} \text{ eV}$$

Wavelength of an Electron

The DeBroglie wavelength of an electron is inversely related to its momentum:

$$\lambda = h/p$$

$$h = 6.626 \times 10^{-34} \text{ J-sec}$$

Frequently we need to know the relation between the electron's wavelength λ and its kinetic energy E . Because the electron has $v \ll c$, p and E are related through the Physics 211 formula:

$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

Valid for all (non-relativistic) particles

For $m = m_e$:
(electrons)

$$h = 4.14 \times 10^{-15} \text{ eV-sec}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$E_{\text{electron}} = \frac{1.505 \text{ eV} \cdot \text{nm}^2}{\lambda^2}$$

(E in eV; λ in nm)

Don't confuse this with $E_{\text{photon}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}$ for a photon.

Wavelengths of Various "Particles"

Calculate the wavelength of

- a. an electron that has been accelerated from rest across a 3-Volt potential difference ($m_e = 9.11 \times 10^{-31}$ kg).
- b. Ditto for a proton ($m_p = 1.67 \times 10^{-27}$ kg).
- c. a major league fastball ($m_{\text{baseball}} = 0.15$ kg, $v = 50$ m/s).

Solution

Calculate the wavelength of

- an electron that has been accelerated from rest across a 3-Volt potential difference ($m_e = 9.11 \times 10^{-31}$ kg).
- Ditto for a proton ($m_p = 1.67 \times 10^{-27}$ kg).
- a major league fastball ($m_{\text{baseball}} = 0.15$ kg, $v = 50$ m/s).

a. $E = eV = 4.8 \times 10^{-19}$ J

Physics 212

$p = \sqrt{(2m_e E)} = 9.35 \times 10^{-25}$ kg m/s

Physics 211

$\lambda = h/p = 7.1 \times 10^{-10}$ m = 0.71 nm

Physics 214

b. $p = \sqrt{(2m_p E)} = 4.00 \times 10^{-23}$ kg m/s

E is the same.

$\lambda = h/p = 1.7 \times 10^{-11}$ m

Mass is bigger $\Rightarrow \lambda$ is smaller.

c. $p = mv = 7.5$ kg m/s

SI units were designed to be

$\lambda = h/p = 8.8 \times 10^{-35}$ m

convenient for macroscopic objects.

QM wave effects are negligible in the motion of macroscopic objects. 10^{-35} m is many orders of magnitude smaller than any distance that has ever been measured (10^{-19} m, at Fermilab).

Summary: Photon & Matter Waves

Everything

$$E = hf$$

$$p = h/\lambda$$

Light ($v = c$)

$$E = pc, \text{ so}$$

$$E = hc/\lambda$$

$$E_{\text{photon}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}$$

Slow Matter ($v \ll c$)

$$KE = p^2/2m, \text{ so}$$

$$KE = h^2/2m\lambda^2$$

For electrons:

$$KE = \frac{1.505 \text{ eV} \cdot \text{nm}^2}{\lambda^2}$$