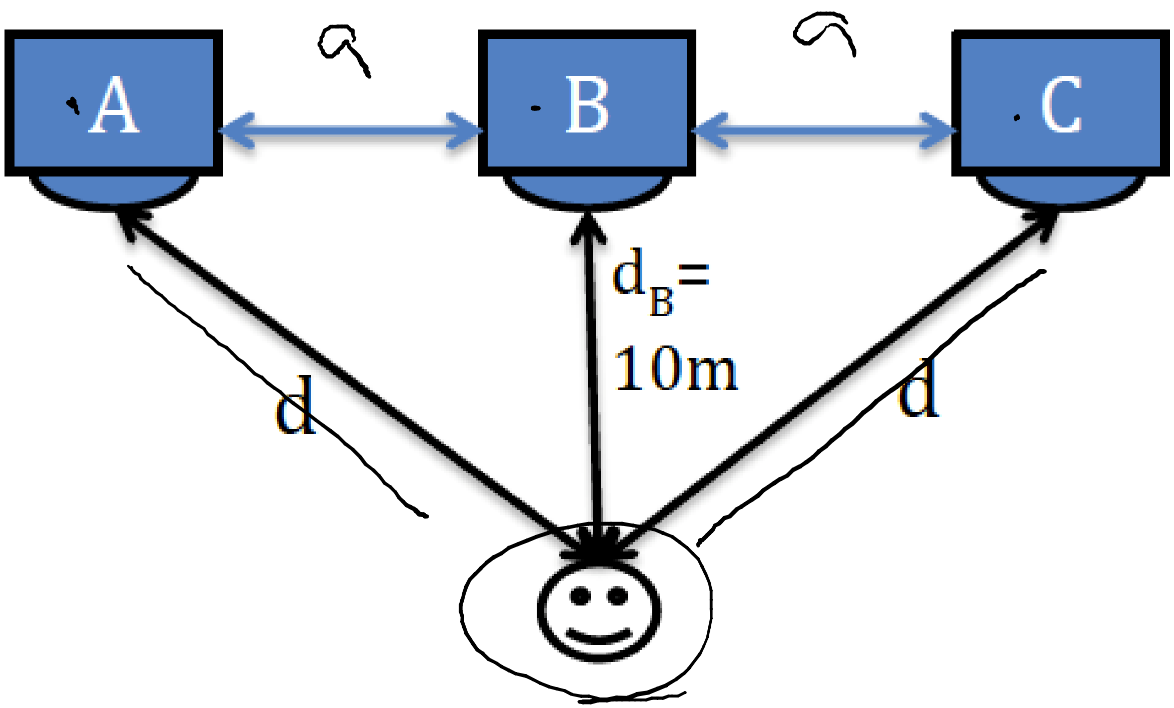
Consider three speakers, A, B, and C, equally spaced along a line, as shown below. A listener is positioned directly across from speaker B, at a position $d_B = 10$m, and hears intensities $I_A = I_0$, $I_B = I_0/4$, and $I_C = I_0$ from A, B, and C, respectively, when one speaker at a time is turned on. The speakers are all driven in phase at a frequency of 800 Hz, and the speed of sound is 330 m/s.



1. What is the minimum distance, *d*, the listener can be from speakers A and C so that she hears all three waves arrive in phase?

a. $d = 10.4$ m
b. $d = 14.1$ m
c. $d = 20.0$ m

$d_B = 10\,m$

$d_A = d_C \qquad V = \lambda f$

$\delta_{AB} = d - d_B = m\lambda \qquad f = 800\,s^{-1}$

$\cancel{\delta_{BC} = d - d_B = m\lambda} \qquad V = 330\,\frac{m}{s}$

$\lambda = \frac{V}{f} = 0.4125\,m$

$d = m\lambda + d_B \qquad m = 1$

$d = \lambda + d_B = 10.4125\,m$

2. What total intensity does the listener hear, assuming the waves arrive in phase?

a. $I_o/4$
b. $9I_o/4$
c. $25I_o/4$

$$\xrightarrow[A]{\sqrt{I_0}} \xrightarrow[C]{\sqrt{I_0}} \xrightarrow[B]{\sqrt{\frac{I_0}{4}}}$$

$$I_T = |A_A + A_B + A_C|^2$$

$$= \left|\sqrt{I_0} + \sqrt{\frac{I_0}{4}} + \sqrt{I_0}\right|^2 = \left(\sqrt{I_0}\right)^2 \left|1 + \frac{1}{2} + 1\right|^2$$

$$= I_0 \left(\frac{5}{2}\right)^2 = \frac{25 I_0}{4}$$

3. The listener now adds a phase shift to speaker B so that she hears a total intensity of $I_{\text{Tot}} = 5I_0$ from the three speakers. What phase shift ϕ_B did she add?

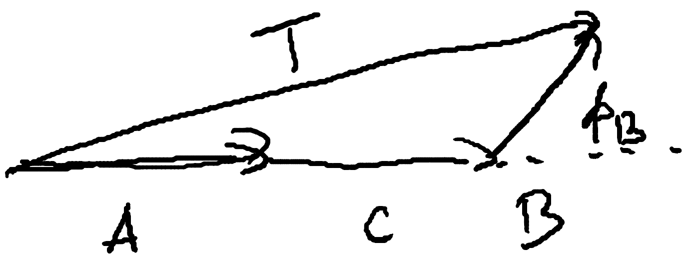
a. It is not possible to add a phase shift that gives this total intensity.

b. 14°

c. 37°

d. 68°

e. 82°



$B = \sqrt{\frac{I_0}{4}}$

$\boxed{\frac{25}{4} I_0 = 6.25 I_0}$

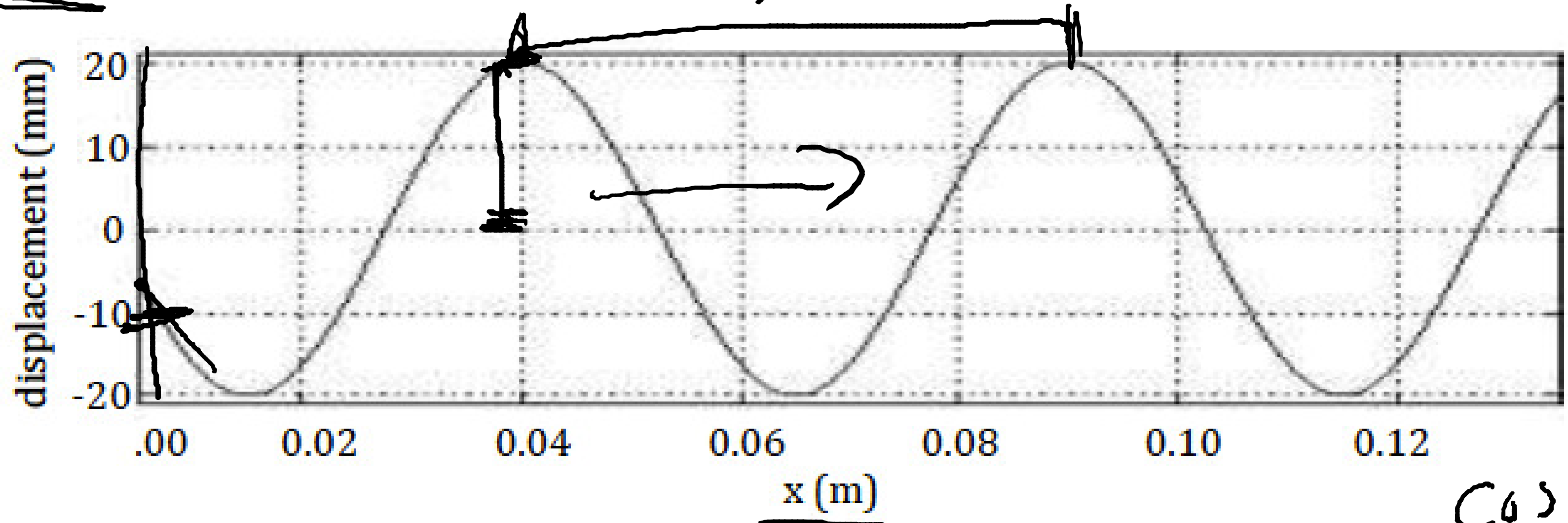
$$T^2 = (A+C)^2 + B^2 + 2(A+C)B\cos\phi_B$$

$$5I_0 = T^2 = (2\sqrt{I_0})^2 + \frac{I_0}{4} + 2(2\sqrt{I_0})\frac{\sqrt{I_0}}{2}\cos\phi_B$$

$$5\cancel{I_0} = 4\cancel{I_0} + \frac{\cancel{I_0}}{4} + 2\cancel{I_0}\cos\phi_B$$

$\cos\phi_B = 3/8$ $\qquad$ $\phi_B = 68°$

The graph below shows the transverse displacement of a string segment as a function of position. Assume the wave is harmonic and has a propagation velocity v = 20 m/s in the $+x$-direction.



$\cos\theta = \frac{1}{2}$

$\theta = \pi/3$

4. What is the approximate period of this wave?

$V = \lambda f$

a. ~~1.25 ms~~
b. 2.50 ms
c. 50.0 ms

$\lambda = |0.04\,m - 0.09\,m|$ $\quad T = \frac{1}{f}$

$= 0.05\,m$

$0.045\,m$

$f = \frac{V}{\lambda} = 400\,Hz$

$T = \frac{1}{400\,Hz} = 2.5 \times 10^{-3}\,s$

5. Which of the following equations could describe this wave?

a. ~~$y(x) = 20\cos[(.05m^{-1})x + (400s^{-1})t + 2\pi/5]\ mm$~~

b. ~~$y(x) = 10\cos[(.05m^{-1})x + (400s^{-1})t + 2\pi/5]\ mm$~~

c. ~~$y(x) = 10\cos[(40\pi m^{-1})x + (800\pi s^{-1})t + 18\pi/5]\ mm$~~

d. $y(x) = 20\cos[(40\pi m^{-1})x + (800\pi s^{-1})t + 18\pi/5]\ mm$

e. $y(x) = 20\cos[(40\pi m^{-1})x - (800\pi s^{-1})t - 8\pi/5]\ mm$

$\cos(kx \pm \omega t)$

$\cos(kx - a)$

$y(x) = A\cos(kx - \omega t)$

$k = \frac{2\pi}{\lambda}$ $\qquad \omega = \underline{2\pi f}$ $\qquad \omega = 800\pi\ s^{-1}$

$f(x)$

$\lambda = 0.05m = \frac{1}{20}\ m$

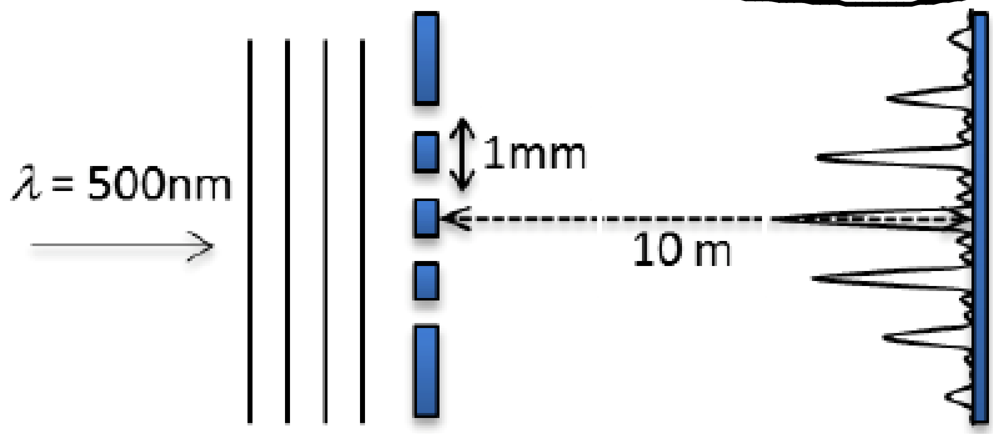
$f(x-a)$

$\frac{2\pi}{\lambda} = 2\pi(20)\ m^{-1}$

$= 40\pi\ m^{-1}$

$f(x - \omega t)$

Consider light of wavelength 500 nm incident on 4 slits, as shown below. The slits are all spaced at 1 mm. ~~An interference pattern~~ appears on a screen placed 10 m away from the slits. (For now, ignore the effect of the finite width of the slits).



Note: Figure not to scale.

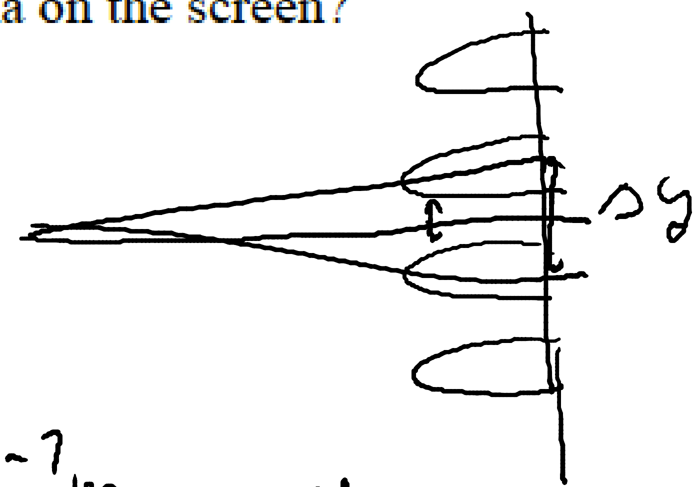
$\lambda = 500 nm = 5 \times 10^{-7} m$

$N = 4 \qquad d = 1 \times 10^{-3} mm$

$L = 10 m$

6. What is the spacing of the principal maxima on the screen?

a. 1.0 mm
b. 5.0 mm
c. 10.0 mm



$$d \sin\theta = \delta = m\lambda$$

$$\sin\theta_m = \frac{m\lambda}{d} = m\,\frac{5\times10^{-7}\,m}{1\times10^{-3}\,m} = m\,(5\times10^{-4})$$

$$\theta_m = (5\times10^{-4})\,m \qquad m << 1000$$

$m=1$

$m=2$

$$\theta_2 - \theta_1 = 5\times10^{-4}$$

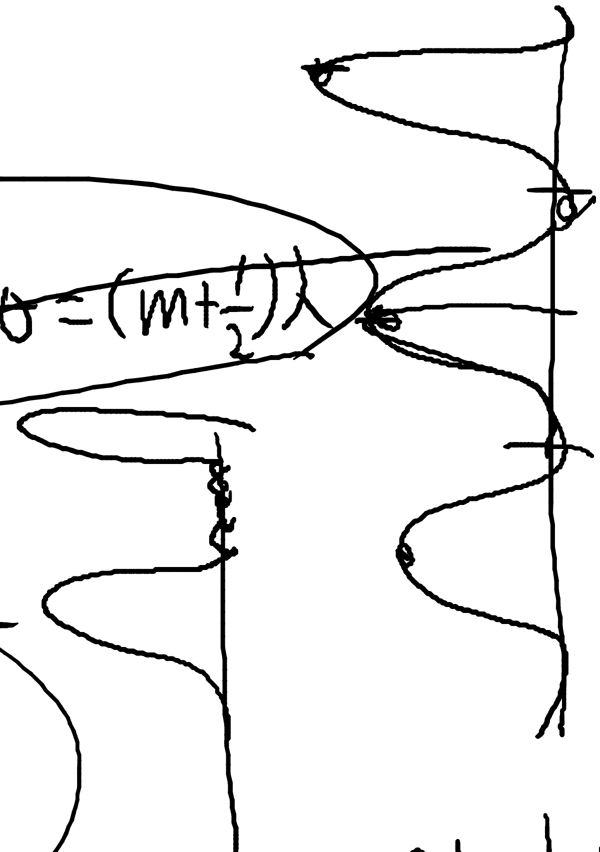
$$y_2 - y_1 = L\theta_2 - L\theta_1 = 10\,m\,(5\times10^{-4})$$
$$= 5\times10^{-3}\,m$$

7. At what angle Θ_{min} from the central principal maximum does the intensity first go to zero?

a. $\Theta_{min} = 0.0072°$
b. $\Theta_{min} = 0.029°$
c. $\Theta_{min} = 0.035°$
d. $\Theta_{min} = 0.057°$
e. $\Theta_{min} = 180°$

$N > 2$

$\delta = d\sin\theta = (m+\frac{1}{2})\lambda$



$N_{slit} > 2$

$$I_N = I_1 \left(\frac{\sin(N\phi/2)}{\sin(\phi/2)}\right)^2$$

$N-1$ minima

$N = 4$ $\quad \frac{N\phi}{2} = \pi \Rightarrow \phi = \frac{2\pi}{N} = \frac{\pi}{2} \Rightarrow \boxed{\phi} = \frac{2\pi\delta}{\lambda}$

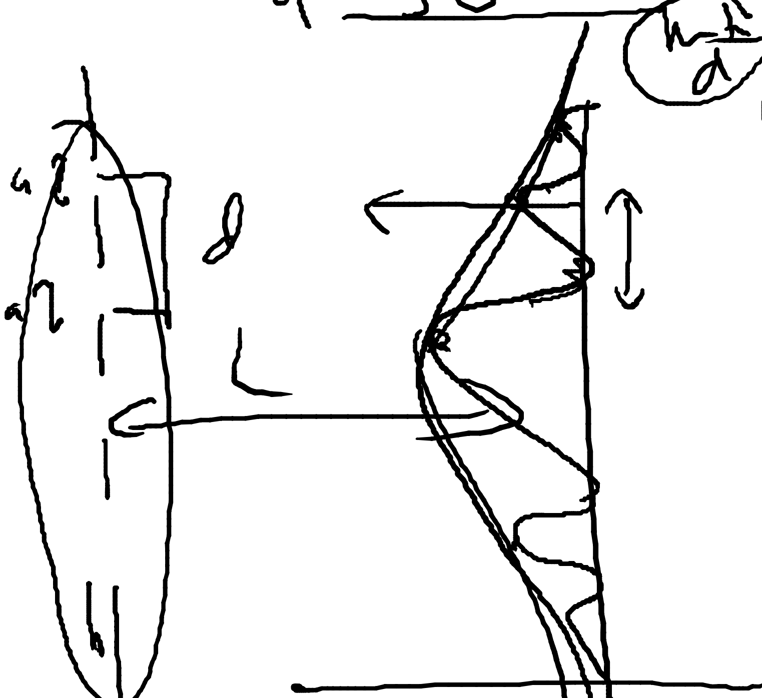
8. Now consider the additional effect of the finite width of the slits on the interference pattern, which is ~~shown in the~~ Figure above. What happens to the ratio of the peak intensity of the 2nd order principal max to that of the central principal max as the slit widths are decreased?

a. $I_{2nd}/I_{central} \rightarrow 1$
b. $I_{2nd}/I_{central} \rightarrow 0$
c. $I_{2nd}/I_{central} \rightarrow \infty$

$m = 0$
$m = 1$
$m = 2$
$m\lambda = d\sin\theta$
$\frac{m\lambda}{d} = \sin\theta$

heights of all maxima for all m are the same

$a \rightarrow 0$



$I \propto N^2, \; W \sim \frac{1}{N}$

$\delta = \frac{\pi}{2} = \frac{2\pi S}{\lambda} = \frac{2\pi d\sin\theta}{\lambda}$

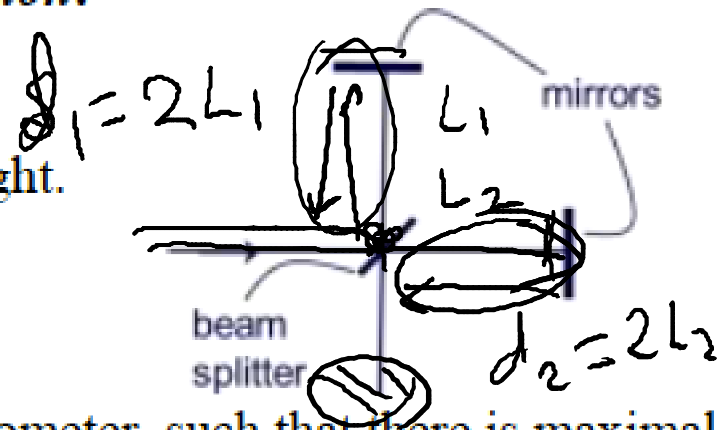
$\sin\theta_{min} = \frac{\lambda}{4d} \rightarrow \frac{1}{N}\frac{\lambda}{d}$

$\theta_{min} = 0.0072^\circ$

$\Delta y = L\tan\theta$

The next two problems refer to the following situation:

Consider a Michelson interferometer as shown at right.



9. After painstakingly tuning your Michelson interferometer, such that there is maximal constructive interference, your lab partner bumps one of the outer mirrors, reducing the path length of that arm of the interferometer. You notice that your photometer (a tool for measuring the intensity of light) reading has not changed. What is the minimum distance that the mirror could have been bumped?

a. $\lambda/2$
b. λ
c. $\lambda/4$

Mirror moves by a

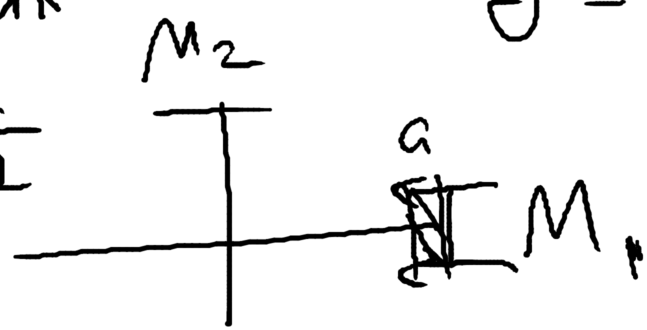
$\delta = 2a$

$2a = \cancel{m}\lambda$

$a = \frac{\lambda}{2}$

$d_1 = d_2 + m\lambda$

$\delta = d_1 - d_2 = m\lambda$



$\delta = d_1 - d_2$

$$\boxed{\delta = |d_1 - 2a| - d_2 = n\lambda}$$

10. Assume that the laser source operates at $\lambda = 500$ nm and 2 mW, so that there is 1mW incident on each mirror. If a mirror reflects all of the light (i.e., none absorbed or transmitted), what force is exerted on the top mirror by the light? Note: the momentum transfer for 100% reflection is double that for complete absorption.

a. 2×10^{-3} kg-m/s^2
b. ~~1×10^{-3} kg-m/s^2~~
c. 6.7×10^{-12} kg-m/s^2
d. ~~4×10^{-19} kg-m/s^2~~
e. 1.3×10^{-27} kg-m/s^2

$E = \frac{hc}{\lambda}$

$p = h/\lambda$

Photon Force

$$\boxed{\text{Force} = ma = \frac{dp}{dt} \qquad \frac{d(mv)}{dt} = ma}$$

$E_{photon} = (\text{Momentum})_{photon}\, c$

$\text{Power} = \frac{\text{Energy}}{\text{sec}} = c\,\frac{\text{Momentum}}{\text{sec}}$

$E_{photon} = \frac{hc}{\lambda} = \frac{1240\,\text{eV nm}}{500\,\text{nm}} = 2.4\,\text{eV}$

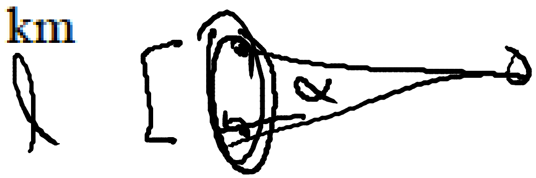
$\frac{\Delta E}{\Delta t} = 1\,\text{mW} \Rightarrow \frac{\Delta p}{\Delta t} = \left(\frac{1\,\text{mW}}{c}\right) \Rightarrow \text{Force} = 2\left(\frac{1\,\text{mW}}{3 \times 10^{8}\,\text{m/s}}\right)$

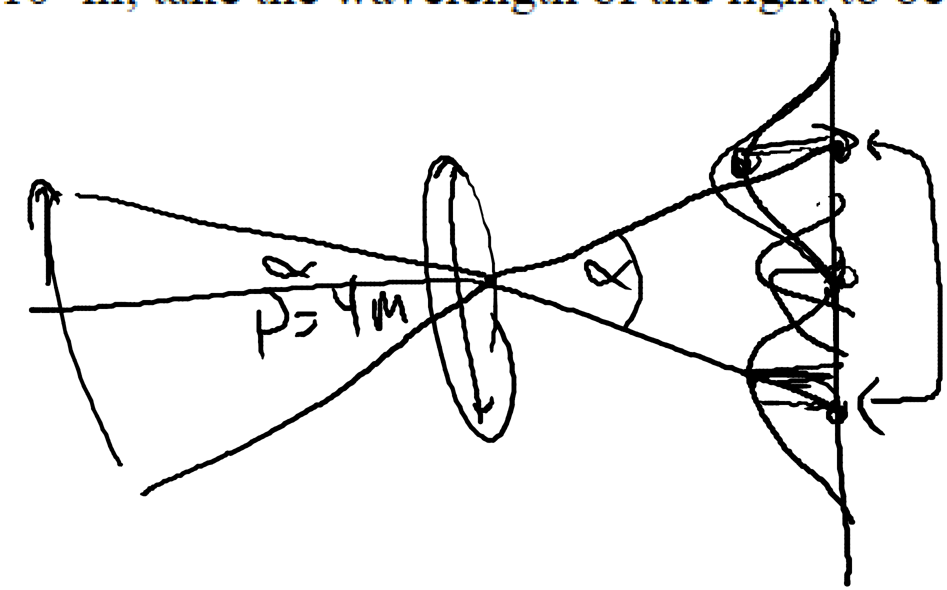
$= 6.6 \times 10^{-12}$ N

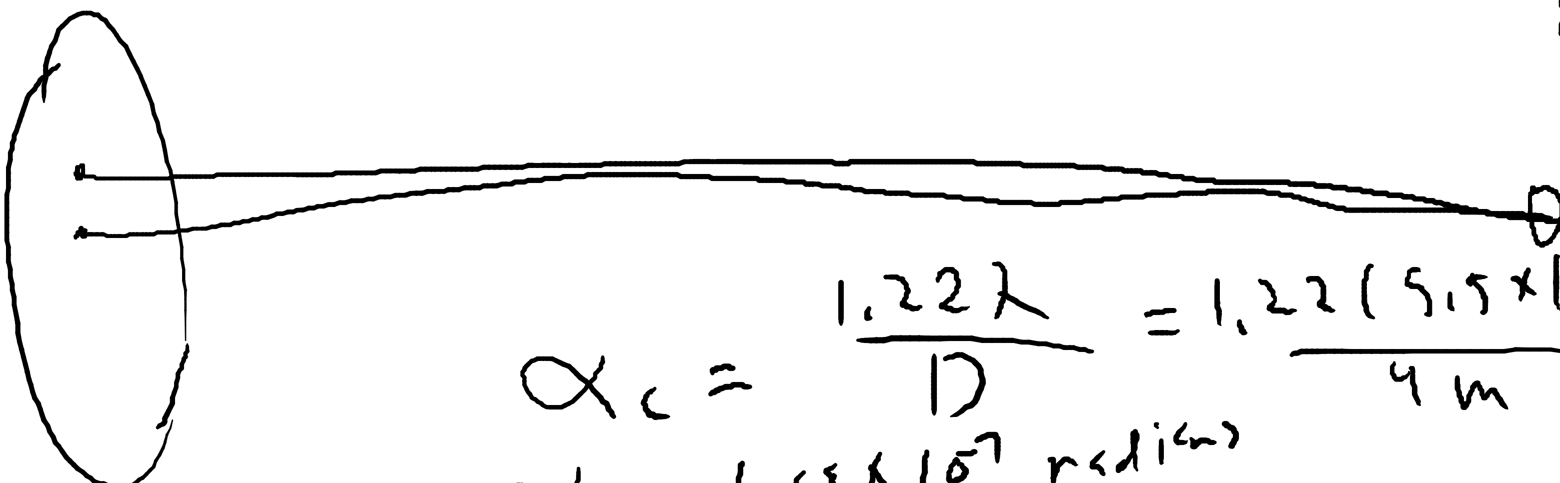
11. Consider a telescope which has a lens with a 400-cm diameter. How far apart must two objects be on the surface of the Moon if they are to be resolvable by this telescope? The Earth-Moon distance is 3.8 x 10^8 m; take the wavelength of the light to be 550 nm.

a. 32 cm
b. 3.2 m
c. 64 m
d. 1 km
e. 10 km

$l = d_{EM} \tan\alpha$







$$\alpha_c = \frac{1.22\lambda}{D} = \frac{1.22(5.5\times10^{-7}\,m)}{4\,m}$$

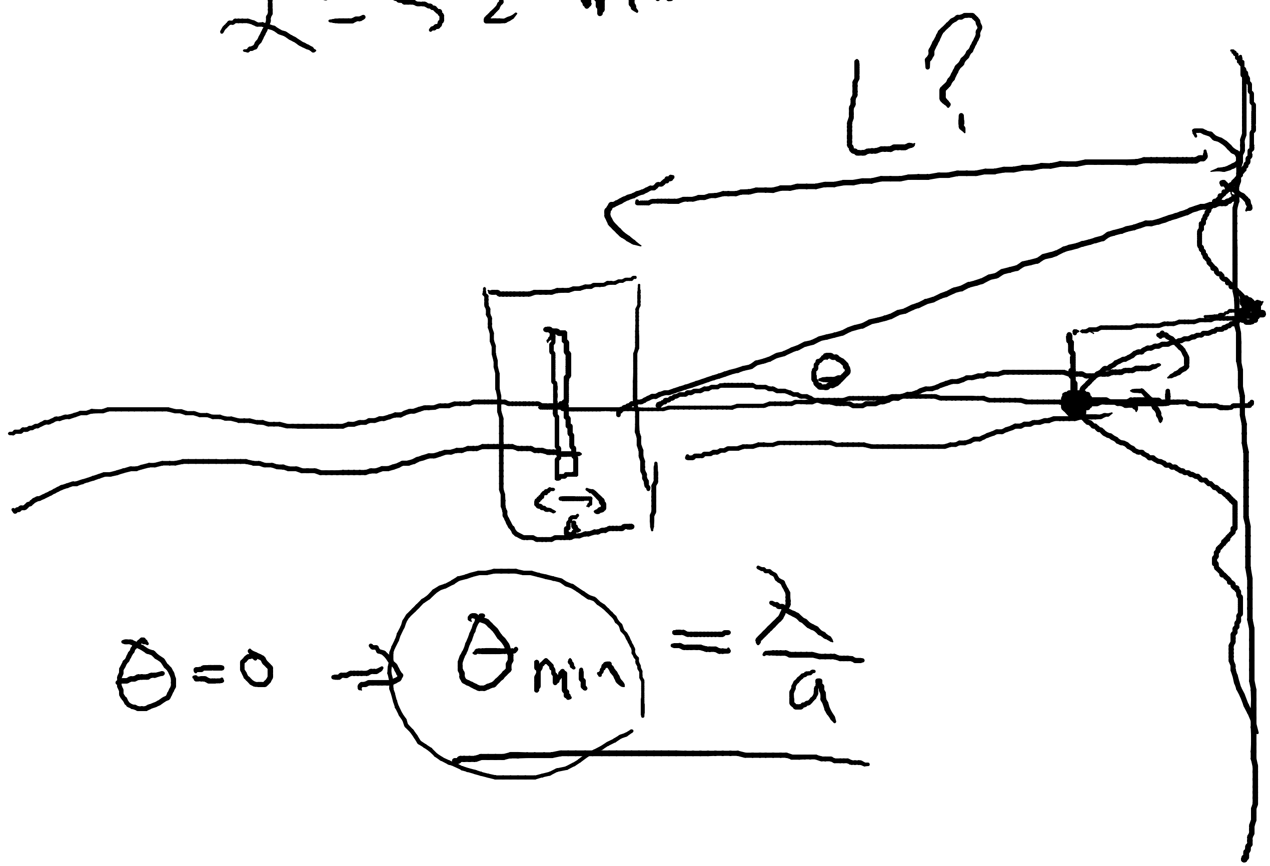
$$\alpha_c = \underline{1.68\times10^{-7}}\ \text{radians}$$

$$l = d_{EM}\alpha_c = 64\,m$$

12. If light of wavelength 520 nm goes through a slit which is 0.4-mm wide and travels to a square screen of area 2m × 2m, how wide is the central diffraction peak?

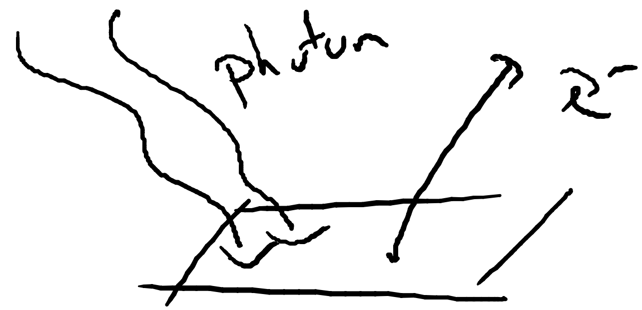
a. The information provided is insufficient to answer this question.
b. 2.6 mm
c. 5.2 mm

$\lambda = 520\,\text{nm}$ $\qquad a = 0.4\,\text{mm}$



$$\theta = 0 \rightarrow \theta_{min} = \frac{\lambda}{a}$$

You wish to design a photocell that operates (i.e., can emit electrons) when illuminated with violet light (wavelength 379 nm). You have the following materials to choose fron and their associated work functions:



(i) Tungsten, $\Phi = 4.5$ eV
(ii) Aluminum, $\Phi = 4.2$ eV
(iii) Barium, $\Phi = 2.5$ eV
(iv) Cesium, $\Phi = 1.9$ eV

13. Which of the following choices would be appropriate for your photocell?

a. (i) and (ii)
b. (iii) and (iv)
c. All of them

$$E^{violet}_{photon} > \Phi$$

$$E_{photon} = \frac{hc}{\lambda} = \frac{1240\,\text{eV nm}}{379\,\text{nm}} = 3.27\,\text{eV}$$

14. If the frequency of the laser for your working photocell were doubled, how would it affect the stopping voltage and work function?

a. The stopping voltage would remain the same and the work function would remain the same.

~~b. The stopping voltage would increase and the work function would decrease.~~

c. The stopping voltage would increase and the work function would remain the same.

$$E_{photon} = hf \rightarrow E'_{photon} = h(2f)$$

Φ

$$\downarrow$$

$$KE = E_{photon} - \underline{\underline{\Phi}}$$

$KE\uparrow$ when $f \rightarrow 2f$

$\underline{\Phi}$

15. A microwave oven operates at a power level of 700 Watts and at a frequency of 2.5 GHz. How many photons per second does this microwave oven produce?

a. ~~2.8 × 10~~^{7} Photons/s
b. 4.2 × 10^{26} Photons/s
c. 2.7 × 10^{27} Photons/s

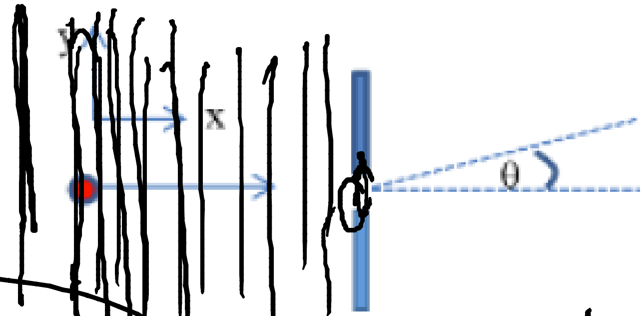
$$\text{Power} = (E_{photon}) \frac{\#\text{photons}}{\text{sec}}$$

$$E_{photon} = hf = (6.626 \times 10^{-34}\ \text{J}\cdot\text{s}) \times 2.5 \times 10^{9}\ \text{s}^{-1}$$

$$= 1.66 \times 10^{-24} \frac{\text{J}}{\text{photon}}$$

$$\frac{700\ \frac{\text{J}}{\text{s}}}{1.66 \times 10^{-24}\ \text{J/photon}} = 4.2 \times 10^{26} \frac{\text{photons}}{\text{sec}}$$

16. A gold atom (mass 3.27×10^{-25} kg) has a velocity of 3 m/s in the x-direction and a velocity of *exactly* 0 m/s in the y-direction. The atom is incident on a single slit of width 3μm (extending in the y-direction). What is the minimum uncertainty in its y-velocity after the slit?



a. ~~0 m/s.~~
b. 2.15×10^{-10} m/s.
c. 1.07×10^{-4} m/s.

$0 \pm \Delta v_y$

$$\Delta P_y \Delta y \sim \hbar$$

$$\Delta P_y = \frac{\hbar}{\Delta y} \qquad \Delta y = 3\mu m$$

$$\Delta P_y = \frac{1.054\times10^{-34}\ J\cdot s}{3\times10^{-6}\ m} \approx 0.3\times10^{-28}\ N\cdot s$$

$$P_y = mV_y \Rightarrow \Delta P_y = m\Delta V_y \qquad 1\times10^{-4}\ m/s$$

$$\Delta V_y = \frac{\Delta P_y}{m} = \frac{0.3\times10^{-28}\ N\cdot s}{3.27\times10^{-25}\ kg} \rightarrow$$

17. In what direction θ is there no chance to find gold atoms after the slit?

~~a. There is no such direction~~
b. $\theta = 0.013°$
~~c. $\theta = 0°$~~

$$\boxed{\lambda_{DB}} = \frac{h}{p} = \frac{6.6\times10^{-34}\,\mathrm{J\,s}}{(3.3\times10^{-25}\,\mathrm{kg})(3\,\mathrm{m/s})}$$

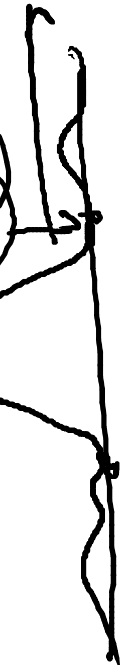
$$\frac{\lambda}{a} \ll 1$$

$$= \boxed{6.2\times10^{-10}\,\mathrm{m}}$$

$$\theta_{min} = \frac{\lambda}{a}$$

$$= \frac{\lambda_{DB}}{a} = \frac{6.2\times10^{-10}\,\mathrm{m}}{3\times10^{-6}\,\mathrm{m}} = 2.1\times10^{-4}$$

$$\theta_{min} = 0.012°$$



You're trying to make your own spectrometer, using either a CD (groove spacing = 1.6 μm) or a DVD (groove spacing = 0.74 μm) as the diffracting element.

18. If you want the best spectral resolution (i.e., ability to distinguish two closely spaced wavelengths) around 500 nm, which should you use, assuming your light source can coherently illuminate up to ~1-cm-wide patch (thereby determining the number of lines), and you are restricted to first-order?

a. ~~CD~~
b. DVD
c. Doesn't matter, they have the same performance specifications for this application.

CD $\quad d = 1.6 \times 10^{-6}$ m

DVD $\quad d = 0.74 \times 10^{-6}$ m

$D_{Beam} = 1$ cm wide

$$N_{CD} = \frac{D_{Beam}}{d} = \frac{10^{-2}\,\text{m}}{1.6 \times 10^{-6}\,\text{m}}$$

$N_{CD} = 6250$ lines/slits

$N_{DVD} = 13514$ lines/slits

$$\Delta\lambda \geq \frac{\lambda}{N m}$$

19. Using the CD, what's the smallest wavelength that could be resolved from 500 nm, again assuming you can coherently illuminate a 1-cm-wide patch. Make sure that you employ the optimal diffraction order.

a. 500.080 nm
b. 500.040 nm
c. 500.027 nm
d. 500.020 nm
e. 500.016 nm

CD

$\lambda = 500\text{nm}$

$N_{slits} = 6250$

$\lambda + \Delta\lambda$

$\Delta\lambda \geq \frac{\lambda}{Nm}$

$\Delta\lambda \geq \frac{0.08}{3}$

$\Delta\lambda \geq 0.027$

$\Delta\lambda \geq \frac{0.08\text{nm}}{m}$

$|\sin\theta| \leq 1$

what is the maximum order

$d\sin\theta = m\lambda$

$\sin\theta = \frac{m\lambda}{d} \Rightarrow \left|\frac{m\lambda}{d}\right| \leq 1$

$\boxed{|m| \leq \left|\frac{d}{\lambda}\right|}$

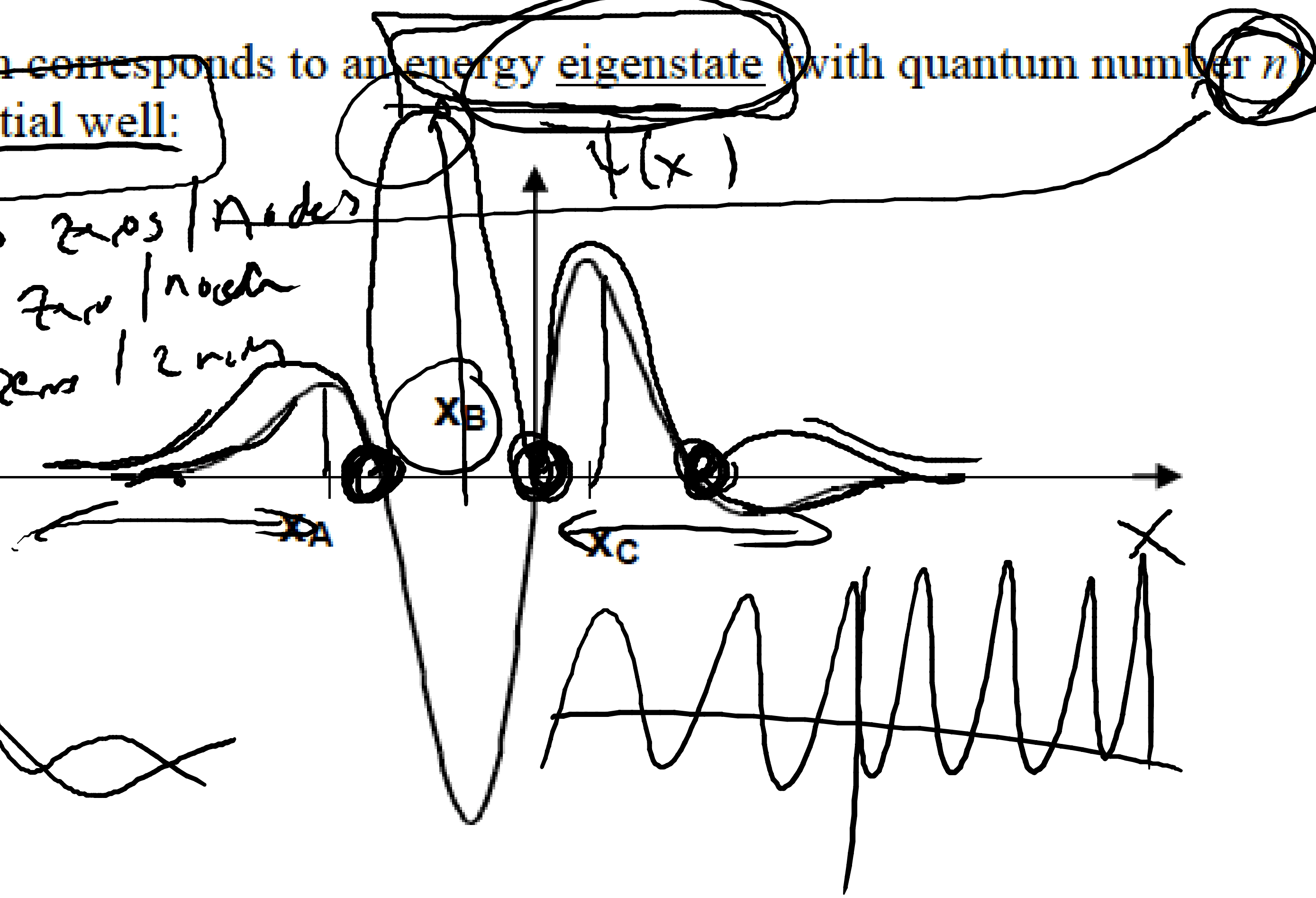
$|m| \leq 3.2$

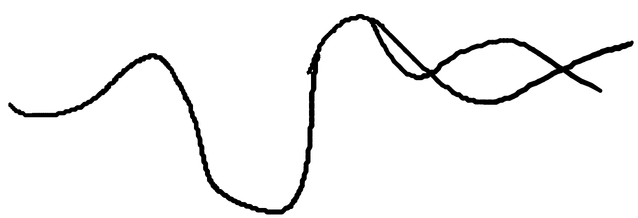
$m_{max} = \pm 3$

$|m| \leq \frac{1.6\times10^{-6}\text{m}}{5\times10^{-7}\text{m}} = 3.2$

This wavefunction corresponds to an energy eigenstate (with quantum number n) of particle in a potential well:

$n=1$ ← no zeros | Nodes
$n=2$ ← 1 zero | node
$n=3$ ← 2 zeros | 2 nodes

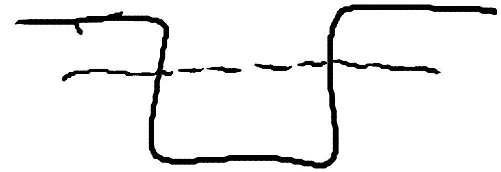




20. If we measure the location of the particle, where is it most likely to be found?

a. at position x_A
b. at position x_B
c. at position x_C

21. Where is the total energy of the particle the highest?



$E_T = KE + U$

a. at position x_A
b. at position x_B
c. at position x_C
d. same at all positions
e. cannot be determined from the information given

22. Which of the following can we conclude about the potential?

~~a. It has infinite walls.~~
~~b. It is a symmetric potential, i.e., $V(-x) = V(x)$.~~
c. ~~It has *at most* three bound states, i.e., no more than three energy eigenstates.~~
~~d. All of the above.~~
e. None of the above.

3 zeros

$n = 3 + 1 = 4$

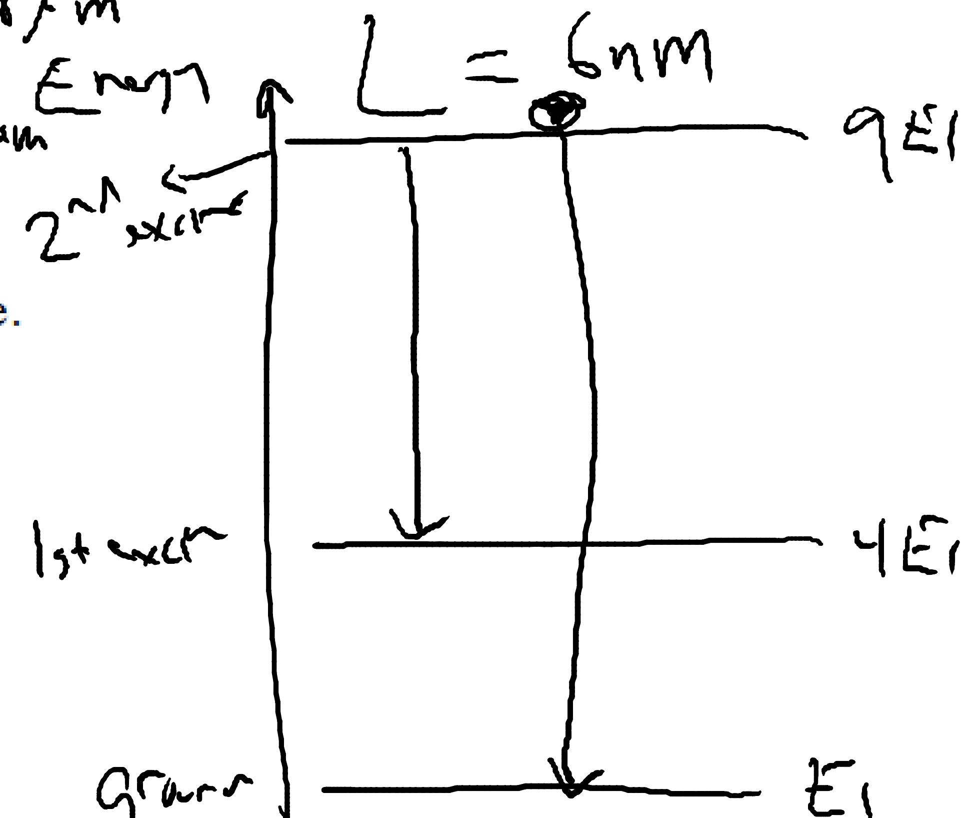
An electron in a quantum well is approximated as a particle in an infinite square well of width 6 nm.

23. What is/are the possible wavelength(s) of a photon emitted by the electron as it decays out of the 2nd excited state? (Hint: It may help to draw the energy level diagram.)

$\lambda_1 = \frac{hc}{8E_1} = 14.8\,\mu m$

$\lambda_2 = \frac{hc}{5E_1} = 23.7\,\mu m$

a. 23.7 μm
b. 14.8 μm
c. 9.5 μm
d. 6 nm
e. 23.7 μm and 14.8 μm are both possible.



$$E_n = n^2 E_1 = n^2 \frac{h^2}{8 m_e L^2} = n^2 \frac{1.505\,eV\,nm^2}{4L^2}$$

$E'_{photon} = 9E_1 - E_1 = 8E_1 = \frac{hc}{\lambda_1}$ $\quad E_1 = \frac{1.505\,eV\,nm^2}{4(36\,nm^2)} = 0.011\,eV$

L

$E''_{photon} = 9E_1 - 4E_1 = 5E_1 = \frac{hc}{\lambda_2}$

24. Compare the initial wavelength of the electron ($\lambda_{electron}$) in the 2^{nd}-excited state of the 6-nm well with the wavelength of a proton (λ_{proton}) also in the 2^{nd}-excited state of an analogous infinite well (also 6 nm wide).

a. $\lambda_{electron} = \lambda_{proton}$

b. $\lambda_{electron} > \lambda_{proton}$

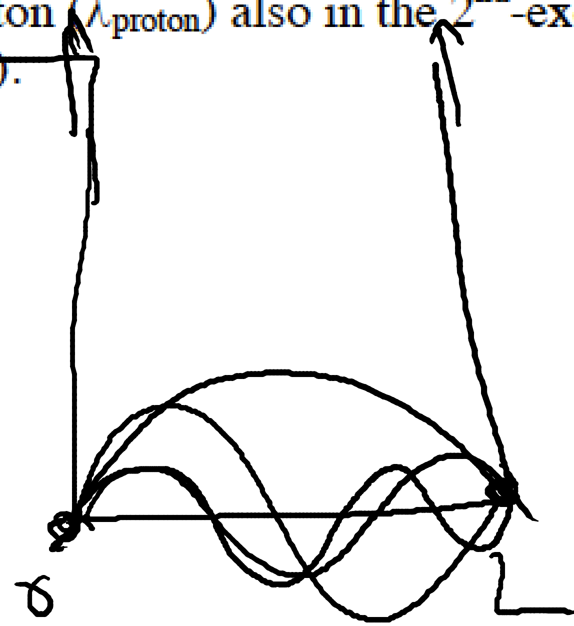
c. $\lambda_{electron} < \lambda_{proton}$

$n=1 \quad \lambda_1 = 2L$

$n=2 \quad \lambda_2 = L$

$n=3 \quad \lambda_3 = \frac{2L}{3}$

$mv = p = \frac{h}{\lambda}$

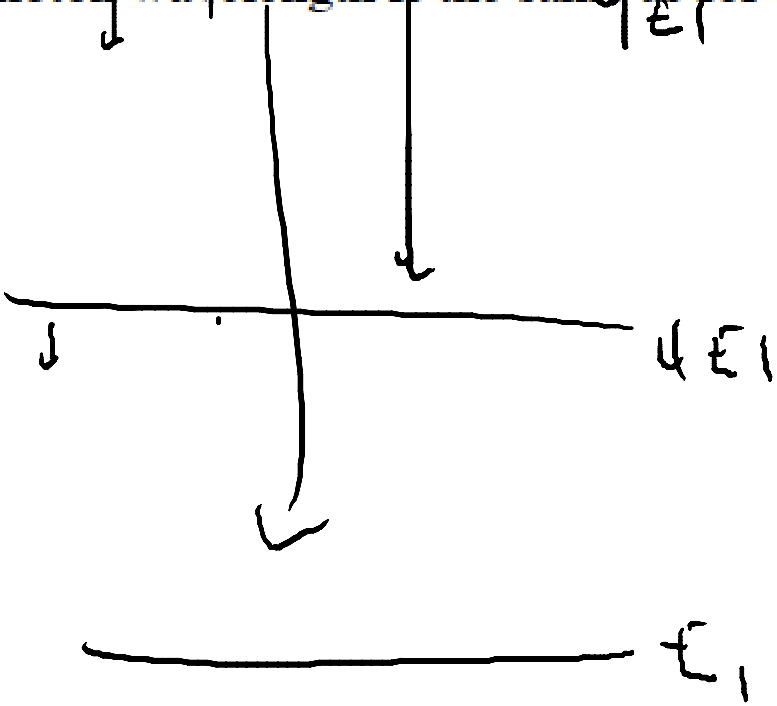


For infinite well
Changing mass does not
change λ

$m_e V_e = m_p V_p$

25. If we now account for the fact that the well actually has finite depth, not infinite, what happens to the wavelength of a photon emitted when the electron decays out of the 2nd excited state?

a. Photon wavelength is longer than for the infinite-well case.
b. Photon wavelength is shorter than for the infinite-well case.
c. Photon wavelength is the same as for the infinite-well case.



$$E \sim \frac{n^2}{L^2}$$

$\lambda_e \uparrow$

$p \to h/\lambda_e$

$\Delta E \downarrow$

$$E = \frac{hc}{\lambda} \qquad \lambda \uparrow$$

$\Delta E = E_{photon}$

26. Compare the probabilities to find the electron between 2 nm and 4 nm (i.e., in the middle third of the well) if it is in the 2nd excited or the ground state? (Hint: Sketch the probability densities $|\psi(x)|^2$.)

$\frac{D}{1.6\,\mu m}$ $D/0.74\,\mu m$

1 cm

a. $P_{ground}(2\text{ nm} \le x \le 4\text{ nm}) \ge P_{2nd\ excited}(2\text{ nm} \le x \le 4\text{ nm})$

b. $P_{ground}(2\text{ nm} \le x \le 4\text{ nm}) = P_{2nd\ excited}(2\text{ nm} \le x \le 4\text{ nm})$

c. $P_{ground}(2\text{ nm} \le x \le 4\text{ nm}) \le P_{2nd\ excited}(2\text{ nm} \le x \le 4\text{ nm})$

$A \sin kx$

$\frac{2}{L}$

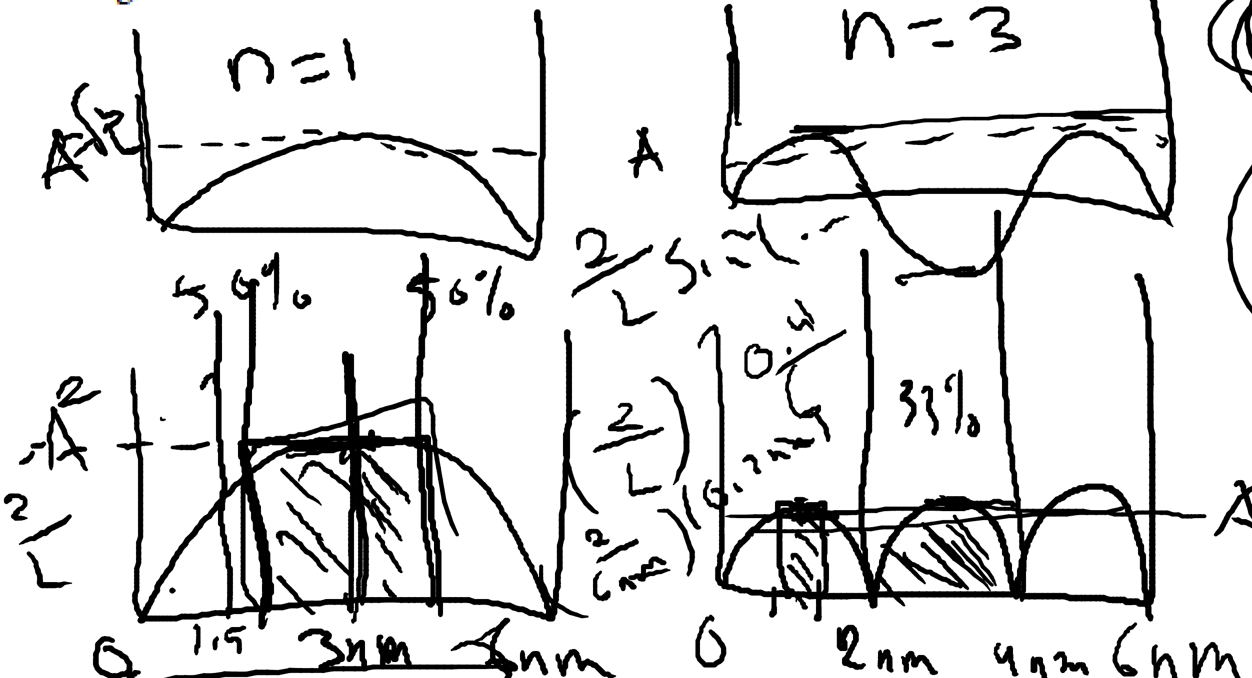
$k_1 = \frac{2\pi}{2L}$

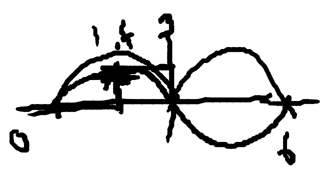
$k_3 = \frac{2\pi}{2L/3}$

$\frac{2}{L}\sin\left(\frac{3\pi}{L}x\right)$

$\frac{2}{L}\sin$

$\sin^2 x = \frac{1}{2}$





$\Delta\lambda \ge \frac{\lambda}{(N_{slits})m}$