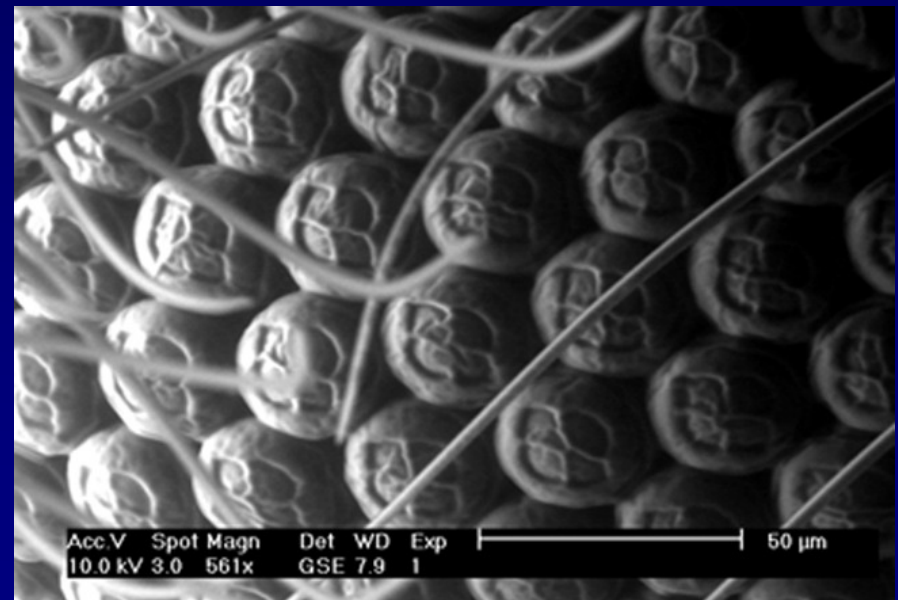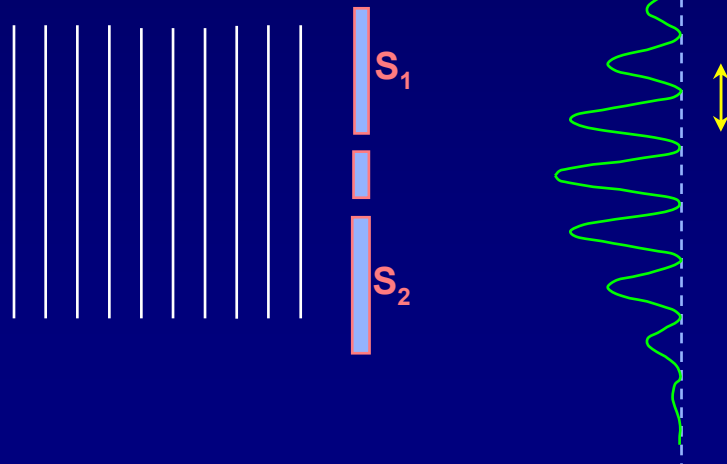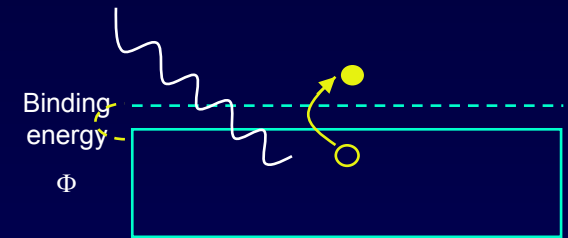# Lecture 9:
Introduction to QM:
Review and Examples

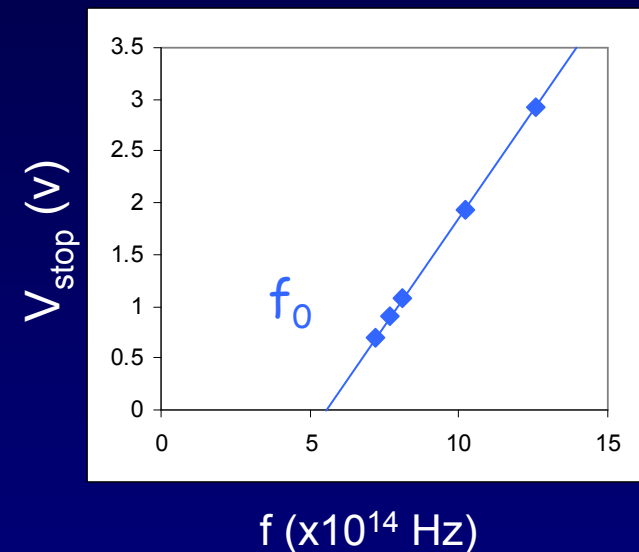# Photoelectric Effect

$$KE_{max} = e \cdot V_{stop} = hf - \Phi$$

The work function:

- Φ is the *minimum* energy needed to strip an electron from the metal.

- Φ is defined as positive.

- Not all electrons will leave with the maximum kinetic energy (due to losses).

Conclusions:

- Light arrives in "packets" of energy (photons).
- E_{photon} = hf
- Increasing the intensity increases # photons, not the photon energy. Each photon ejects (at most) one electron from the metal.

Recall: For EM waves, frequency and wavelength are related by f = c/λ.
Therefore: E_{photon} = hc/λ = 1240 eV-nm/λ

Binding energy

Φ

V_{stop} (v)

f_0

f (x10^{14} Hz)

Photoelectric Effect Example

1. When light of wavelength λ = 400 nm shines on lithium, the stopping voltage of the electrons is V_{stop} = 0.21 V. What is the work function of lithium?

Photoelectric Effect: Solution

1. When light of wavelength λ = 400 nm shines on lithium, the stopping voltage of the electrons is V_{stop} = 0.21 V. What is the work function of lithium?

$$\Phi = hf - eV_{stop}$$
$$= 3.1eV - 0.21eV$$
$$= 2.89 \text{ eV}$$

Instead of hf, use hc/λ: 1240/400 = 3.1 eV

For V_{stop} = 0.21 V, eV_{stop} = 0.21 eV

Act 1

1. If the workfunction of the material increased, how would the graph change?
   a. Increased slope
   b. Increased f_o
   c. Both a and b

V_{stop} (v)

f_0

$f (\times 10^{14}$ Hz)

2. We now shine on light with frequency $2f_0$. What effect does doubling the intensity (i.e., the power) of the incident light have on the current of emitted electrons?
   a. doubles
   b. stays the same
   c. decreases

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   - c. Both a and b

V_{stop} (v)

f_0

f (×10^{14} Hz)

The y intercept moves down, so the slope is unchanged and f_0 increases → the photons need more energy to be able to free the electrons from the increased binding.
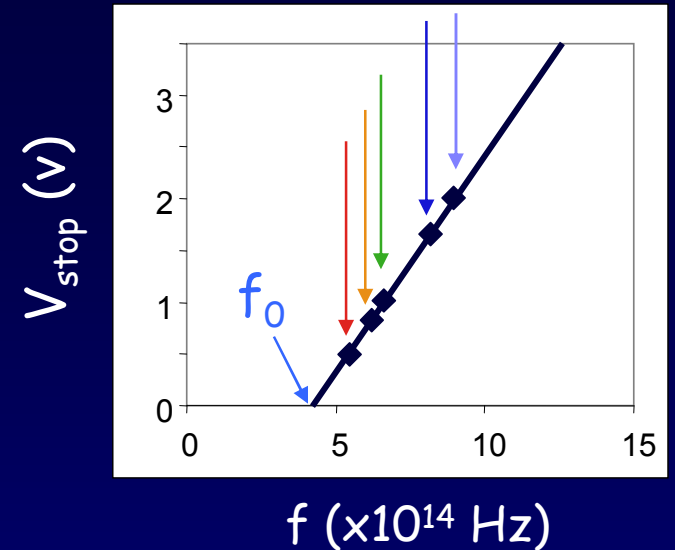
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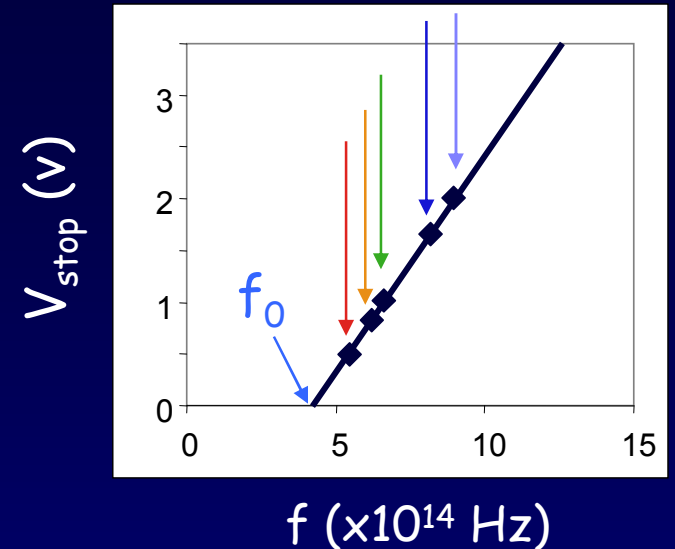
2. We now shine on light with frequency $2f_0$. What effect does doubling the intensity (i.e., the power) of the incident light have on the current of emitted electrons?
   a. doubles
   b. stays the same
   c. decreases

Because the frequency is higher than f_0, each incident photon has a chance to emit an electron. Doubling the number of photons doubles the number of photoelectrons.

Measurement

How does what we measure determine whether we observe wave or particle properties?

Waves have wavelength, λ, and frequency, f. So, if we measure momentum (wavelength) or energy (frequency), we have observed the wave properties of our object.

Particles have position (and trajectories). If we measure position (*e.g.*, which slit it went through) we have observed a particle property. That's why the "which slit" measurement destroys the interference pattern.

Note that particle and wave properties are incompatible. One can't simultaneously measure both wavelength and position. This is the basis of Heisenberg's "uncertainty principle". (more later)

Exercise

We can use our rules for quantum mechanical inter-ference to understand classical interference too!
Consider a Michelson interferometer, into which is directed an 8-mW laser with a 1-cm beam diameter.

We now put an iris in arm 1, centered on the beam, that reduces its diameter to only 0.71 cm, so that the power coming to the detector just from that arm is only 1 mW (and still 2 mW from the other path, whose beam is still 1 cm in diameter).
As we move the arm 1 mirror outward, which of the following curves might describe the power measured on the detector?
(Hint: what's required for interference.)

a. dash dot curve (varies from 0 to 8 mW)
b. red curve (varies from 1 to 5 mW)
c. solid curve (varies from 0.17 to 5.83 mW, with an average of 3 mW)
d. dashed curve (constant at 3 mW)
e. orange curve (constant at 4 mW)
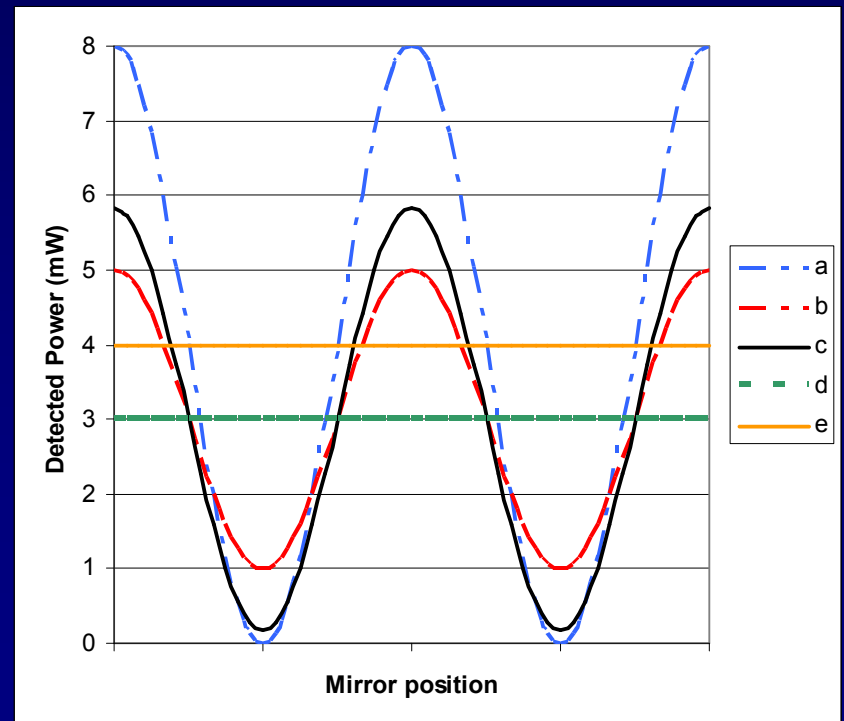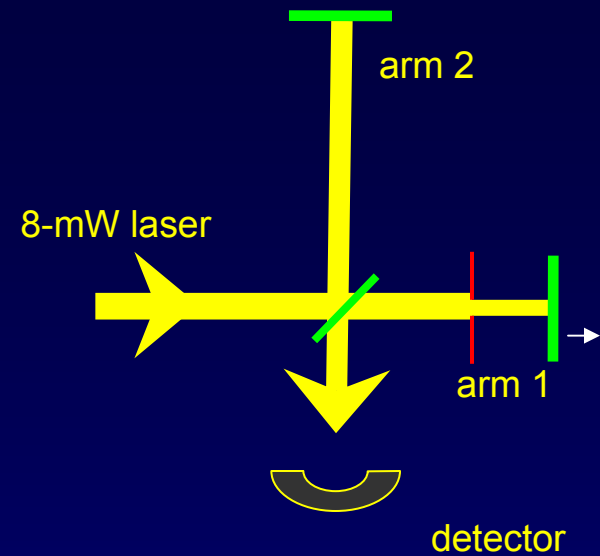
Solution

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(Hint: what's required for interference.)

b. red curve (varies from 1 to 5 mW)

Interference can only occur if the contributing processes are indistinguishable. In this problem, that's only the case for photons inside the 0.71-cm diameter disk, which could have come from either arm. Inside that disk, we have perfect interference (0 → 4 mW). But the detector also sees the non-interfering 1 mW from the outer ring from arm 2. This adds as a background.
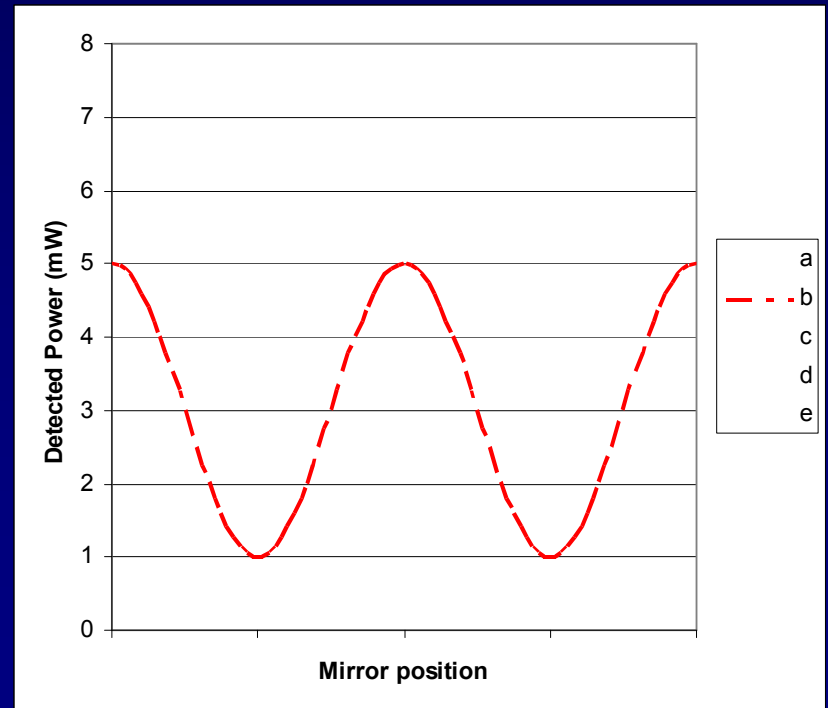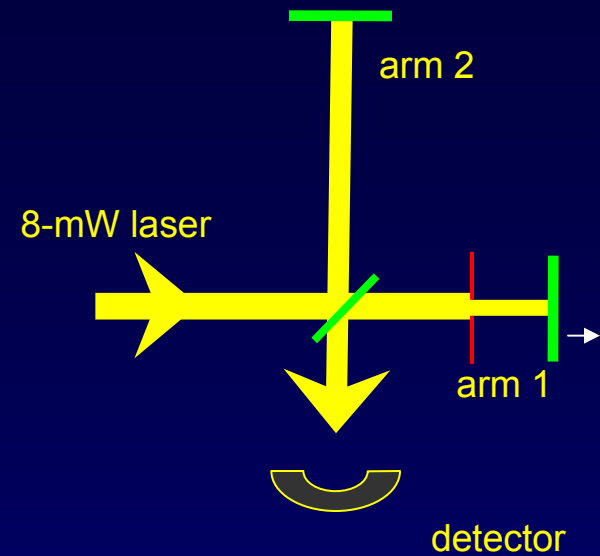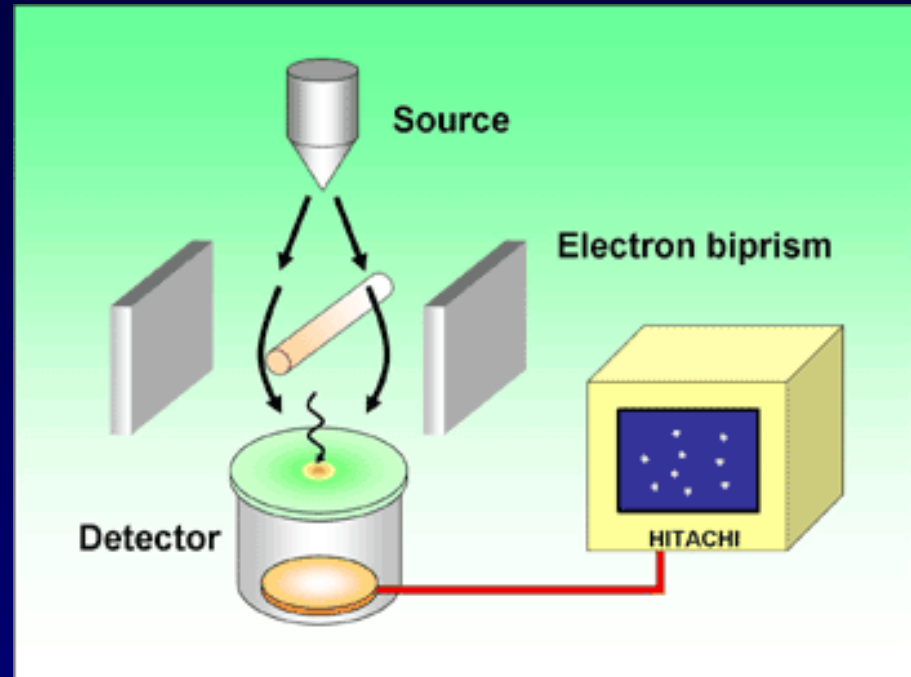
"Double-slit" Experiment for Electrons

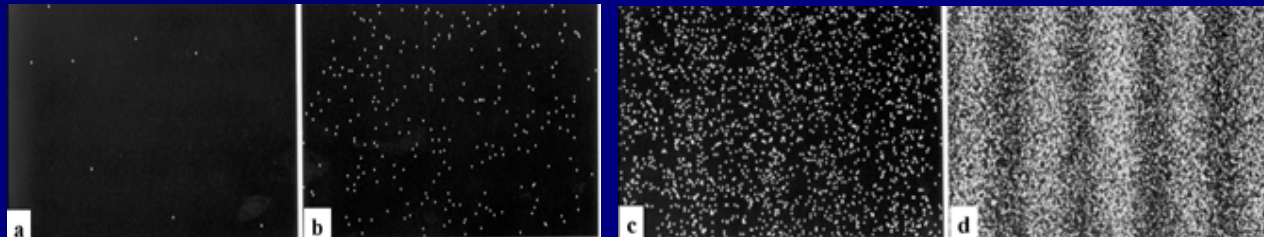Electrons are accelerated to 50 keV
→ λ = 0.0055 nm

Central wire is positively charged →
bends electron paths so they
overlap.

A position-sensitive detector records
where they appear.

<< 1 electron in system at any time

Video by A. TONOMURA (Hitachi) --pioneered electron holography.
http://www.hqrd.hitachi.co.jp/rd/moviee/doubleslite.wmv

Exposure time: 1 s   10 s   5 min   20 min

See also this Java simulation: http://www.quantum-physics.polytechnique.fr/index.html
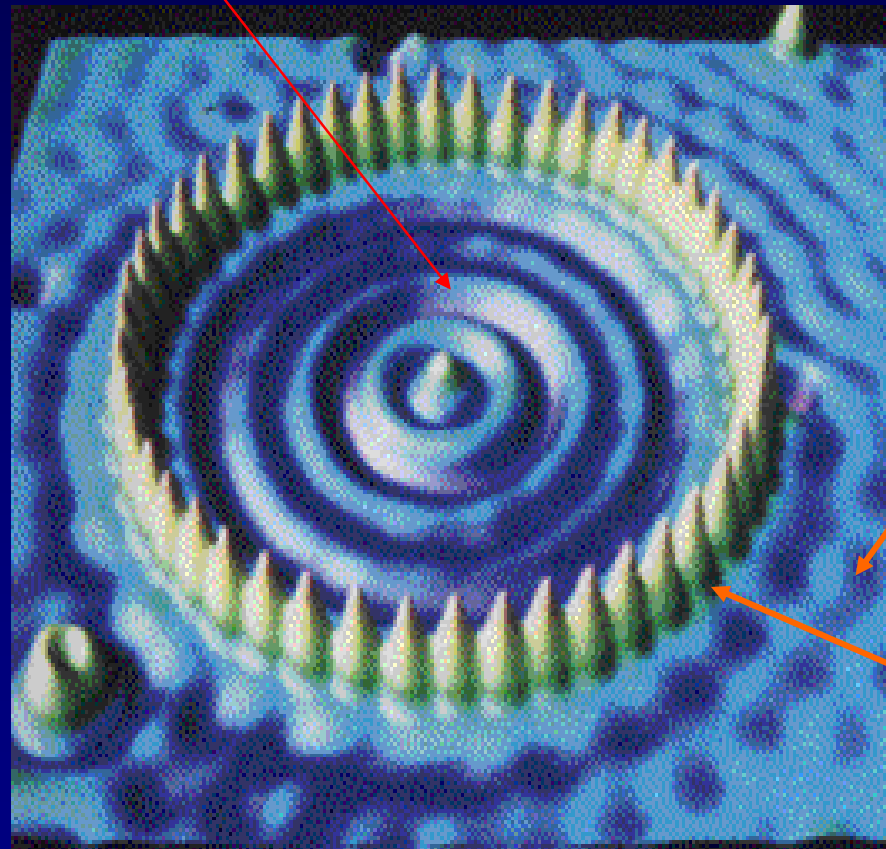
Observation of an electron wave "in a box"

Image taken with a scanning tunneling microscope (more later)
(Note: the color is not real! – it is a representation of the
electrical current observed in the experiment)

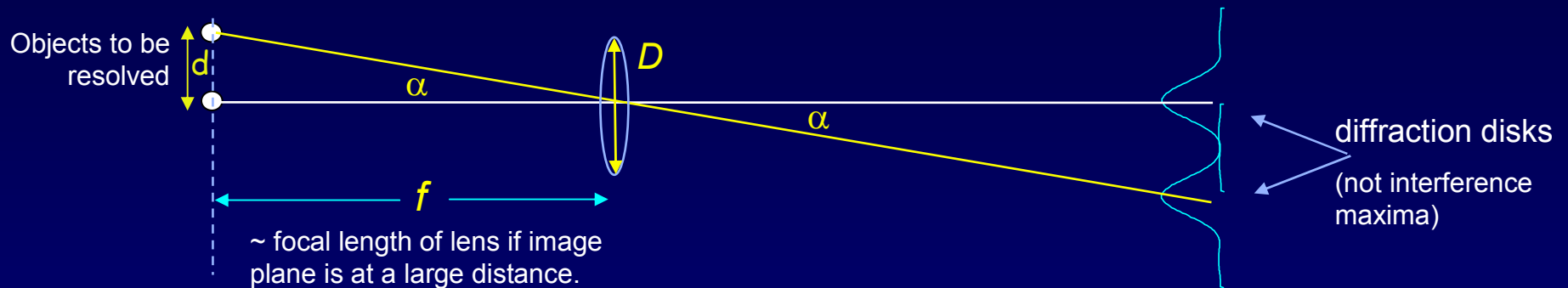Real <u>standing waves</u> of electron
density in a "quantum corral"

IBM
Almaden

Cu

Single
atoms (Fe)

Application of Matter Waves:
Electron Microscopy

The ability to resolve tiny objects improves as the wavelength decreases.
Consider the microscope:

Objects to be resolved — d

D

α α

α

f

diffraction disks

(not interference maxima)

~ focal length of lens if image plane is at a large distance.

Rayleigh's criterion:
$$\alpha_c = 1.22 \frac{\lambda}{D}$$

The minimum d for which we can still resolve two objects is α_c times the focal length:
$$d_{min} \approx f\alpha_c = 1.22\lambda \frac{f}{D}$$

the "f-number"

The objective lens of a good optical microscope has $f/D \cong 2$, so with $\lambda \sim 500$ nm the microscope has a resolution of $d_{min} \sim 1\ \mu$m.

We can do much better with matter waves because electrons with energies of a few keV have wavelengths much less than 1 nm.

The instrument is known as an "electron microscope".

Application of Matter Waves:
Electron Microscopy

Scientists and engineers - such as those here at the Materials Research Lab and the Beckman Institute - use "electron microscopy" to study nanometer-scale structures in materials and biological systems

Cu-In alloy

Compound eye of a fly

Imaging technology at Beckman: http://www.itg.uiuc.edu/
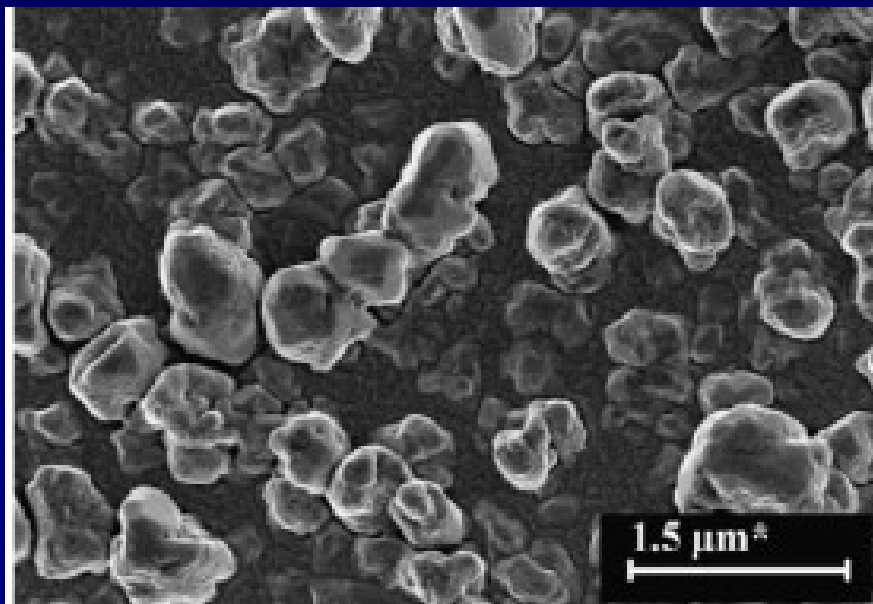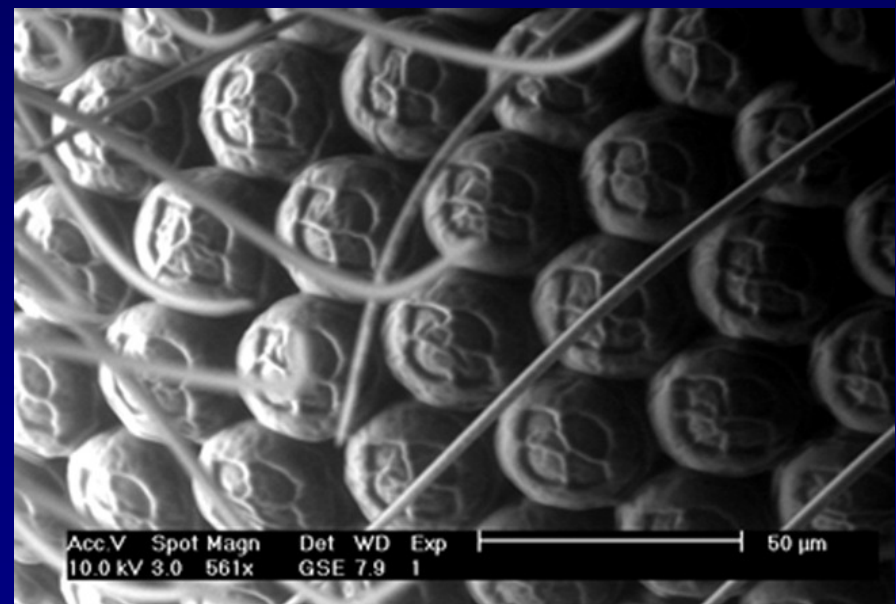
Example: Imaging a Virus

You wish to observe a virus with a diameter of 20 nm, much too small to observe with an optical microscope. Calculate the voltage required to produce an electron wavelength suitable for studying this virus with a resolution of d_{min} = 2 nm. The "f-number" for an electron microscope is quite large: $f/D \approx 100$.

Hint: First find λ required to achieve d_{min}.
    Then find E of an electron from λ.

electron gun
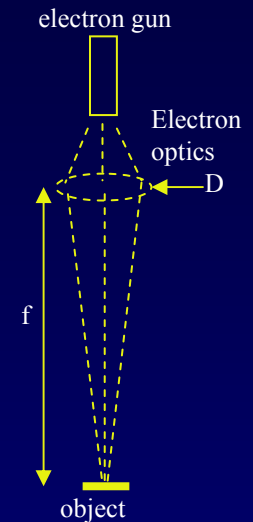
Electron optics

D

f

object
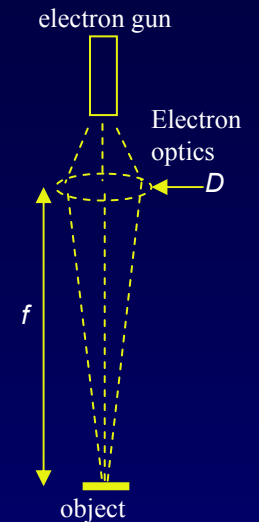
Solution

You wish to observe a virus with a diameter of 20 nm, much too small to observe with an optical microscope. Calculate the voltage required to produce an electron wavelength suitable for studying this virus with a resolution of d_{min} = 2 nm. The "f-number" for an electron microscope is quite large: $f/D \approx 100$.

Hint: First find λ required to achieve d_{min}.
     Then find E of an electron from λ.

electron gun

Electron optics

D

f

object

$$d_{min} \approx 1.22 \frac{\lambda}{D} f$$

$$\lambda \approx d_{min} \left(\frac{D}{1.22f} \right) = 2nm \left(\frac{D}{1.22f} \right) = 0.0164 \ nm$$

Note:
    1.22λ/D is the diffraction angle, θ
    f is the lever arm,
    So, θf is the spot size.

$$E = \frac{h^2}{2m\lambda^2} = \frac{1.505 \ eV \cdot nm^2}{(0.0164 \ nm)^2} = 5.6 \ keV$$

To accelerate an electron to an energy of 5.6 keV requires 5.6 kilovolts .
(The convenience of electron-volt units)
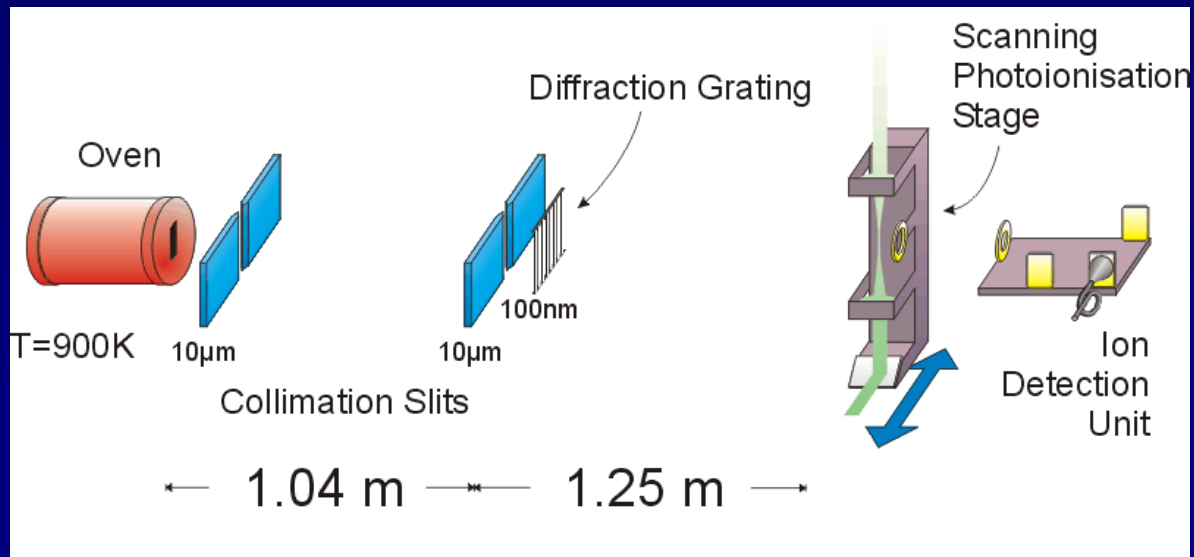
Interference of larger particles

- Matter-wave interference has now been demonstrated with electrons, neutrons, atoms, small molecules, BIG molecules, & biological molecules
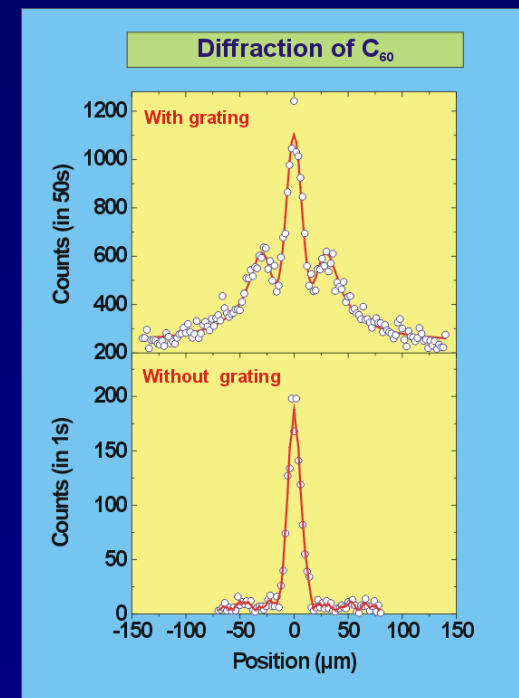- Recent Example: Interference of C_{60}, a.k.a. "fullerenes", "buckyballs"

Mass = (60 C)(12 g/mole) = 1.2×10^{-24} kg

$$\frac{\langle p^2 \rangle}{2m} = K.E. \approx \frac{3}{2}kT \implies \langle p \rangle = \sqrt{3kTm} = 2.1 \times 10^{-22} \, kg \, m/s$$

λ = h/p = 2.9 pm   (c.f. C_{60} is ~ 1 nm across!)

Diffraction Grating

Scanning
Photoionisation
Stage

Oven

100nm

T=900K   10µm   10µm
Collimation Slits

Ion
Detection
Unit

← 1.04 m → ← 1.25 m →

Diffraction of C_{60}

With grating

Counts (in 50s)
1200
1000
800
600
400
200

Without grating

Counts (in 1s)
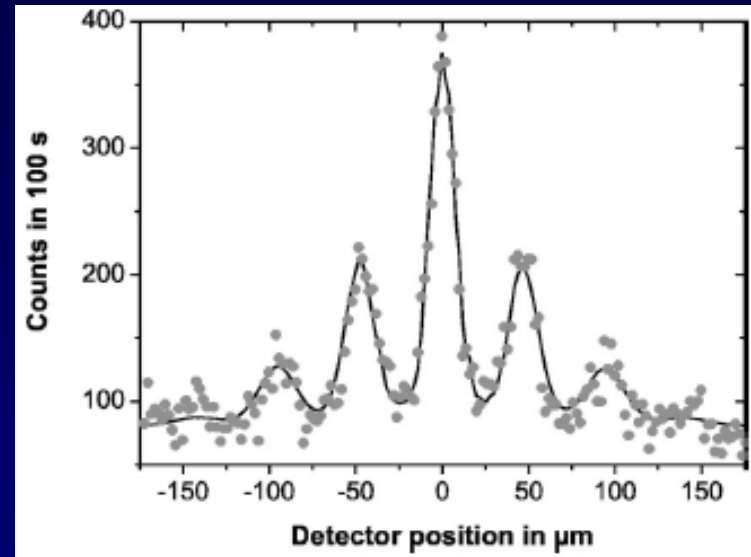200
150
100
50
0
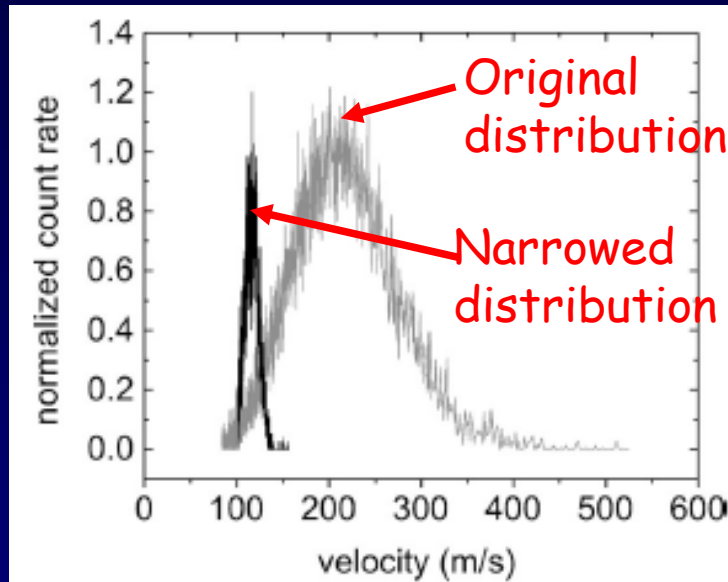-150 -100 -50 0 50 100 150
Position (µm)

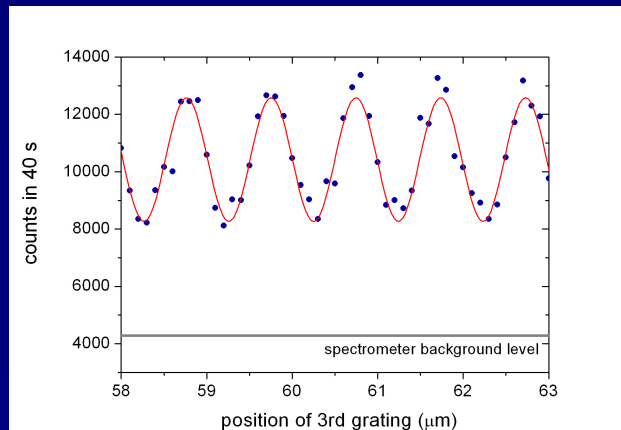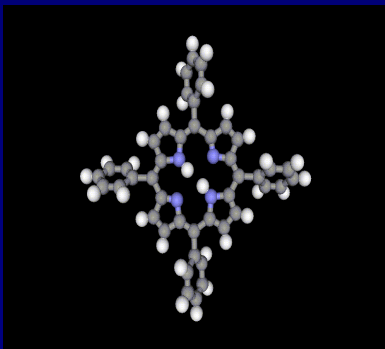[A. Zeilinger (U. Vienna), 1999]

FYI: More on Interference of larger particles

- Using a velocity selector, they could make the atoms more monochromatic → improved interference:

Original distribution

Narrowed distribution

- In <u>2003</u> interference was observed with porphyrin, a bio. molecule:

Now they're trying to do something like this with a virus!

Where do we go from here?

Two approaches pave the way:

Uncertainty principle

- In quantum mechanics one can only calculate a probability distribution for the result of a measurement.

- The Heisenberg uncertainty principle provides a way to use simple arguments and a simple inequality to draw important conclusions about quantum systems.

Schrödinger equation (next week)

- This differential equation describes the evolution of the quantum wave function, Ψ. Ψ itself has no uncertainty.

- $|\Psi|^2$ will then tell us the probabilities of obtaining various measurement results. That's where the uncertainty enters.
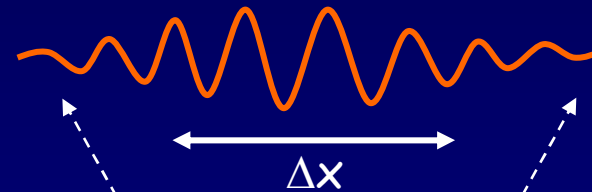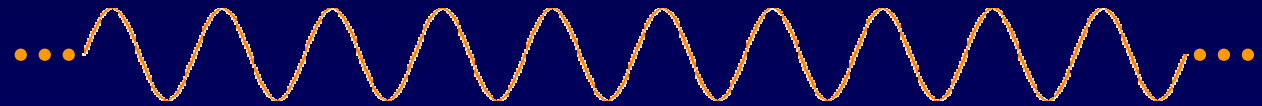
Heisenberg Uncertainty Principle

All QM objects (we think that includes everything have wave-like properties.
One mathematical property of waves is:

$\Delta k \cdot \Delta x \geq 1$ (See the supplementary slide for some discussion) $k = 2\pi/\lambda$

Examples:

- Infinite sine wave:
  A definite wavelength must extend forever.

- Finite wave packet:
  A wave packet requires a spread*
  of wavelengths.

Δx

We need a spread of wavelengths in order to get destructive interference.

Using $p = h/\lambda = \hbar k$, we have:

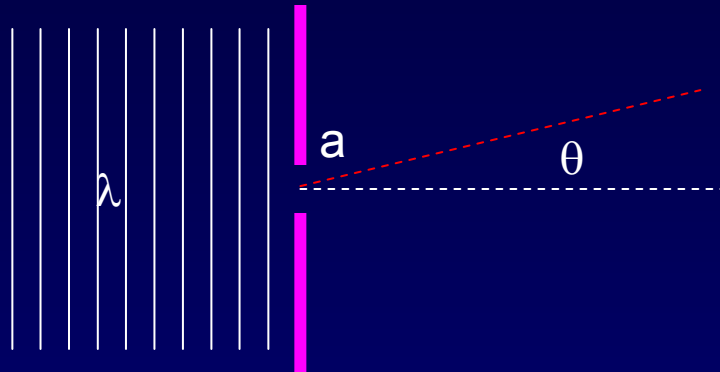$$\hbar \, (\Delta k \cdot \Delta x \geq 1) \Rightarrow (\hbar \Delta k) \cdot \Delta x \geq \hbar \Rightarrow \boxed{\Delta p \cdot \Delta x \geq \hbar}$$

This relation is known as the Heisenberg Uncertainty Principle.
It limits the accuracy with which we can know the position and momentum of objects.

* We will not use the statistically correct definition of "spread", which, in this context, we also call "uncertainty".
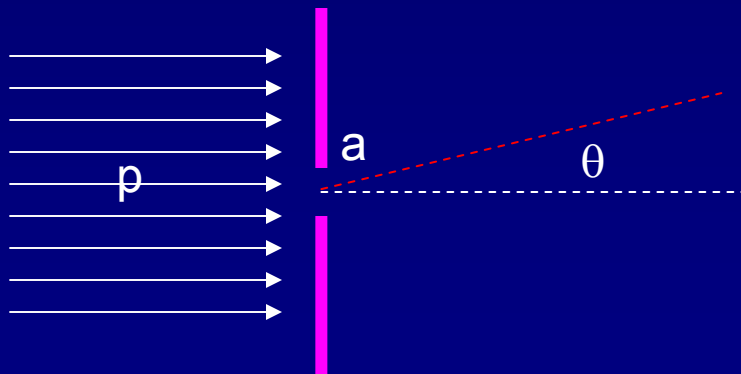
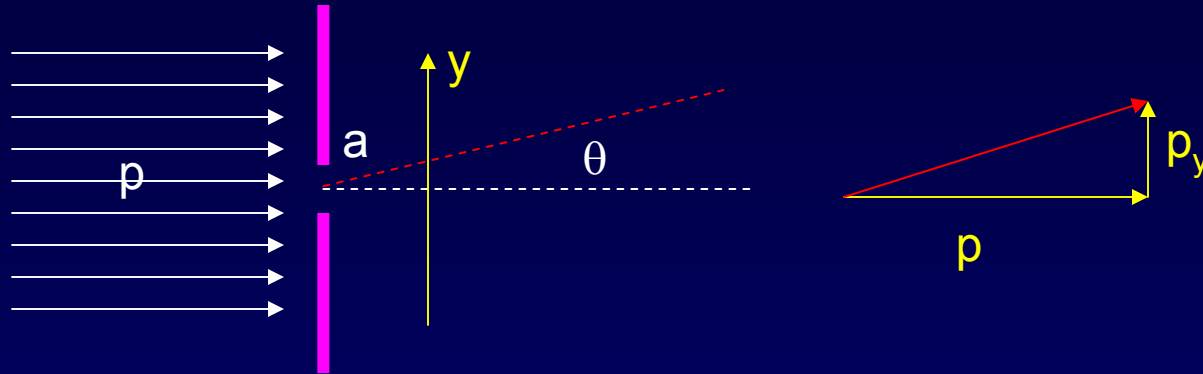# Diffraction and the Uncertainty Principle

Remember single-slit diffraction:

Wavelength: λ
Slit width: a
Diffraction angle: $\theta = \lambda/a$
angle to first zero

Let's analyze this problem using the uncertainty principle.

Suppose a beam of electrons of momentum p approaches a slit of width a. How big is the angular spread of motion after it passes through the slit?

Solution

Consider the momentum uncertainty in the y-direction.

- Before the slit, the y-position is not known, so the uncertainty of p_y can be zero. We know that $p_y = 0$.

- Just after the slit, the y-position has an uncertainty of about $a/2$.
  Therefore p_y must have an uncertainty $\Delta p_y \geq 2\hbar/a$.
  This corresponds to a change of direction by an angle, $\theta = \Delta p_y / p = 2\hbar/ap$.

  Using $p = h/\lambda$, we have $\theta = \lambda/(\pi a)$.

This is almost the diffraction answer: $\theta = \lambda/a$. The extra factor of π is due to our somewhat sloppy treatment of the uncertainty.

The important point is that the uncertainty principle results because matter behaves as a wave.

Example

The position of an electron in the lowest-energy state of a hydrogen atom; is known to an accuracy of about Δx = 0.05 nm (the radius of the atom). What is the minimum range of momentum measurements? Velocity?

Solution

The position of an electron in the lowest-energy state of a hydrogen atom; is known to an accuracy of about Δx = 0.05 nm (the radius of the atom). What is the minimum range of momentum measurements? Velocity?

$\Delta x \Delta p \geq \hbar$ Heisenberg's Uncertainty Principle

$\Delta p \geq \hbar / \Delta x$ $\hbar = h / 2\pi$

$= 2.1 \times 10^{-24} \, \mathrm{J \cdot s/m}$

$\boxed{= 2.1 \times 10^{-24} \, \mathrm{kg \cdot m/s}}$

$\Delta v = \Delta p / m_e$ $m_e = 9.1 \times 10^{-31} \mathrm{kg}$

$\boxed{= 2.3 \times 10^{6} \, \mathrm{m/s}}$

Why "Uncertainty"?

"Uncertainty" refers to our inability to make definite predictions.

Consider this wave packet:
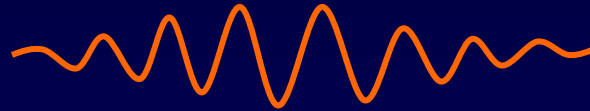
- Where is the object?
- What is its momentum?

The answer is, We don't know. We can't predict the result of either measurement with an accuracy better than the Δx and Δp given to us by the uncertainty principle.

Each time you look, you *find a local blip* that is in a different place (in fact, it is your looking that *causes* the wavefunction to "collapse"!).

If you look many times, you will find a probability distribution that is spread out

But you're uncertain about where that local blip will be in any one of the times you look -- it could be anywhere in the spread.

An important point: You never observe the wave function itself.

The wave merely gives the probabilities of obtaining the various measurement results.
A measurement of position or momentum will always result in a definite result.
You can infer the properties of the wave function by repeating the measurements
(to measure the probabilities), but that's not the same as a direct observation.

Uncertainty Principle –Implications

The uncertainty principle explains why electrons in atoms don't simply fall into the nucleus: If the electron were confined too close to the nucleus (small Δx), it would have a large Δp, and therefore a very large average kinetic energy ($\approx (\Delta p)^2/2m$).

The uncertainty principle does <u>not</u> say "everything is uncertain". Rather, it tells us what the limits of uncertainty are when we make measurements of quantum systems.

Some classical features, such as paths, do not exist precisely, because having a definite path requires both a definite position and momentum. One consequence, then, is that electron orbits do not exist. The atom is not a miniature solar system.

Other features can exist precisely. For example, in some circumstances we can measure an electron's energy as accurately as technology allows.

Serious philosophical issues remain open to vigorous debate, e.g., whether all outcomes or only one outcome actually occur.
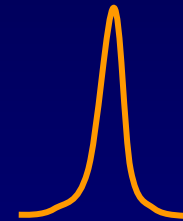
Supplement: Wave Uncertainty

Mathematically, one can produce a localized function by superposing sine waves with a "spread" of wave numbers, Δk: $\Delta k \cdot \Delta x \geq 1$.

This is a result of Fourier analysis, which most of you will learn in Math.

It means that making a short wave packet requires a broad spread in wavelengths.
Conversely, a single-wavelength wave would extend forever.

So far, this is just math. The physics comes in when we make the wavelength-momentum connection: $p = h/\lambda = \hbar k$.

Example:
How many of you have experienced a close lightning strike (within a couple hundred feet)? If you were paying attention, you may have noticed that the sound, which is a very short pulse, is very weird. That weirdness is a result of the very broad range of frequencies that is needed to construct a very short pulse. One doesn't normally experience such a broad frequency range.