MAKING SENSE OF THE EQUATION SHEET
Interference & Diffraction

INTERFERENCE

$\delta = r_1 - r_2 = d\sin\theta$. Equation for path length difference. r1 – r2 is completely general. Use $\delta \sin\theta$ only when the two sources are far away from the observation point.

$\frac{\phi}{2\pi} = \frac{\delta}{\lambda}$ is completely general whenever you have waves from two sources interfering.

$\frac{\phi}{2\pi} = \frac{\delta}{\lambda} = \frac{d\sin\theta}{\lambda} = \frac{d\theta}{\lambda} = \frac{d}{\lambda}\frac{y}{L}$ applies to interference from multiple slits. ϕ is the phase difference between waves from successive slits at the point of observation. d is the slit separation. λ is the wavelength. θ is the position on the screen measured as an angle. y is the position on the screen measured as a distance. L is the distance from the slits to the screen.

$\sin\theta = \pm\frac{n\lambda}{d}$ applies to interference from multiple slits. θ is the angular position of the nth order peak. Note that: $\sin\theta = \theta = \pm\frac{n\lambda}{d}$ for small angles and that $\Delta\theta = \frac{\lambda}{d}$ where $\Delta\theta$ is the angular separation between successive peaks.

$I = 4I_1\cos^2\left(\phi/2\right)$ applies only to the superposition of **2** waves.

DIFFRACTION

$\delta_a = a\sin\theta$ applies to diffraction. δ_a is the path length difference between the top and bottom of the slit of width *a*.

$\frac{\beta}{2\pi} = \ldots$ applies to diffraction. Here β is the phase difference between the waves coming from the top and the bottom of the slit.

$\sin\theta = \pm\frac{m\lambda}{a}$ applies to diffraction. θ is the angular position of the mth order **minimum** caused by diffraction.

INTERFERENCE PLUS DIFFRACTION

$I_1 = I_0 \left\{ \frac{\sin(\beta/2)}{\beta/2} \right\}^2$ gives the shape of the diffraction pattern (the envelope).

$I_N = I_1 \left\{ \frac{\sin(N\phi/2)}{\sin(\phi/2)} \right\}^2$ gives the shape of the interference pattern (the peaks). N is the number of slits.

Note that: $I = I_0 \left\{ \frac{\sin(\beta/2)}{\beta/2} \right\}^2 \left\{ \frac{\sin(N\phi/2)}{\sin(\phi/2)} \right\}^2$ gives the total intensity pattern.

RESOLUTION OF LENSES, GRATINGS, ETC

$\theta_0 = \frac{\lambda}{a}$ is the minimum angular separation of two objects resolvable through a 1D slit of width *a*.

$\theta_0 = 1.22\frac{\lambda}{D}$ is the minimum angular separation of two objects resolvable through a lens or circular aperture of diameter D. α_c can also be taken to mean the minimum resolvable angle.

$\frac{\Delta\lambda_{\min}}{\lambda} = \frac{1}{Nm}$ applies to resolution of two interference peaks through a diffraction grating. $\Delta\lambda$ is the minimum resolvable wavelength difference. N is the number of slits. m is the order of the peak.

MAKING SENSE OF THE EQUATION SHEET
Quantum Physics, Part I

ENERGY & MOMENTUM

$KE_{\max} = eV_{stop} = hf - \Phi = h(f - f_0)$ applies to the photoelectric effect. The maximum kinetic electrons coming off the metal is KE_{max}. V_{stop} is the stopping voltage. hf is the energy of the photon. Φ is the work function of the metal.

Note: Multiplying any voltage V by electric charge e gives energy in eV numerically equal to the voltage.

For example: If $V = 69$ volts, then $eV = e(69 \text{ volts}) = 69 \text{ eV}$.

$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ gives the kinetic energy for any massive particle. Note that a photon is not a massive particle.

$E = pc = hf = \frac{hc}{\lambda} = \frac{1240\text{eV}\cdot\text{nm}}{\lambda}$ gives the energy of a photon. For $\frac{1240\text{eV}\cdot\text{nm}}{\lambda}$ use nanometers for wavelength.

$\lambda = h / p$ applies to both massive particles and photons.

$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$ gives the kinetic energy of any massive particle. $KE = \frac{1.505\text{eV}\cdot\text{nm}^2}{\lambda^2}$ is for electrons.

SCHRODINGERS EQUATION

$-\frac{\hbar}{2m}\frac{\partial^2\Psi}{\partial x^2} + U(x)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$ is the time dependent schrodinger equation. Here capital psi Ψ is a function of x and t.

$\Psi(x,t) = \psi(x)e^{-i\omega t}$ is the time dependent solution to the schrodinger equation. Lowercase psi $\psi(x)$ is solution to the time independent schrodinger equation.

$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi = E\psi$ is the time independent schrodinger equation. E is the energy of the particle.

$\psi^*(x)\psi(x) = |\psi(x)|^2$ is the probability density function, it gives the probability per unit length that the particle can be found at x. The * denotes complex conjugate.

$P_{ab} = \int_a^b |\psi(x)|^2 \, dx$ gives the probability that the particle can be found between x=a and x=b.

$\Psi(x,t) = Ae^{i(kx-\omega t)}$ is the solution to the schrodinger equation for a free particle (the potential energy U(x) is zero). Note that $\hbar\omega = \frac{\hbar^2 k^2}{2m} = E$, the energy of the particle.

$\Delta x \Delta p \geq \hbar$ is the Heisenberg uncertainty principle. The uncertainty in momentum multiplied by the uncertainty in position must be greater than or equal to $\hbar$.

$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$ gives the n[th] state wavefunction for a particle in an infinite square well of length L.

$E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 = \left(\frac{h^2}{8mL^2}\right) n^2 = E_1 n^2$ gives the n[th] state energy for a particle in an infinite square well of length L.

$n\lambda = 2L$ gives the n[th] state wavelength of the wavefunction for a particle in an infinite square well of length L.

MAKING SENSE OF THE EQUATION SHEET
Quantum Physics, Part II

THE FINAL STUFF

$$T \sim e^{-2KL}$$

$$\text{where } K^2 = \frac{2m}{\hbar^2}\left(U_o - E\right)$$

T is the probability that a particle of energy E can tunnel through a potential energy barrier of length L and height U_0.

$$t_0 = \frac{h}{2\left(E_2 - E_1\right)}$$

This equation gives the half-period of the time-dependent wavefunction that results from a superposition of two stationary states.

$$U\left(r\right) = -\frac{\kappa e^2}{r}$$

The "coulomb potential", or in other words, the potential that an electron in a hydrogen atom "feels". e is the electric charge (and here we assume there is a single proton; otherwise it would be e(Ze)). r is the distance to the nucleus. $\kappa = 1/4\pi\varepsilon_0$ is a constant.

$$\psi\left(x, y, z\right) = \sqrt{\frac{8}{abc}}\sin\left(\frac{n_1\pi}{a}x\right)\sin\left(\frac{n_2\pi}{b}y\right)\sin\left(\frac{n_3\pi}{c}z\right)$$

This is the wavefunction for a particle in an 3-dimensional infinite square well of lengths a, b, c, in the x, y, z directions respectively. n1, n2, and n3 are independent of each other, but must be >1.

$$E\left(n_1, n_2, n_3\right) = \frac{h^2}{8m}\left(\frac{n_1^2}{a^2} + \frac{n_2^2}{b^2} + \frac{n_3^2}{c^2}\right)$$

Allowed energies for the particle in 3D infinite square well.

$$\psi_{1s}\left(r, \theta, \phi\right) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

The ground state wavefunction for the electron in the hydrogen atom. a_0 is the bohr radius.

$$E_n = \frac{-13.6 \text{ eV}}{n^2}$$: Energy levels for the hydrogen atom.

$$E_n = -13.6\text{ eV}\ \frac{Z^2}{n^2}$$

Energy levels for an electron subject to Z positive charges. Note that Z=1 gives the hydrogen equation.

$$P(r)\,dr = 4\pi r^2 \left|\psi(r)\right|^2 dr$$

You probably won't have to use this equation. What it means it that $P(r)$ probability per unit of radial distance is equal to $4\pi r^2 \left|\psi(r)\right|^2$. To find probability over a whole range of r, integrate with respect to r.

$$\psi_{nlm}(r,\theta,\phi) = \mathrm{R}_{nl}(r)\,Y_{lm}(\theta,\phi)$$

General form of hydrogen wavefunctions. R is the radial wavefunction and Y is the spherical harmonic. They are independent of each other.

$$L^2 = l(l+1)\hbar^2$$

Very important. L is total angular momentum. *l* is the familiar quantum number.

$$L_z = m\hbar$$

Also important. Angular momentum in z direction is proportional to *m* quantum number.

$$Y_{00}, Y_{1\pm1}, Y_{10} = \ldots$$

The spherical harmonics for $l=0$ and $l=1$.

$$U = -\mu B$$

Potential energy of a particle in a magnetic field is equal to magnetic moment times field strength.

$F_z = -\mu_z \frac{dB_z}{dz}$: Follows directly from above. Take derivate w.r.t z.

$\mu_z = -\frac{e}{m_e} S_z$ Magnetic moment of electron. e is electric charge. m_e is mass. S_z is the spin of the electron in the z direction.

$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$: energy levels of the harmonic oscillator.

$S^2 = s(s+1)\hbar^2$: *S* is the spin angular momentum. *s* is the spin quantum number.

$S_z = m_s\hbar$: S_z is the z component of the spin angular momentum. m_s is another quantum number related to spin *s*, just like m_l relates to *l*.