Shhh...

The next two questions pertain to the following situation:

1. A laser light beam (the wavelength is 500 nm) hits a screen with three parallel slits with equal spacing of 30 microns between them. Some photons pass through the slits and travel to an imaging screen, which is 2 m away. At which angle is the first secondary maximum observed?

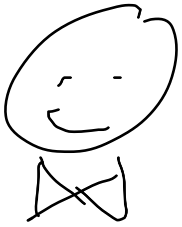
a. 0.004 rad

[✓ b. 0.008 rad]

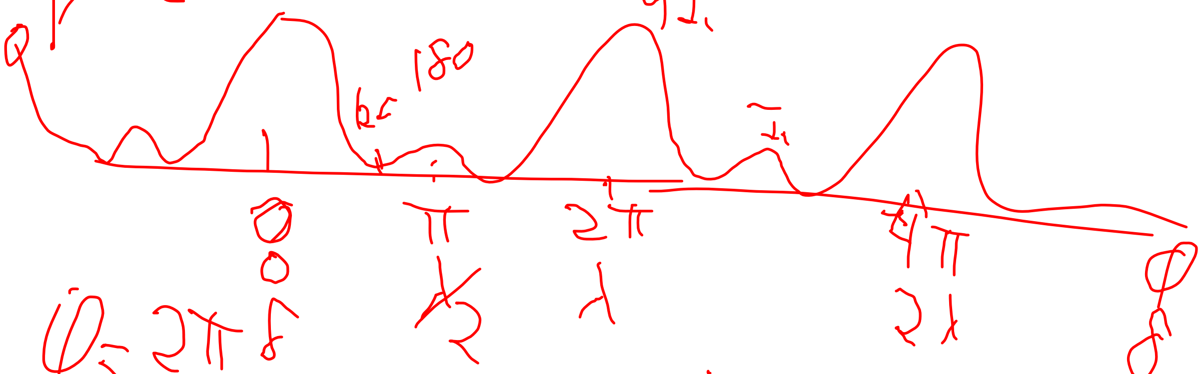
c. 0.012 rad

Hi

Thank you



$y = L\tan\theta$



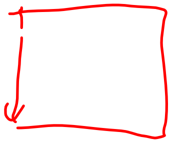
$\phi = \frac{2\pi\delta}{\lambda}$

d

$\delta = d\sin\theta = \frac{\lambda}{2}$

$\sin\theta = \frac{\lambda}{2d} = \frac{0.5\mu m}{2\cdot 30} = 0.008$

$\sin\theta \sim \theta$



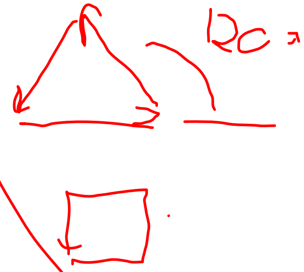
$\left(\frac{\sin N\phi/2}{\sin \phi/2}\right)^2$

2. What is the intensity at this secondary maximum, if the intensity obtained with just one slit is I? Assume the slits are so narrow that the diffraction effects can be neglected. (Hint: draw phasors.)

[✓ a. I]
b. 2I
c. 4I
d. I/2
e. I/3

$I_{principal} = (3A)^2 = 9A^2 = 9I_1$

Total transmitted light ~ $N(A^2)$



$I = I_1$

Add more slits (w same spacing)

$I_{Max} = N^2 I_1$

Prin. Max same angle

$d \sin\theta_n = n\lambda$

More secondary max $(N-2)$

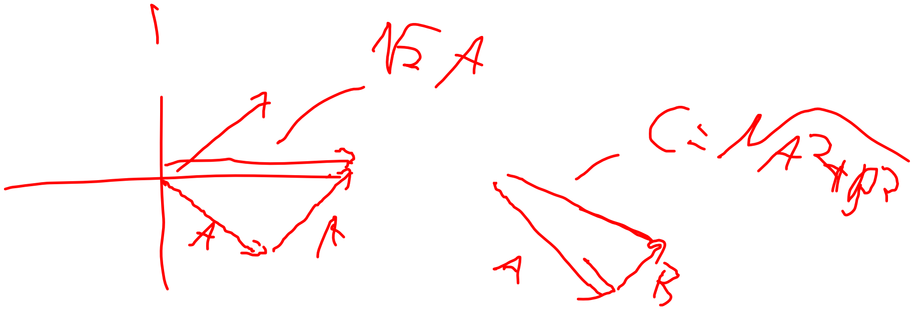
Peaks narrower $\sim \frac{1}{N}$

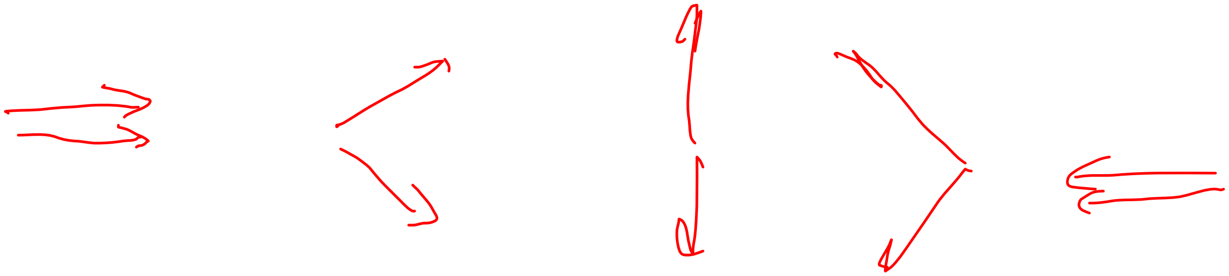
3. Two waves

$$y = A\cos(200t - 10x - \pi/4)$$
$$y = A\cos(200t - 10x + \pi/4)$$

are superposed. What is the resulting wave?

a. $2A\cos(200t-10x+\pi/8)$
b. $\sqrt{2}\,A\cos(200t-10x+\pi/8)$
[✓ c.] $\sqrt{2}\,A\cos(200t-10x)$
d. $1.85\,A\cos(200t-10x+\pi/4)$
e. none of these





The next two questions pertain to the following situation:

4. What is the wavelength of a photon which has energy barely sufficient to extract one electron from a metal with a work function of 5 eV.

$$E_{photon} = \phi = \frac{hc}{\lambda} = \frac{1240\,eV\text{-}nm}{\lambda}$$

[✓ a. 250 nanometers]
b. 415 nanometers
c. 310 nanometers

$$\lambda = \frac{1240}{5} = 248\,nm$$

$$E_{photon} = \phi + KE$$

$$\boxed{\lambda = 500\,nm}$$

5. We now send in 200-nm light (i.e., each photon has more than enough energy to liberate an electron). If we decrease the intensity I of the light, what will happen to the current of the emitted electrons?

$$\#electron/sec \propto \#photons/s$$

a. The current will decrease as √I.
[✓ b. The current will decrease as I.]
c. The current will stay the same, it does not depend on the intensity of the incident light.

$$I \propto power = \frac{energy}{time} = \cancel{\phi} E_{photon} \times \frac{\#photon}{sec}$$

$(2000)^2$

6. A spectrometer is used to analyze the light coming from a distant source. The spectrometer has a 6-mm wide diffraction grating with 2000 lines. The light is normally incident on the grating and illuminates the whole grating. The third-order principle maximum is observed at an angle of 50° from normal incidence. What is the wavelength of the light coming from the source?

$d_m = d\sin\theta_m = m\lambda$

a. 192 nm
b. 354 nm
~~c. 383 nm~~
[✓ d. 766 nm]
e. 873 nm

$d = \frac{6\,mm}{2000} = \frac{6000\,\mu m}{2000} = 3\,\mu m$

$\lambda = \frac{3\,\mu m \sin 50}{3} = 766\,nm$

~~$d\sin\theta \sim d\theta = 3\lambda \Rightarrow \lambda \sim 0.8 \cdot 3 \sim 3\,\mu m$~~

7. Suppose the correct answer to the previous problem were 650 nm. The source is moving away from the observer so its color is Doppler shifted to the red side of the spectrum. What is the smallest wavelength change that could be detected using this spectrometer, assuming that now only half of the number of lines is illuminated? (Hint: what is the highest order that can be used?)

a. Δλ=77 nm
b. Δλ=0.5 nm
[✓ c. Δλ=0.16 nm]

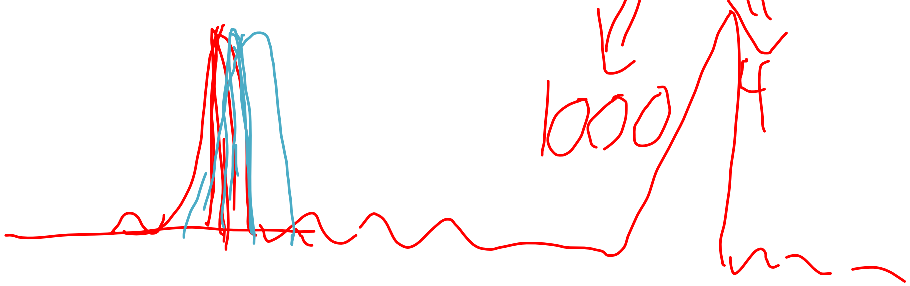
$\frac{\Delta\lambda}{\lambda} > \frac{1}{Nm}$, $N = 1000$, $m = 4$

$\Delta\lambda > \frac{650\,nm}{1000 \cdot 4} = 0.16\,nm$

$d\sin\theta_m = m\lambda$

$\sin\theta_m = \frac{m\lambda}{d} \leq 1$

$m < \frac{d}{\lambda} = \frac{3\,\mu m}{0.65} = \boxed{4}.65$



8. An array of three identical loudspeakers separated by distances d is arranged as pictured. A receiver is placed at position R far from the array. The emitters radiate waves of frequency 1 kHz with the same individual intensity at R. The sound speed in this air is 340 m/s. All the loudspeakers are driven with the same phase.

$\phi_{12} = \frac{2\pi d}{\lambda}$

$\phi_{23} = \frac{2\pi d}{\lambda}$

What is the minimum distance d such that the intensity at R is zero? (Hint: draw phasor diagram.)

a. d = 0.34 m
b. d = 0.68 m
c. d = 0.17 m
[✓ d. d = 0.11 m]
e. none of these

$\lambda f = v$

$\phi = 120 = \frac{360 d}{\lambda}$

$d = \frac{\lambda}{3} = \frac{0.34}{3}$

$\lambda = \frac{340\,m/s}{1000} = 0.34$

$= 0.11\,m$

$d \sin\theta = m\lambda$

Decrease d => smaller angle between max

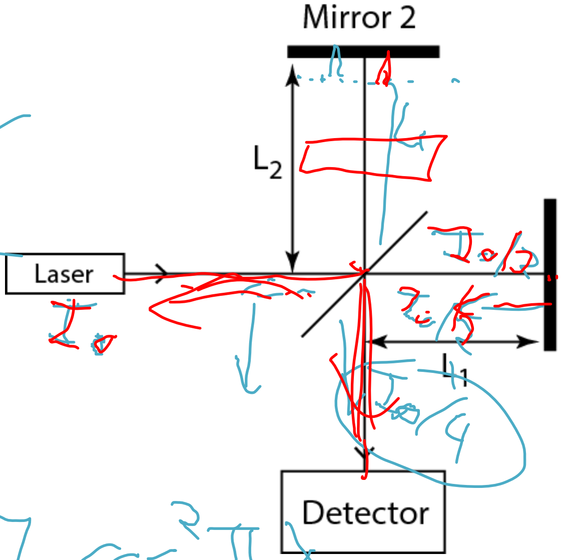
The next three questions pertain to the following situation:

$I = 4I_1 \cos^2 \frac{\phi}{2}$

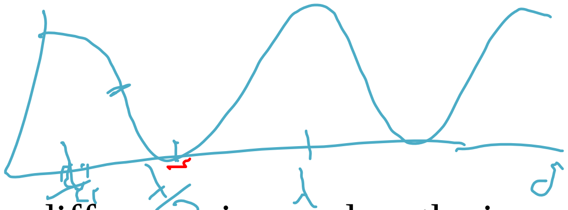
$= 4\frac{I_0}{4} \cos^2 \frac{\phi}{2}$

$= I_0 \cos^2 \frac{\phi}{2}$

$= I_0 \cos^2 \frac{\pi \delta}{\lambda} = I_0 \cos^2 \frac{\pi}{4}$



$\phi = \frac{2\pi\delta}{\lambda}$



A Michelson interferometer operating in vacuum is set up such that the difference in arm lengths is $L_2 - L_1 = \lambda/8$, where λ is the wavelength of the laser. Here L_1 (L_2) is the separation between the beam splitter and mirror 1 (2). The laser intensity is I_0.

$\delta = 2(L_2 - L_1) = \frac{2\lambda}{8} = \frac{\lambda}{4}$

$L_1 = 1000\ nm$

$L_2 = 1001\ nm$

9. What is the intensity at the detector?

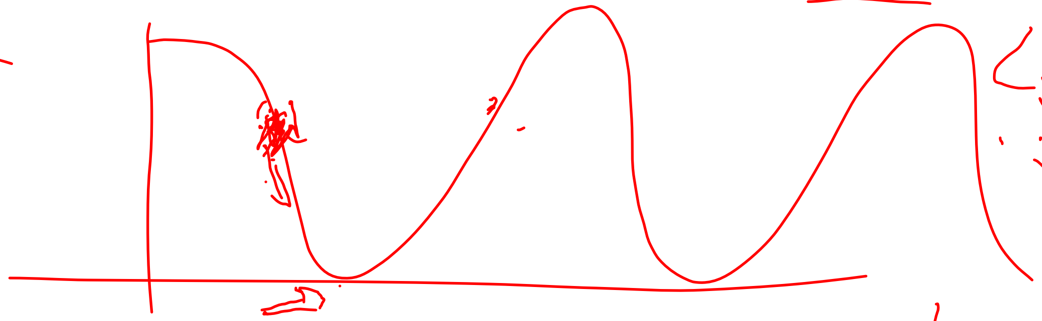
$I_0 \cos^2 \frac{\pi}{4} = \frac{I_0}{2}$

a. $I = I_0$
b. $I = I_0/4$
[✓ c. $I = I_0/2$]

10. Now suppose we insert a thin transparent slab of thickness $\delta x << \lambda$ and ~~index of refraction n =~~ 1.1 into arm 2 of the interferometer. How does the intensity change? Note, $L_2 > L_1$. (Hint: it may help you to sketch $I(L_2)$.)

a. increases
[✓ b. decreases]
c. stays the same

$\phi = \frac{2\pi\, \delta x}{\lambda}$



$L_2 - L_1 = \frac{\lambda}{8}$

$\lambda f = \lambda_n = \frac{c}{n}$

$\lambda \propto \frac{1}{n}$

$L_2 - L_1$

$\delta_2 = 2\pi \frac{2\delta x\, n}{\lambda} - 2\pi \frac{2\delta x}{\lambda} = \delta_1$

$= 2\pi \frac{2\delta x \cdot 1}{\lambda}$

$\delta x\, n - \Delta x = \delta x$

$\Delta x = \delta x (n-1)$

11. If we want to make the intensity at the detector equal to the case in which the slab was absent, how much do we need to move mirror 1 relative to its initial position?

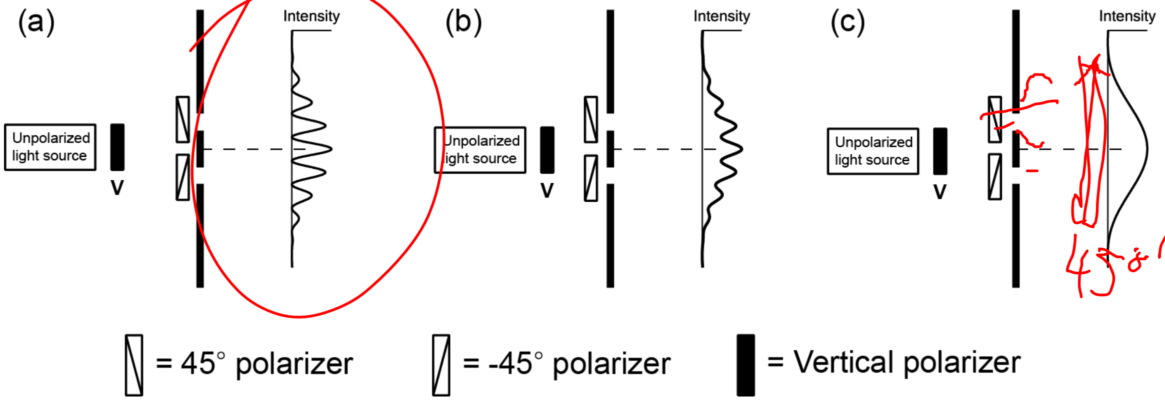
Initially: $L_2 - L_1 = \frac{\lambda}{8}$

with slab $L_2 - L_1 > \frac{\lambda}{8}$

~~a. 2δx (to the right)~~
b. nδx (to the right)
~~c. δx (to the left)~~
[✓ d. (n-1) δx (to the right)]
~~e. (n-1) δx (to the left)~~

The next two questions pertain to the following situation:

Two slits are illuminated by vertically polarized photons. A 45° polarizer is placed in front of the upper slit and a -45° polarizer is placed in front of the lower slit.



Interference requires indistinguishable processes

12. Which of the figures shown above best illustrates the intensity pattern built up on the screen? Note the intensity scale for the various choices need not be the same.

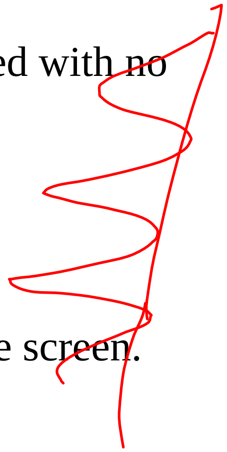
a.
b.
[✓ c.]

Disting: $\text{Prob} = P_1 + P_2$

Indist: $\text{Prob} = |A_1 + A_2|^2$

13. Which of the following would recover the usual 2-slit intensity pattern observed with no polarizers in front of the slits?

a. There already <u>is</u> such a pattern, i.e., nothing new needs to be added.
b. Add a vertical polarizer in front of the viewing screen.
c. Rapidly and randomly switch between a +45° and a -45° polarizer in front of the screen.
d. b or c would both work.
e. none of the above.



Only top slit photon

Only bottom slit

The next two questions pertain to the following situation:

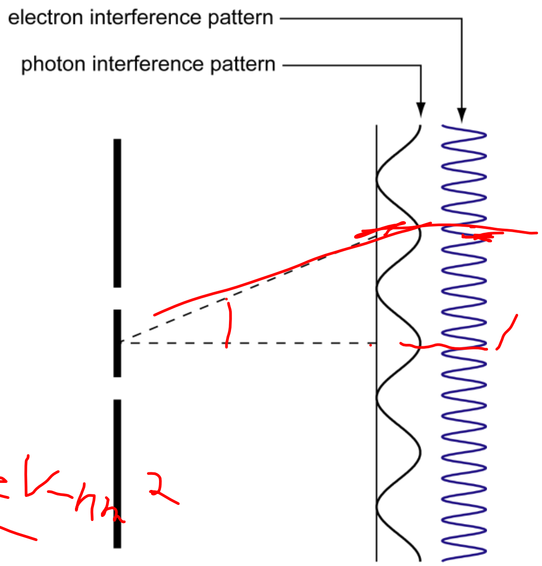
Two slits are illuminated with 5-eV photons as well as monochromatic electrons accelerated to 0.247 eV. The interference pattern produced by the photons and electrons are viewed simultaneously on a screen located a distance L from the slits. Note that the patterns are not drawn to the same scale as each other.

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV-nm}}{\lambda}$$

$$\lambda = \frac{1240}{5} = 248\,\text{nm}$$

$$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{1.505\,\text{eV-nm}^2}{\lambda^2}$$

$$\lambda = \sqrt{\frac{1.505}{0.247}} = 2.47\,\text{nm}$$



$$d\sin\theta_{phot} = \lambda_{photon}$$

$$\sin\theta = \boxed{\frac{\lambda_{photon}}{d}}$$

$$d\sin\theta_{m,electron} = m\lambda_{elec}$$

$$m = \frac{d\sin\theta_{m,elect}}{\lambda_{electron}} \leq \frac{d\sin\theta_{phot}}{\lambda_{elec}} < \frac{d\,\lambda_{phot}/d}{\lambda_{elec}}$$

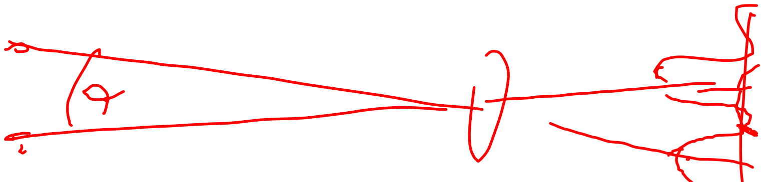
14. Calculate the number of interference maxima produced by the electrons that lie between the m = 0 and m = 1 maxima of the interference pattern produced by the photons.

$$m < \frac{\lambda_{phot}}{\lambda_{elect}} = \frac{248}{2.47} = 100.5$$

a. 1
b. 4
c. 20
[✓ d. 100]
e. 420

15. You would like to image a binary star system using an optical telescope. The two stars have an angular separation of 4.23×10^{-7} rad. The telescope uses a lens with a diameter of 1.7 m and a filter to select a particular range of wavelengths. What is the longest wavelength light for which the two stars can be clearly resolved?

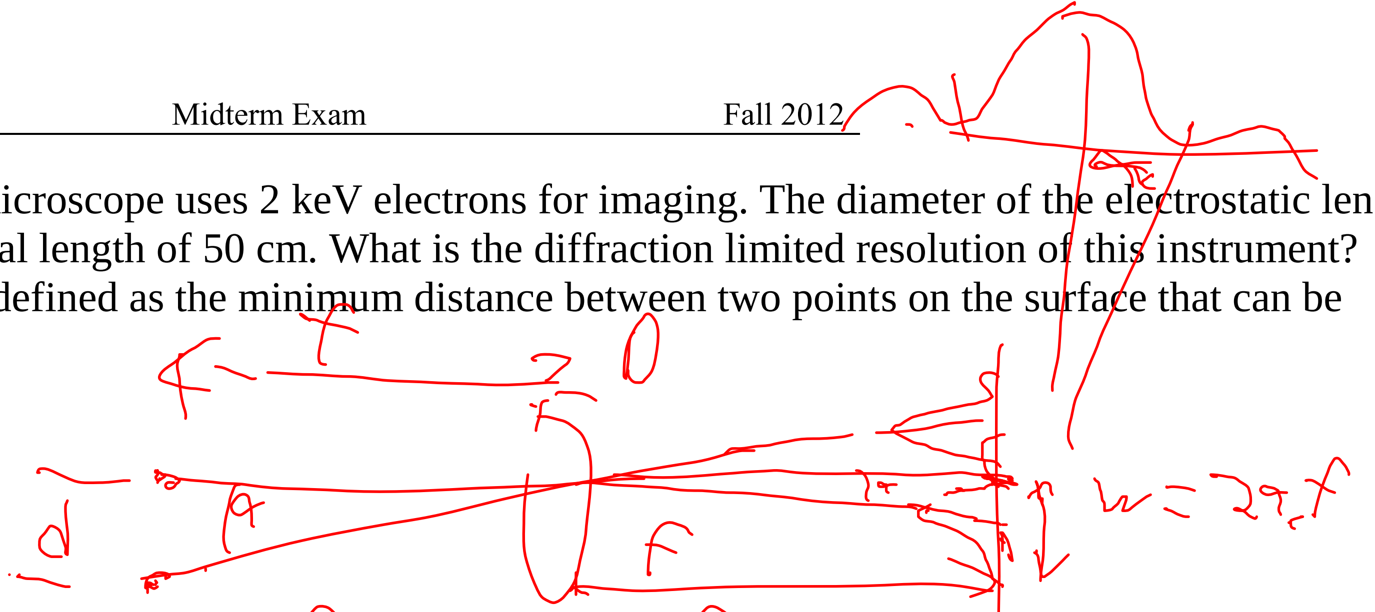
a. There is no "longest wavelength"—the ~~larger~~ Smaller the wavelength, the better the resolution.
b. 515 nm
c. 550 nm
[✓ d. 585 nm]
e. 620 nm



$$\alpha > \alpha_c = \frac{1.22\lambda}{D}$$

$$\lambda < \frac{D\alpha}{1.22} = \frac{1.7\,(4.23\times10^{-7}\,\text{rad})}{1.22} = 589\text{ nm}$$

16. An electron microscope uses 2 keV electrons for imaging. The diameter of the electrostatic lens is 5 cm with a focal length of 50 cm. What is the diffraction limited resolution of this instrument? The resolution is defined as the minimum distance between two points on the surface that can be clearly resolved.



a. 0.027 nm
b. 0.33 nm [✓]
c. 1.45 nm
d. 3.22 nm
e. 4.76 nm

$$d = \alpha f > \alpha_c f = 1.22 \frac{\lambda f}{D} = \frac{50}{5}(1.22)(0.027\text{ nm})$$

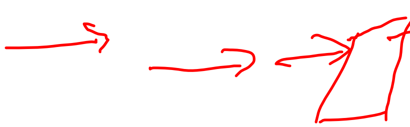
$$E = \frac{h^2}{2m\lambda^2} = \frac{1.505}{\lambda^2}$$

$$\lambda = \sqrt{\frac{1.505}{2000}} = 0.027\text{ nm}$$

$$= 0.33\text{ nm}$$

17. A solar sail is a device that uses the light from the sun to accelerate a spacecraft. This type of propulsion system relies on the momentum exchange between photons and a large sail attached to the craft. Assuming the sail is 10×10 m^2 in area and the power from the sun is 500 W/m^2, estimate the change in momentum of the spacecraft after 30 days. In your estimate, assume that the light from the sun is always normally incident on the sail, and that the photons are completely absorbed by the sail (in practice, a reflective sail would give an extra factor of 2 enhancement). Hint: Solve symbolically first.

Intensity



$A = 100\,m^2$

a. Δp = 236 kg m/s
[✓ b. Δp = 432 kg m/s]
c. Δp = 578 kg m/s
d. Δp = 699 kg m/s
e. Δp = 1023 kg m/s

$p = \frac{h}{\lambda}$

$\Delta p = p_{photon}$ # photons absorbed

$\frac{\#\,photons\ absorbed}{s} \times \Delta t$

$\Delta p = \frac{h}{\lambda} \cdot 500\frac{W}{m^2} \cdot 100\,m^2 \cdot \frac{\lambda}{hc}\,\Delta t$

$I = \frac{P}{Area}$

$P = \frac{energy}{time} = E_{phot} \left(\frac{\#\,phot}{s}\right)$

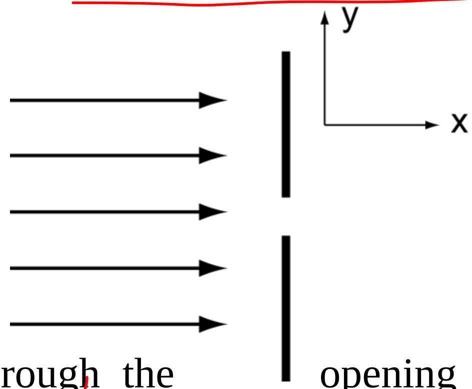
$\frac{\#\,phot}{s} = \frac{P}{E_{phot}} = \frac{P\lambda}{hc}$

$\frac{\#\,photon}{s \cdot m^2} \cdot Area$

$= (I \cdot Area)\frac{\lambda}{hc}$

$\frac{500 \cdot 100}{3 \times 10^8}\ 1\,mon \times \frac{30\,day}{1\,mon} \times \frac{24\,hr}{1\,day}\ \frac{3600\,s}{1\,hr}$

18. Electrons are accelerated to 150 eV and made incident on an aperture with a 10-nm diameter. Estimate the minimum range of transverse (y-component) velocities that the electrons possess after passing through the aperture.



a. 1×10^6 m/s

b. 1×10^5 m/s

[✓ c. 1×10^4 m/s]

d. 1×10^3 m/s

e. 0 (any electrons that actually pass through the opening do not acquire any transverse momentum.)

$\Delta y \Delta p_y > \hbar$

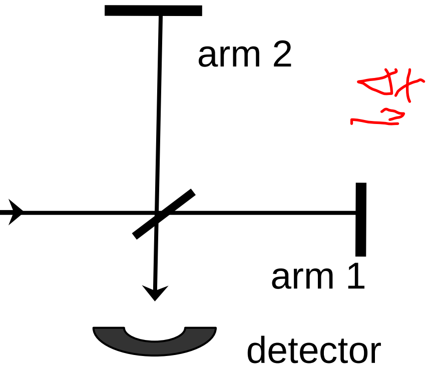
$\Delta p_y = m \Delta v_y$

$\Delta v_y = \frac{\hbar}{m \Delta y}$

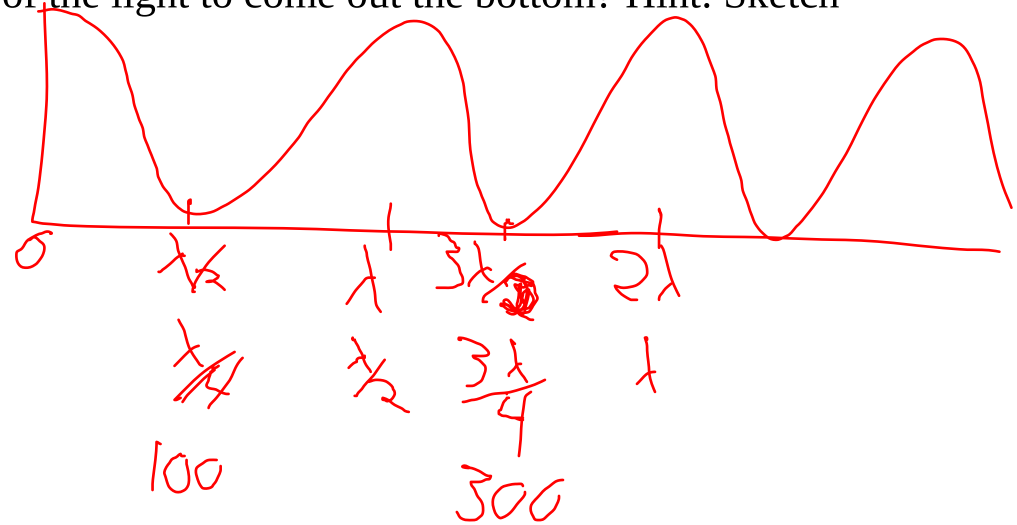
$= \frac{6.6\times10^{-34}/2\pi}{10\,\mathrm{nm}\;9\times10^{-31}\,\mathrm{kg}} = 2000\,\mathrm{m/s}$

The next two questions are related.

A blue laser (400 nm) is directed into a Michelson interferometer, initially with equal path lengths (each arm is 1-m long), so that all the light exits via the bottom port.



19. Which of the following displacements δx of the right mirror will cause *none* of the light to come out the bottom? Hint: Sketch I(δx).



$\delta = \delta x$

path length difference

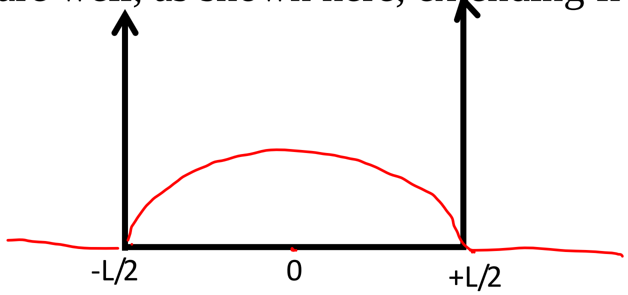
[✓ a. 200 nm]
b. 300 nm
c. 400 nm
d. 150 nm
e. 800 nm

20. Consider that we start in the condition that no light is coming out the bottom. As we now decrease the reflectivity of the right mirror, what will happen to the intensity on the detector?

a. The intensity will decrease as the reflectivity decreases.
[✓ b.] The intensity will increase as the reflectivity decreases.
c. The intensity will remain unchanged as the reflectivity decreases.

The next six questions are related.

Consider an electron in an infinite square well, as shown here, extending from x = -L/2 to x = +L/2 (i.e., of width L).



21. Which of the following describes the ground state wavefunction of the electron inside the well? Hint: *draw the wavefunction*.

~~a. $\sqrt{\frac{2}{L}}\sin(\frac{p}{L}x)$~~

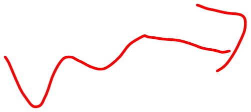
b. $\sqrt{\frac{2}{L}}\sin(\frac{2p}{L}x)$

[✓ c. $\sqrt{\frac{2}{L}}\cos(\frac{p}{L}x)$]

$E_1 = \frac{h^2}{8mL^2}$ $E_n = n^2 E_1$

22. What wavelength of photon could excite the electron from the ground state (n = 1) to the *second* excited state, assuming the *total width* of the well is 3 nm?

a. 30,000 nm
[✓ b. 3700 nm]
c. 3300 nm
d. Either b or c would work.
e. None of the above



$E_{photon} + E_1 = E_3$

$E_{photon} = E_3 - E_1$

$= 9E_1 - E_1 = 8E_1$

$= 8\left(\frac{h^2}{2m}\right)\frac{1}{4L^2} = \frac{2(1.505\ \text{eV-nm}^2)}{3^2}$

$E = \frac{hc}{\lambda} = \frac{1240}{\lambda}$ $= 0.33\ \text{eV}$

$\lambda = \frac{1240}{0.33} = 3708\ \text{nm}$

23. (How) would the answer to the previous question change if the electron were replaced by a muon (same charge -|e|, but $m_\mu = 209\ m_e$)?
The wavelength of the exciting photon would.

[✓ a. increase.]
b. decrease.
c. stay the same.

m ↑
E ↓
λ ↑

$E_1 = \frac{h^2}{8mL^2}$

What λ dB, muon ?
6nm

24. Let P("center") be the probability that the electron is found between $x = -L/10$ and $x = +L/10$. Compare P("center") if the particle is in the ground, excited, or second excited states. Hint: sketch the solutions carefully.

symmetric even

a. $P(\text{“center”})_{\text{ground}} > P(\text{“center”})_{\text{excited}} > P(\text{“center”})_{\text{2nd excited}}$

[✓ b.] $P(\text{“center”})_{\text{ground}} > P(\text{“center”})_{\text{2nd excited}} > P(\text{“center”})_{\text{excited}}$

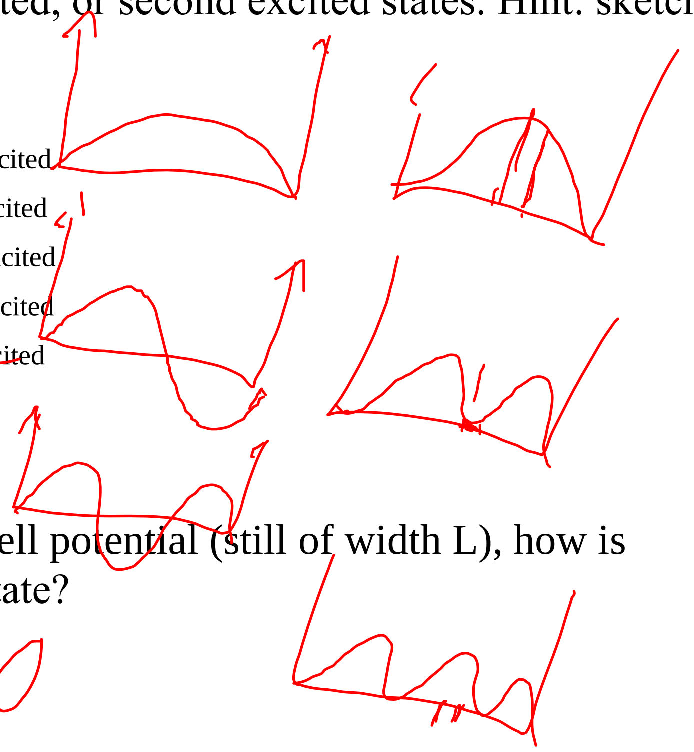
c. $P(\text{“center”})_{\text{ground}} = P(\text{“center”})_{\text{2nd excited}} > P(\text{“center”})_{\text{excited}}$

d. $P(\text{“center”})_{\text{ground}} < P(\text{“center”})_{\text{excited}} < P(\text{“center”})_{\text{2nd excited}}$

e. $P(\text{“center”})_{\text{ground}} < P(\text{“center”})_{\text{2nd excited}} < P(\text{“center”})_{\text{excited}}$

antisymmetric odd

even



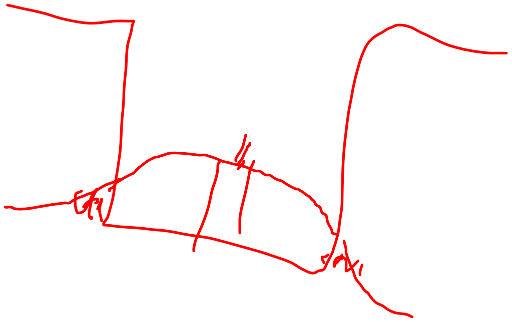
25. If we lower the walls so this is now a finite square-well potential (still of width L), how is P("center") affected if the particle is still in the ground state?

Pcenter, 1st excited ~ 0

a. P("center") increases

[✓ b.] P("center") decreases

c. P("center") ~~increases~~ stays the same.



26. Let the electron initially be in the ground state of the well. If we perform a measurement and localize the wavefunction of the electron in the region x = -L/10 and x = + L/10, what might we say about the energy of the electron after the measurement?

[✓ a. It likely has a higher energy than it did before the measurement.]
b. It likely has a lower energy than it did before the measurement.
c. It definitely has the same energy as it did before the measurement.

$E \approx \frac{1}{L^2}$

HUP

$\Delta x \Delta p \geq \hbar$