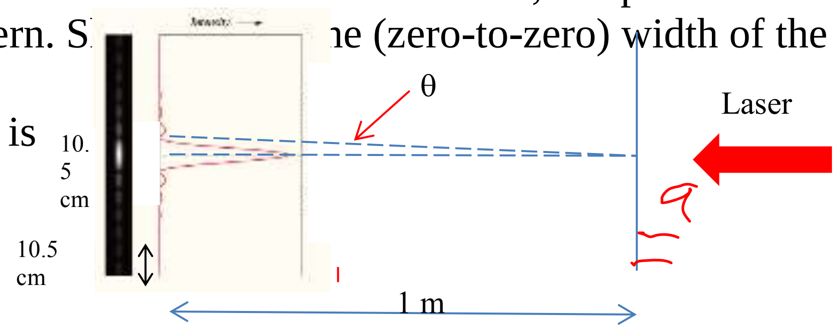
The next two problems refer to the following situation:

In order to measure the width of a single slit in a sheet of plastic, Alice shines a red laser with a wavelength of 650 nm at the slit. On the other side of the slit, she places a screen 1 meter away to measure the interference pattern. S ne (zero-to-zero) width of the central lobe on the screen to be 10.5 cm.

1. The width of the single slit is



a. 12.4 μm

b. 4.6 μm

c. 6.2 μm

W

L

$\theta_{min} = \frac{\lambda}{a}$

$W = 2\theta_{min} L$

$= \frac{2\lambda L}{a}$

$a = \frac{2\lambda L}{W} = 2(\ldots)$

$650\,nm \cdot 1\,m$

2. Now immerse the whole system in water (index of refraction is n=1.4). What will be the new central lobe width?

a. 7.5 cm
b. 10.5 cm
c. 14.7 cm

water ⇒ slower = more relative phase

10.5 cm

need smaller θ ⇒ smaller width

≈ 12.4 mm

$$w = \frac{2\lambda L}{a} \qquad \lambda f = \frac{c}{n}$$

$$= 2\frac{c}{fn}\frac{L}{a} = \frac{2cL}{fa}\frac{1}{n} = \frac{w}{n}$$

$$\frac{10.5}{1.4} = 7.5$$

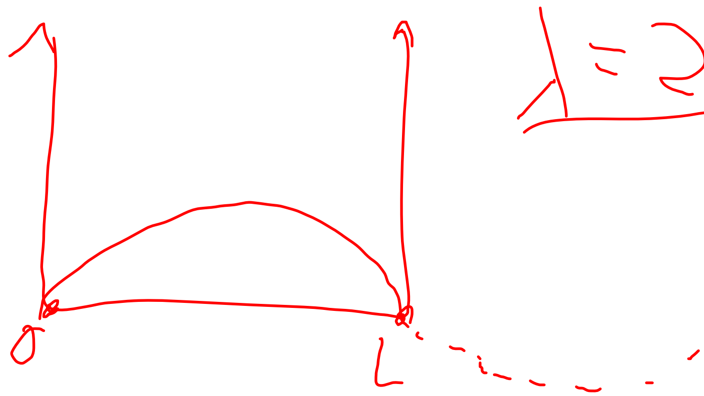
3. An electron and an anti-proton (same charge but ~1800x the mass) are each in the respective ground states of two identical1-D potential wells. Compare the deBroglie wavelengths of the particles, and the wavelength of a photon needed to excite the particle to the first excited state:

$\lambda \propto m$

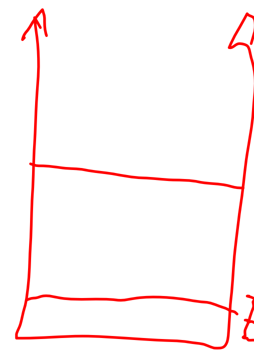
a. ~~$\lambda_{\text{electron deBroglie}} = \lambda_{\text{anti-proton deBroglie}}, \lambda_{\text{photon to excite electron}} = \lambda_{\text{photon to excite anti-proton}}$~~
b. $\lambda_{\text{electron deBroglie}} = \lambda_{\text{anti-proton deBroglie}}, \lambda_{\text{photon to excite electron}} < \lambda_{\text{photon to excite anti-proton}}$
c. ~~$\lambda_{\text{electron deBroglie}} < \lambda_{\text{anti-proton deBroglie}}, \lambda_{\text{photon to excite electron}} < \lambda_{\text{photon to excite anti-proton}}$~~
d. ~~$\lambda_{\text{electron deBroglie}} > \lambda_{\text{anti-proton deBroglie}}, \lambda_{\text{photon to excite electron}} = \lambda_{\text{photon to excite anti-proton}}$~~
e. ~~$\lambda_{\text{electron deBroglie}} > \lambda_{\text{anti-proton deBroglie}}, \lambda_{\text{photon to excite electron}} > \lambda_{\text{photon to excite anti-proton}}$~~

$E_p \sim \frac{1}{m}$

$= \frac{hc}{\lambda}$



$\lambda = 2L$



$E_2 = 4E_1$

E_1

$E = \frac{p^2}{2m} = \frac{h^2}{\lambda^2 2m}$

$E_p = E_2 - E_1 = 3E_1 = \frac{3}{8}\frac{h^2}{mL^2}$

$E_1 = \frac{h^2}{4L^2 2m}$

The next two questions refer to the following situation:

A neon lamp has spectral lines at wavelengths 693 nm and 703 nm. Bob wants to resolve these two spectral lines using a diffraction grating to view the second-order spectrum. The 1-cm by 1-cm grating has 100 slits/mm.

$m=2$

4. What approximate minimum beam diameter is needed to resolve the 2 lines?

- a. 0.1 mm
- b. 0.35 mm
- c. 0.6 mm
- d. 2.7 mm
- e. the beam diameter does not affect the resolution.

$d = \frac{1\,mm}{100} = 10\,\mu m$

2nd

1st

0

θ

$$d \sin\theta_n = n\lambda$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{Nm}$$

#slits or lines

order = 2

$\Delta\lambda = 10\,nm$

$\lambda \approx 698\,nm$

$$N = \frac{\lambda/\Delta\lambda}{m} = \frac{698/10}{2} = 35 \text{ lines}$$

$$w = N \cdot d = 35 \cdot 10\,\mu m = 350\,\mu m$$

5. What is the largest angle at which one could observe the **703-nm** light diffracting from the grating? (The width of each slit is 0.5 μm, *i.e.* you can neglect single-slit diffraction).

$d \sin\theta = m\lambda$ Biggest m?

$\sin\theta = \frac{m\lambda}{d} \le 1$

$m \le \frac{d}{\lambda} = \frac{10\,\mu m}{0.703\,\mu m} \approx 14.2$

$\Rightarrow m = 14$

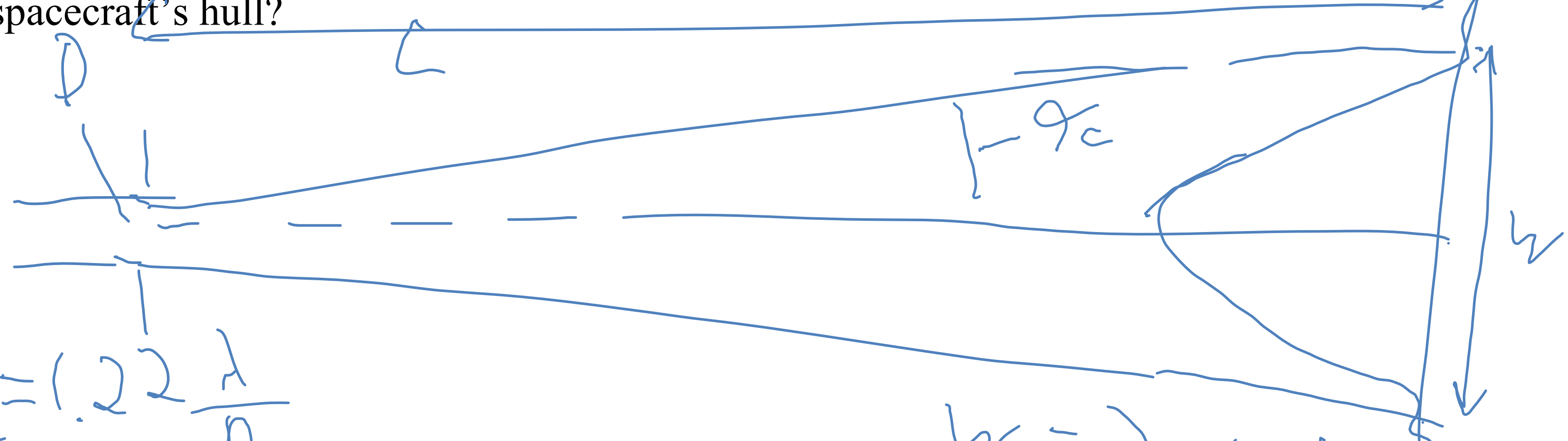
$\theta = \sin^{-1}\frac{14 \cdot \lambda}{d} = \sin^{-1}\left(\frac{14 \cdot 0.703\,\mu m}{10\,\mu m}\right)$

a. 1°
b. 4°
c. 8°
d. 80°
e. 90°

6. Consider a laser beam of wavelength 600 nm with a circular aperture of diameter 3.0 cm. The beam shines on a spacecraft a distance 200 km away.

What is the approximate **diameter** of the center spot size (*i.e.* the zero-intensity to zero- intensity width) on the spacecraft's hull?

a. 5 m
b. 10 m
c. 20 m



$\theta_c = 1.22\frac{\lambda}{D}$

$w = 2\theta_c \cdot L$

$= 2.44\frac{\lambda L}{D} \approx 9.76\,m$

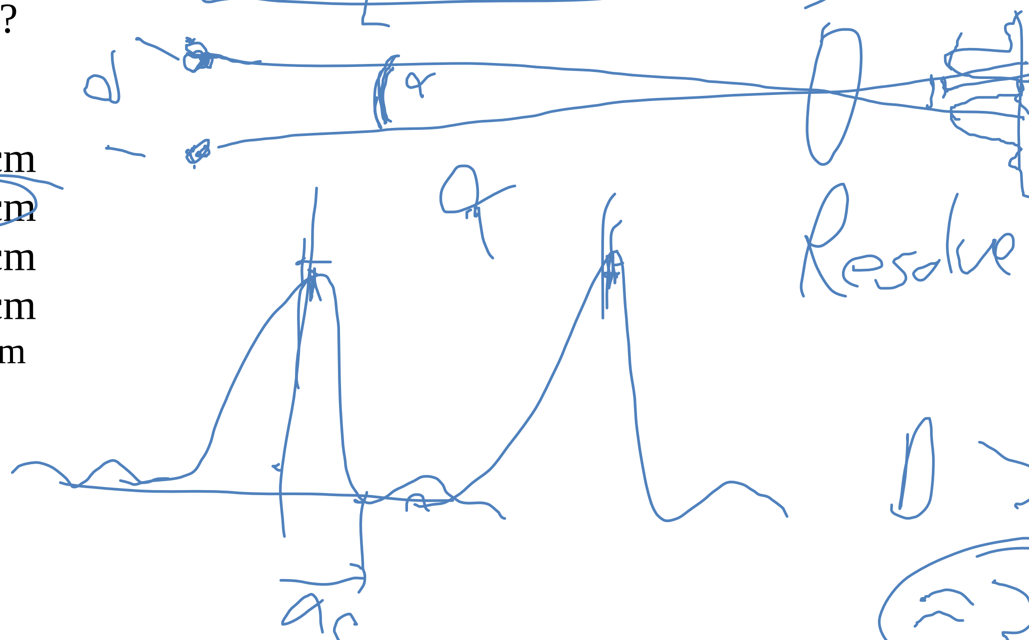
***The next two questions refer to the following situation*:**

1000 m

For a telescope to be able to resolve two objects that are one kilometer away, it is determined that the two objects must be separated by at least three centimeters.

$d = 0.03\ m$

7. What is the approximate diameter of the telescope (assuming visible light has a wavelength of 500 nm)?



   a. 1 cm
   b. 2 cm
   c. 3 cm
   d. 4 cm
   e. 5 cm

Resolve if $\theta > \theta_c$

$$\frac{d}{L} > \frac{1.22\lambda}{D}$$

$$D > \frac{1.22\lambda L}{d}$$

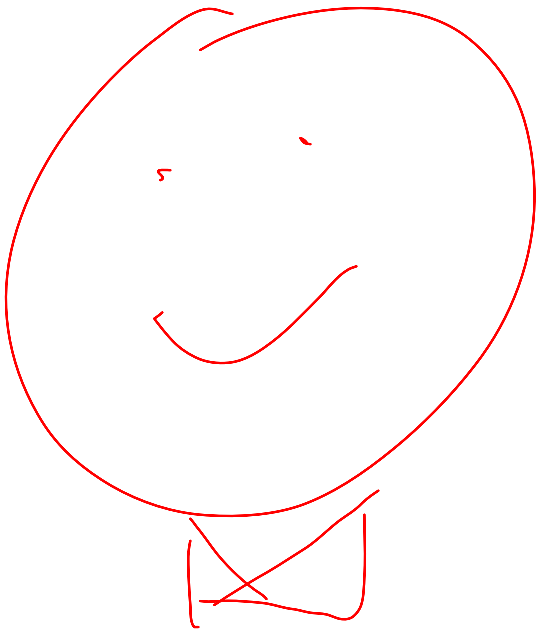
$\approx 2\ cm$

8. If the wavelength of the light illuminating (and thus emitted by the object) is doubled, the diameter of the telescope tripled, but everything else is unchanged, are the two objects still resolvable?

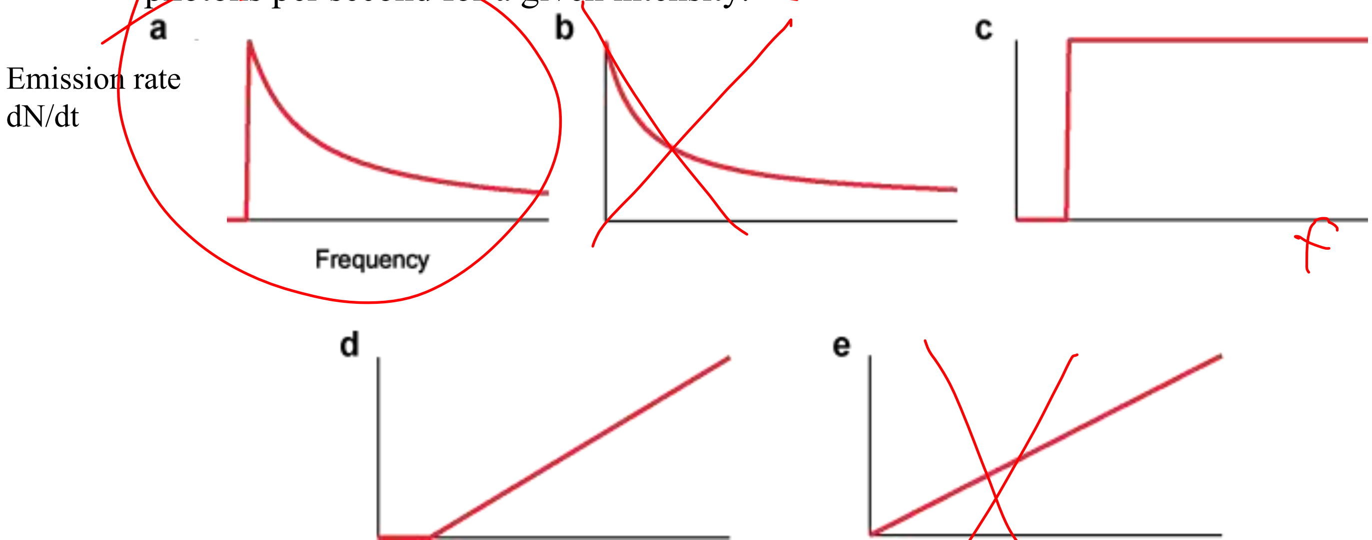
$$\frac{d}{L} > \frac{1.22\lambda 2}{D3}$$

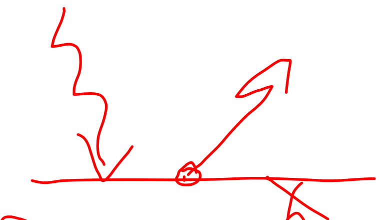
Yes

a. Yes
b. No
c. Not enough information to determine



9. A laser beam with **FIXED INTENSITY** (and fixed diameter) is incident on a metal surface. If we vary the photon frequency (while keeping the intensity the same), which of the graphs below best describes the electron emission rate dN/dt, *i.e*. the number of electrons emitted per second, as a function of frequency? (Assume that the photo-emission probability is either 0 or 1, i.e., it does not depend on photon frequency as long as an electron can be emitted). Hint: Calculate the number of photons per second for a given intensity.





$E_{photon} = \phi + KE$

$= hf$

$f > \phi / h \quad f_c$

have photo-electron

$P_{ower} = E_{photon} \times \# phot/s$

$\# phot/s = \frac{Power}{E_{photo}} = \frac{Power}{hf} \Rightarrow \frac{dN}{dt} \propto \frac{1}{f}$

10. A laser of wavelength 410 nm is incident on a metal with a workfunction of 1.7 eV. There is a stopping voltage of 0.5 volts applied. What is the highest energy of an ejected electron, *immediately after it got ejected* ?

a. 0.8 eV
b. 4.7 eV
c. 3 eV
d. 1.3 eV
e. No electrons are ejected.

$E_{photon} = \phi + KE$

$KE_{max} = E_{photon} - \phi = \frac{hc}{\lambda} - \phi = \frac{1240\,eV\cdot nm}{410} - 1.7\,eV$

$= 1.32\,eV$

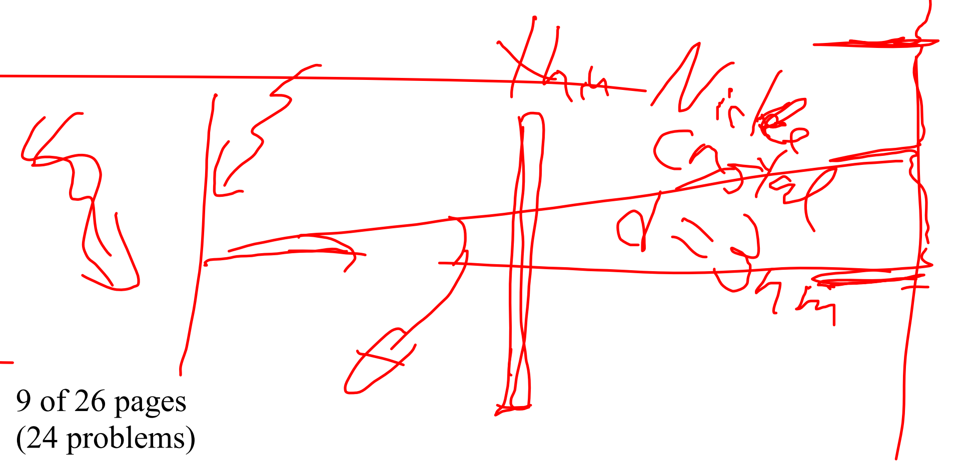
Electron will reach collector with $KE_i - eV$

$= 1.32\,eV - 0.5\,eV =$

Turn off V:

$d\sin\theta = m\lambda$

$\sin\theta = \frac{m\lambda}{d} \le 1$ $\quad m \le \frac{d}{\lambda}$



$m < \frac{d}{\lambda} = \frac{3\text{nm}}{1.07\text{nm}}$

$= 2.8$

$\Rightarrow m = 2$

$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{1.505\,\text{eV-nm}^2}{\lambda^2}$

$\lambda = \sqrt{\frac{1.505}{E}} = \sqrt{\frac{1.505\,\text{eV-nm}^2}{1.3\,\text{eV}}}$

$= 1.07\,\text{nm}$

$\Rightarrow$ 5 peaks

Increase λ_{laser}: width of pattern?

$\lambda_{laser} \uparrow$ $\quad E_{photon} \downarrow$

$KE_{electron} \downarrow$ $\quad \lambda_{electron} \uparrow$ $\quad$ Pattern bigger

11. A 600-nm photon incident on a metal surface ejects an electron. The ejected electrons are stopped by applying a 0.63 V potential difference between the metal and a conducting plane placed above the metal surface. What is the work function of the metal?

a. Φ = 0.63 eV
b. Φ = 1.54 eV
c. Φ = 2.07 eV
d. Φ = 1.44 eV
e. Φ = 3.15 eV

.

$$E_{photon} = \phi + KE$$
$$= \phi + eV_{stop}$$
$$\phi = E_{photon} - eV_{stop} = \frac{1240\ \text{eV-nm}}{600} - 0.63\ \text{eV}$$
$$= 1.44\ \text{eV}$$

12. A red laser of wavelength 662 nm shines on a black bead, which experiences a force of 1 μN as a result. Let R be the rate (number of photons per second) at which photons are emitted by the laser. Assuming idealized situations (full absorption of light by the bead, environment is a vacuum, etc.), calculate R.

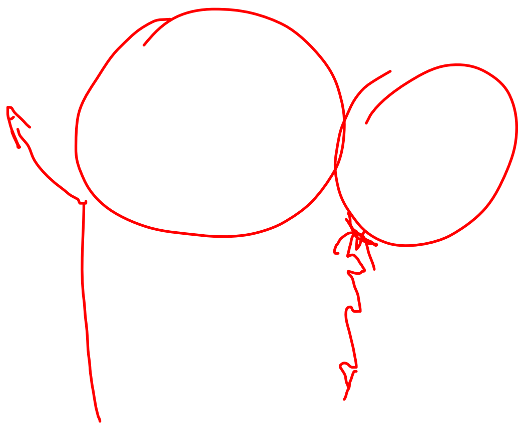
a. 1.5×10^{27} photons / second
b. 1.0×10^{21} photons / second
c. 4.5×10^{8} photons / second

$$F = \frac{dp}{dt} = p_{photon} \times \#photon/sec \quad R$$

$$R = \frac{F}{p_{photon}} = \frac{10^{-6} N}{h/\lambda}$$

$$= \frac{10^{-6} \cdot 662 \times 10^{-9} m}{6.6 \times 10^{-34} Js}$$

$$= 10^{21}/s$$



13. A typical X-ray has a wavelength of 1 Angstrom (0.1 nm). Compare the photon energy to the kinetic energy of an electron having a deBroglie wavelength equal to the photon wavelength.

a. $(K.E.)_{electron} / E_{photon} = 0.012$
b. $(K.E.)_{electron} / E_{photon} = 0.150$
c. $(K.E.)_{electron} / E_{photon} = 0.413$
d. $(K.E.)_{electron} / E_{photon} = 0.657$
e. $(K.E.)_{electron} / E_{photon} = 1.000$

$$\cancel{KE} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{1.505\ \text{eV-nm}^2}{\lambda^2}$$

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV-nm}}{\lambda}$$

$$\frac{1.505/\cancel{\lambda^2}}{1240/\cancel{\lambda}} = \frac{1.505\ \text{eV-nm}^2}{1240\ \text{eV-nm}\ (0.1\ \text{nm})} = \cancel{0.012} = 0.012$$

14. In a hydrogen molecule, if the electron were to "fall into" the nucleus and be trapped in the nucleus it would have to be contained within a space the size of a proton (radius of a proton = 10^{-15}m). This, of course, does not happen. Here we will show why this scenario is unreasonable. If an electron were confined inside a proton, what would be its approximate minimum kinetic energy?

a. $E = 10^{-12}$ eV
b. $E = 10^{-9}$ eV
c. $E = 30$ eV
d. $E = 10^{7}$ eV
e. $E = 10^{10}$ eV

$$KE = \frac{p^2}{2m} \approx \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{2m(\Delta x)^2} = \frac{h^2}{2m(2\pi)^2(\Delta x)^2}$$

$$\Delta x \Delta p \geq \hbar$$

$$\frac{\hbar^2}{2m} = \frac{1.505\ \text{eV-nm}^2}{(2\pi)^2(2\times10^{-15}\ \text{m})^2}$$

$$= \frac{(6.626\times10^{-34}\ \text{Js}/2\pi)^2}{2m_e (10^{-15})^2} \times \frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}}$$

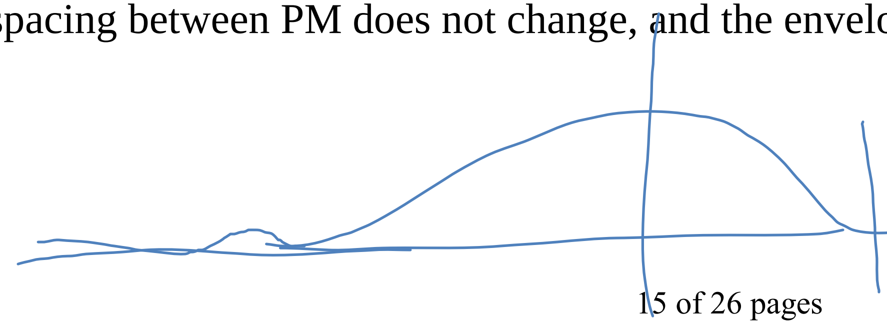
$m_e = 9.1\times10^{-31}$ kg

$= 3.8\times10^{10}$ eV

$\Rightarrow -13.6$ eV

15. A distant screen is illuminated by light passing through a one-dimensional array of slits. Initially, the beam of light illuminates six slits. If we block every other (*i.e*. every alternate) slit while keeping the beam size constant, how does the pattern on the screen change?
PM = Principal maxima

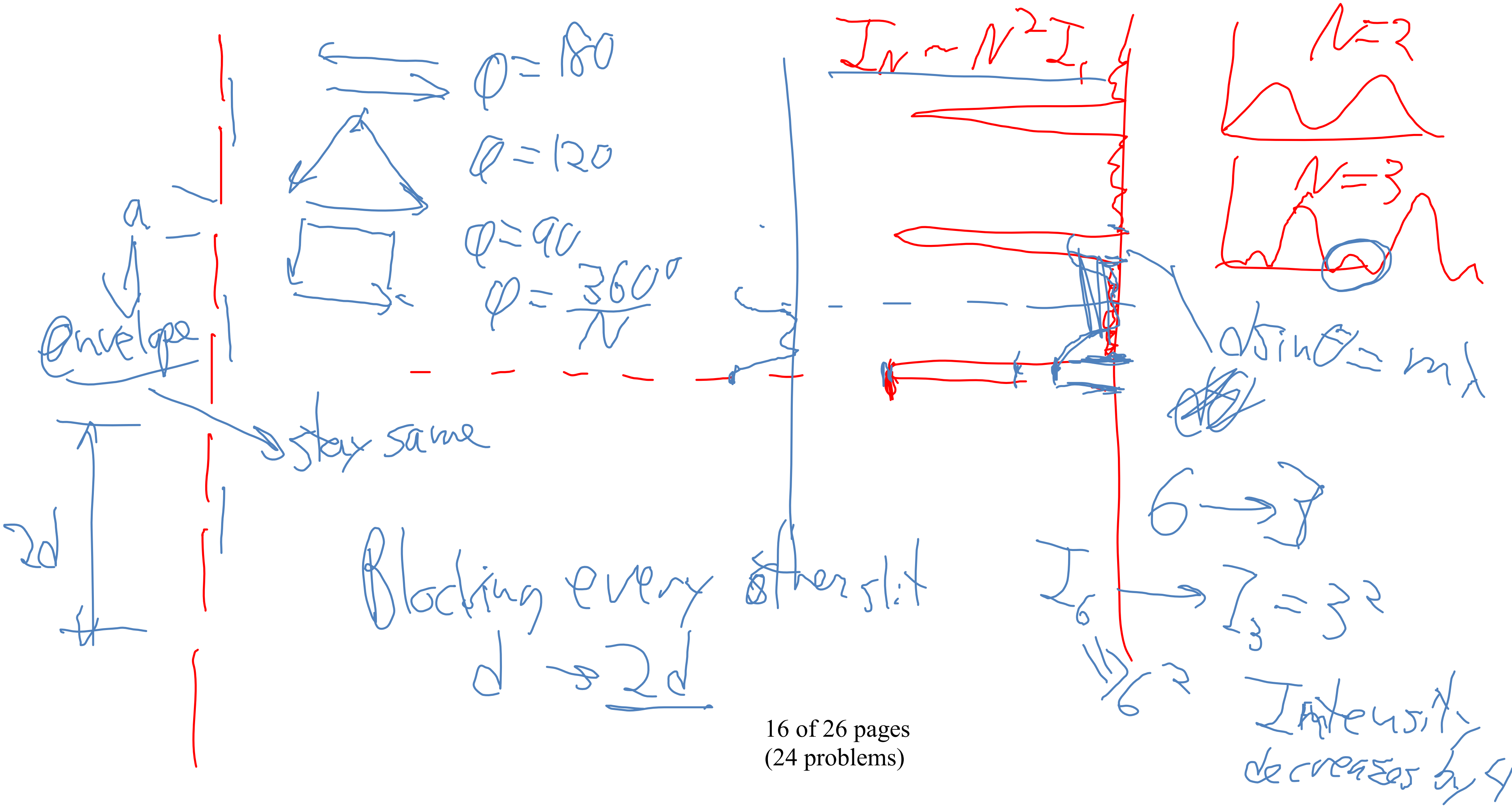
a. The width of the PM increases by a factor of two, the peak intensity of each PM decreases by a factor of four, the spacing between PM decreases and the envelope of the intensity pattern does not change.
b. The width of the PM increases by a factor of two, the peak intensity of each PM decreases by a factor of four, the spacing between PM increases, and the envelope of the intensity pattern becomes broader.
c. The width of the PM increases by a factor of two, the peak intensity of each PM decreases by a factor of four, the spacing between PM does not change, and the envelope of the intensity pattern does not change.

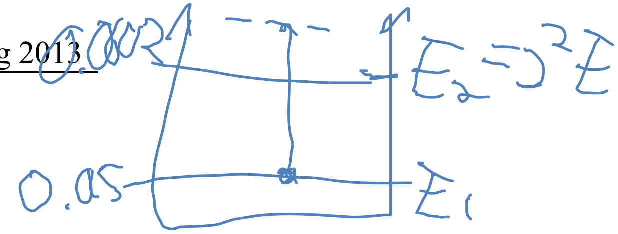


$\theta_{min} = \frac{\lambda}{a}$

d. The width of the PM increases by a factor of two, the peak intensity of each PM decreases by a factor of two, the spacing between PM increases, and the envelope of the intensity pattern does not change.

e. Without more specific information, the *only* general statement that can be made is that the integrated intensity of the diffraction pattern will increase proportionally with the number of slits.





The next three problems refer to the following situation:

An electron trapped in a carbon nanotube (CNT) may be approximated as a particle in an infinite 1-D potential square well (whose width equals the length of the CNT).

Excitation: $E_{photon} = E_2 - E_1$

16. We wish to excite the electron out of the ground state by shining light on the CNT. If the length of the CNT is 5 nm, and we shine on light of wavelength 20.6 µm, what is the final state of the electron? (Hint: Draw an energy level diagram.)

$E_{photon} = \frac{1240\ eV\text{-}nm}{20600\ nm} = 0.0602\ eV$

a. still in the ground state
b. first excited state
c. second excited state

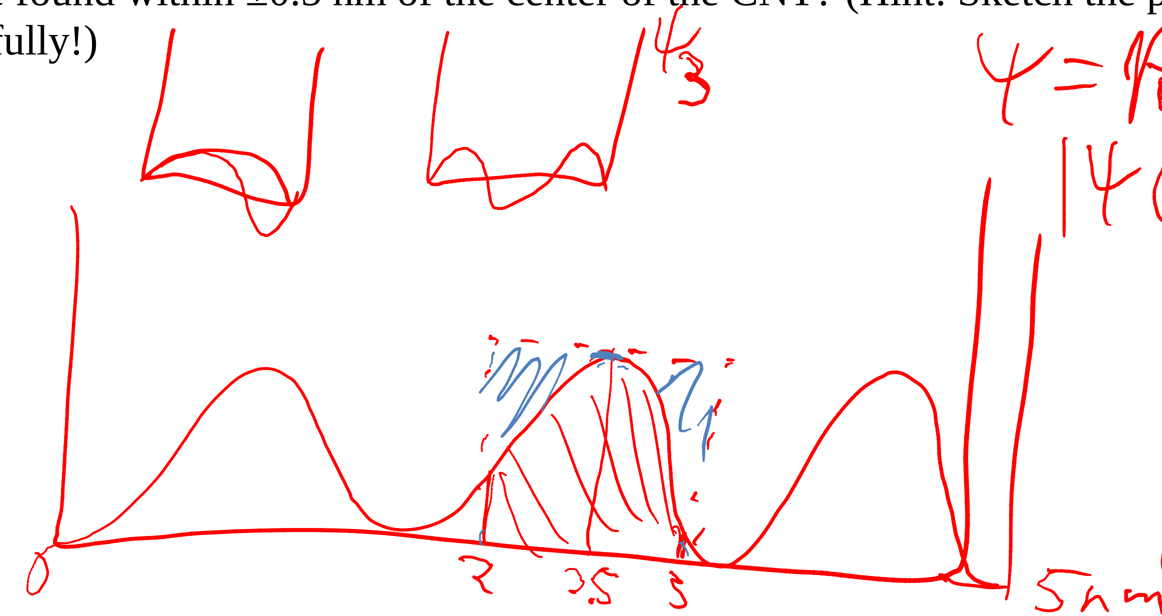
$E_1 = \frac{h^2}{8mL^2} = \frac{h^2}{2m}\frac{1}{4L^2} = \frac{1.505\ eV\text{-}nm^2}{4 \cdot 5^2} = 0.01505\ eV$

$E_2 = 4 \cdot E_1 = 0.0602\ eV$

$n=3$

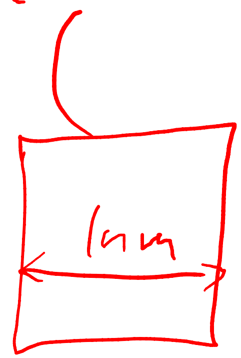
17. Let's say the electron is in the 2^{nd} excited state of the CNT. What is the probability that the electron could be found within ±0.5 nm of the center of the CNT? (Hint: Sketch the probability distribution carefully!)

a. 5%
b. 30%
c. 40%



$\psi = \sqrt{\frac{2}{L}} \sin kx$

$|\psi(x=2.5\text{ nm})|^2 = \frac{2}{L}$



Area $= \frac{2}{5\text{ nm}} \cdot 1\text{ nm}$

≈ 0

18. Let's say that the CNT is now placed next to a 'quantum-well semiconductor device', which we approximate as a <u>finite</u> 1-D potential square well. The CNT starts in its first excited state, and emits a photon, which is then absorbed by the quantum well device, exciting it from its ground state to its first excited state. .4

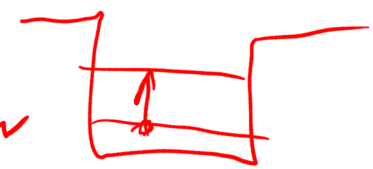
$E \sim \frac{1}{L^2}$

What can we conclude about the width of the quantum well potential (w_{qw}) compared to the width of the carbon nanotube (w_{CNT})?

CNT (infinite) $\rightarrow$ photon $\rightarrow$ QW (finite)

a. $w_{qw} > w_{CNT}$
b. $w_{qw} = w_{CNT}$
c. $w_{qw} < w_{CNT}$

$E_{photon} = (E_2 - E_1)_{CNT} = (E_2 - E_1)_{QW}$



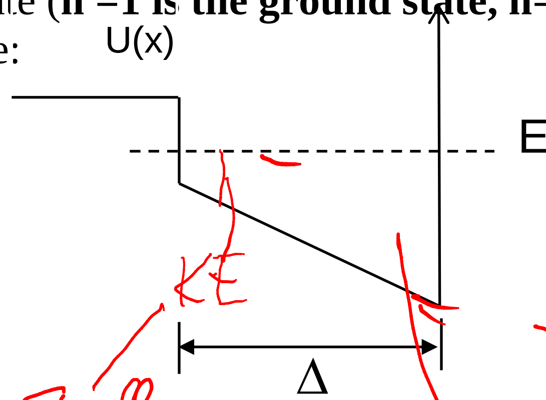
Same width? $E_{finite\ well} < E_{infinite}$

To increase $E_{finite\ well} \Rightarrow$ shrink the well
reduce w

$n = 6$

$n=2$ = first excited
$n=1$

19. Consider a particle in the 5th excited state (**n =1 is the ground state, n=2 is the first excited state, etc.**) of the potential well shown here:
Which of the following best represents the probability distribution of the particle?



$n=1$ ground state

$\Rightarrow |\psi|^2$ has 1 lobe

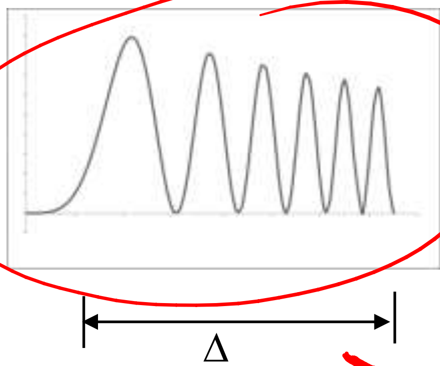
KE

small $\Rightarrow$ big λ

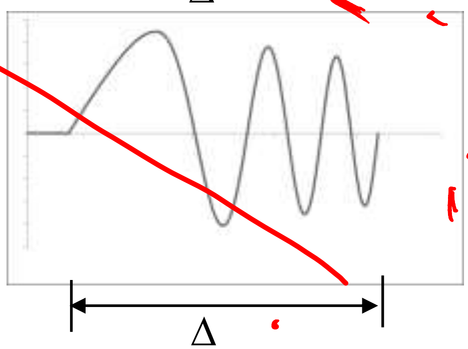
big KE
small λ

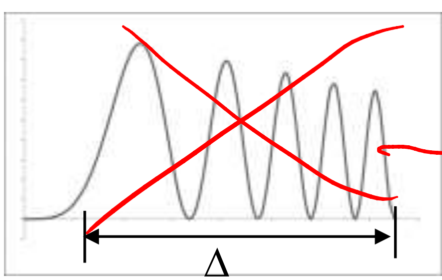
$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

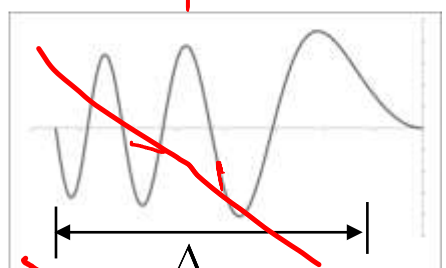
a

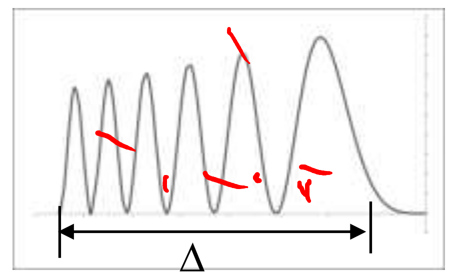


b









d

e

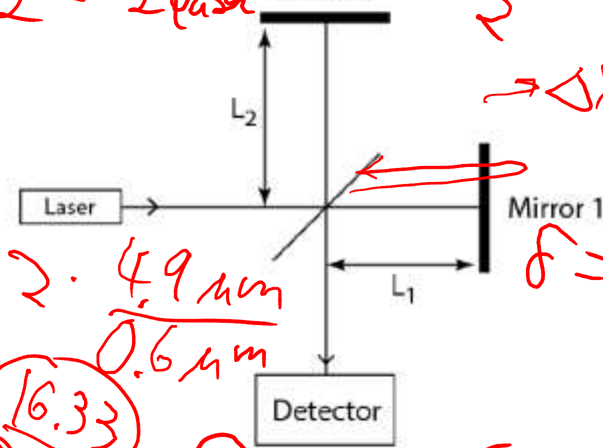
only 5 lobes → need 6

C

The next three problems refer to the following situation:

A Michelson interferometer is initially set to have length L_1 and L_2 equal. Both arms are in vacuum. The wavelength of the laser is 600 nm. Mirror 1 is then moved to the right by 4.9 μm.
The power of the laser is 8 mW. (The intensity is 8mW/1mm^2.)

$I = I_{pasa} \cos^2 \frac{\phi}{2}$

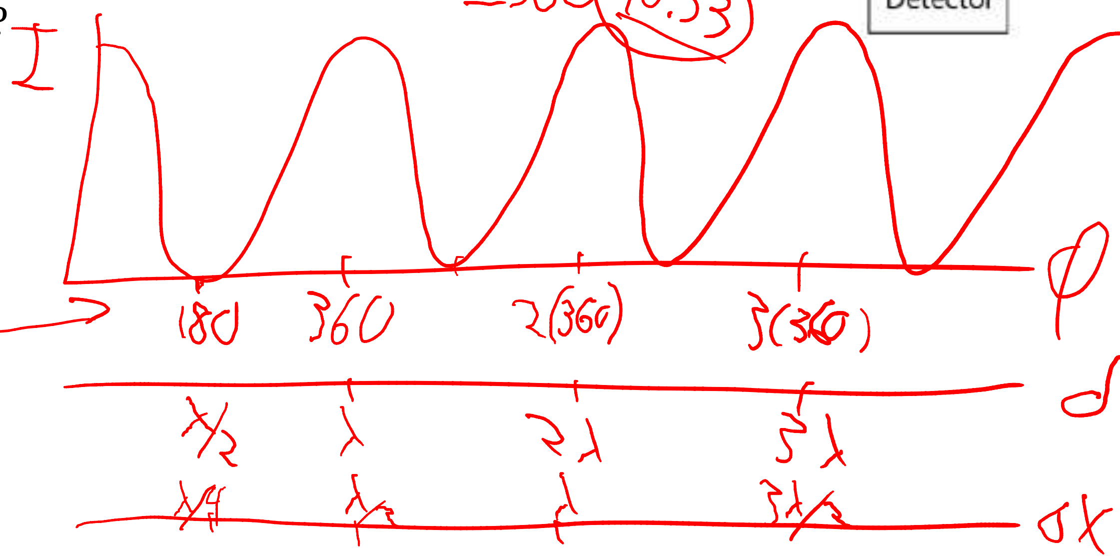


$\rightarrow \Delta x$

$\delta = 2\Delta x$

$\phi = 360 \frac{\delta}{\lambda} = 360 \frac{2 \cdot \Delta x}{\lambda} = 360 \frac{2 \cdot 4.9\,\mu m}{0.6\,\mu m} = 360(16.33)$

20. While mirror 1 is moving, how many times is the minimum power detected?



Maximum

$\phi \ldots N(360)$

$\delta \ldots N\lambda$

$N\lambda/2$

a. 4
b. 8
c. 16

$\phi = 2\pi \frac{\delta}{\lambda} = 360° \frac{\delta}{\lambda}$

top

21. The mirror is now moved so that NO light exits to the detector, and ~~the path lengths are locked at~~ that value. However, over time Mirror 2 degrades so that it only reflects 25% of the power of the incident light (while Mirror 1 is still perfect). What is the new power at the detector?

Initially

$\tilde{I}_{top} = \frac{1}{2}$ mW

$\tilde{A}_{top} = \sqrt{\frac{1}{2}}$

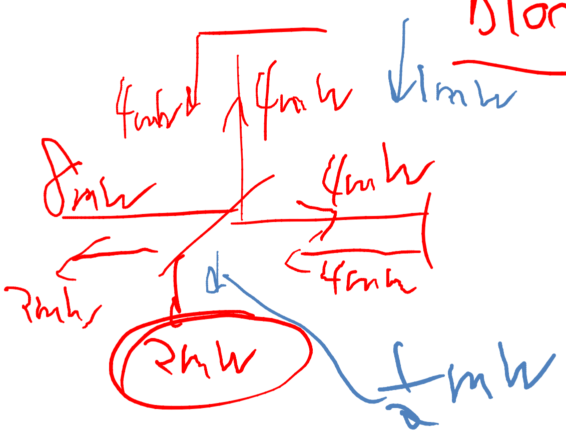
a. 0 mW
b. 0.5 mW
c. 1.5 mW
d. 2.5 mW
e. 4.5 mW

Block top

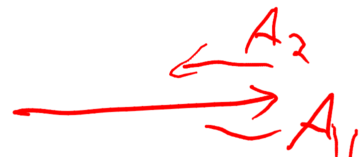
Block right arm

$I_{top} = 2$ mW $\Rightarrow A_{top} = \sqrt{2\text{ mW}}$

$I_{right\ arm} = 2$ mW $\Rightarrow A_{right} = \sqrt{2\text{ mW}}$



After degrades:

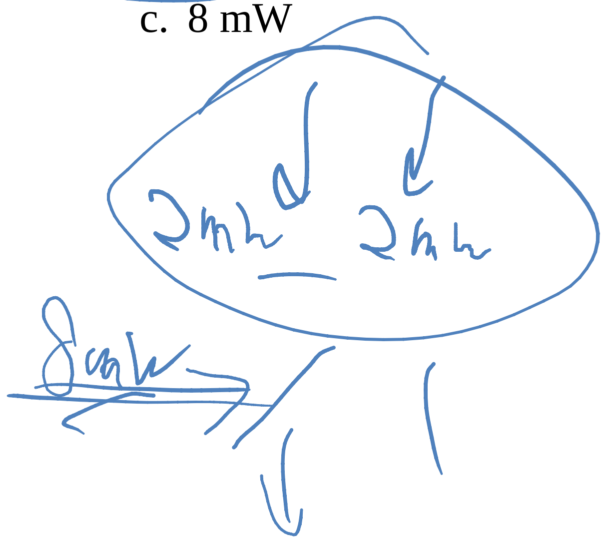


$A_{tot} = A_1 - A_2$
$= \sqrt{2} - \sqrt{\frac{1}{2}}$

I_{tot}
$= A_{tot}^2$
$= \frac{1}{2}$ mW

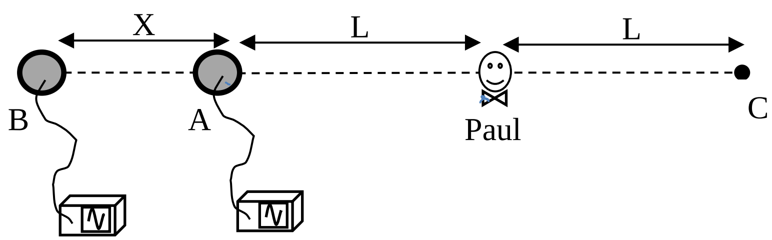
22. We replace the bad mirror with one that reflects 100% of the intensity, but the replacement has an unusual property: it shifts the color of the light that reflects off it, so the wavelength is 599 nm from that arm (and still 600 nm in the other arm). What is now the power at the detector, assuming the path lengths are once again equal?

a. ~~0 mW~~
b. 4 mW
c. 8 mW



Different colors
⇒ processes are distinguishable
⇒ no interference
Add probabilities
Add intensities

The next two problems refer to the following situation:



Initially: Destructive interference

$\phi = \frac{2\pi X}{\lambda} = \pi + 2\pi N$

$X = \lambda\left[\frac{1}{2} + N\right] = \frac{v}{f}\left[\frac{1}{2} + N\right]$

Paul is located away from two speakers (A and B) as shown in the above. When each speaker is turned on *separately* Paul hears the same frequency. However, when A and B are turned on *together*, he hears nothing. The function generator driving speaker A is in phase with the function generator for speaker B.

Final

23. Keeping the sound intensity constant, we gradually increase the frequency of both speakers simultaneously. When the frequency increases by 25 Hz, Paul hears the first loudest sound peak (i.e., for even higher frequencies the sound he hears becomes softer again). Assuming that the velocity of sound is 330 m/s and Paul's hearing is perfect, what is X (the distance between the speakers)?

$\tilde{\phi} = \frac{2\pi X}{\tilde{\lambda}} = \frac{2\pi X f'}{v}$

a. 0.83 m
b. 1.65 m
c. 3.3 m
d. 4.0 m

$$x = \frac{1}{2}\frac{v}{\Delta f} = \frac{330\,m/s}{2 \cdot 25\,s^{-1}} = 6.6$$

e. 6.6 m

Initially $\varphi_i = \pi + N2\pi = 2\pi \frac{x}{\lambda} = 2\pi \frac{x f}{v}$

Finally $\varphi_f = 2\pi + N2\pi = 2\pi \frac{x f'}{v}$

$$\varphi_f - \varphi_i = \pi = 2\pi \frac{x}{v}(f' - f) = 2\pi \frac{x}{v} 25\,Hz$$

24. At this point, i.e., when Paul is hearing the maximum intensity from A and B combined, another speaker is placed at point C to completely cancel out the sound he hears. If the power heard from each speaker A and B separately is 10 W, what is the power heard from speaker C only?

$A_C = 2\sqrt{10}$

$\sqrt{10}$ (A_A) $\sqrt{10}$ (A_B)

$I_C = (2\sqrt{10})^2 = 40\,W$

a. 10 $\sqrt{2}$ W
b. 20 W
c. 40 W