Useful equations

Physical constants

speed of light	c	2.998×10^{8} m/s
Planck constant	h	6.626×10^{-34} J s
		4.135×10^{-15} eV s
	$\hbar$	1.054×10^{-34} J s
		0.658×10^{-15} eV s
electron volt	eV	1.602×10^{-19} J
electron charge	e	1.602×10^{-19} C
Bohr radius	a_0	0.05292 nm
electron mass	m_e	9.109×10^{-31} kg
		0.511 MeV/c^2
proton mass	m_p	1.673×10^{-27} kg
		938.3 MeV/c^2
neutron mass	m_n	1.675×10^{-27}kg
		939.6 MeV/c^2
hydrogen mass	m_H	1.674×10^{-27} kg

Waves

Symbol	Name	SI units
k	Wave number	m^{-1}
λ	Wavelength	m
ω	Angular frequency	rad/s
ϕ	Phase	radians
T	Period	s
f	Frequency	s^{-1}
I	Intensity	W/m^2
A	Amplitude	$\sqrt{W/m^2}$

$$f = \omega/2\pi = 1/T$$
$$k = 2\pi/\lambda$$
$$I_{avg} = \frac{|A|^2}{2}$$

Intensity of superposition of two waves of equal magnitude

$$I_{\text{total}} = 2A^2\cos^2\left(\frac{kr_1+\phi_1-kr_2-\phi_2}{2}\right)$$

Trigonometric identities

$$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$\cos^2\alpha+\sin^2\alpha = 1$$
$$A^2+B^2+2AB\cos\phi = C^2$$

Complex numbers

$$i = \sqrt{-1}$$
$$e^{i\theta} = \cos(\theta)+i\sin(\theta)$$
$$z = x+iy$$
$$z^* = x-iy$$
$$|z|^2 = zz^* = x^2+y^2$$

Diffraction

$$a\sin\theta_0 = \lambda$$
$$D\sin\theta_0 = 1.22\lambda$$

Photons

$$p = \hbar k = h/\lambda$$
$$E = hf = \hbar\omega$$

Wave functions

$$\rho(x,t) = \Psi(x,t)\Psi^*(x,t)$$
$$P(a<x<b) = \int_a^b \rho(x,t)dx$$
$$\int_{-\infty}^{\infty} \rho(x,t)dx = 1$$

Measurement rule

$$\Psi = a\Psi_1 + b\Psi_2$$
$$P(1) = \frac{|a|^2}{|a|^2+|b|^2}$$

Quantum matter

Wave function of momentum p

$$\Psi(x) = Ae^{ikx}$$
$$p = \hbar k$$

Heisenberg Uncertainty Principle

$$\Delta x \Delta p \geq \hbar/2$$

Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + U(x)\Psi(x) = E\Psi(x)$$

Infinite square well eigenstates

$$\Psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L}) & \text{if } 0<x<L \\ 0 & \text{otherwise,} \end{cases}$$
$$E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

Harmonic oscillator

Ground state

$$\Psi_0(x) = Ae^{-\alpha x^2}$$
$$\alpha = \frac{1}{2\hbar}\sqrt{mk}$$
$$E = \frac{\hbar^2\alpha}{m} = \frac{\hbar}{2m}\sqrt{mk}$$

Spectrum

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega, n = 0,1,2,\ldots$$

Time dependent Schrödinger equation

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t)$$

Time dependence of energy eigenstates

$$\Psi(x,t) = e^{-i\omega_n t}\Psi_n(x)$$
$$E_n = \hbar\omega_n$$

Double wells

$$\Delta E = Ce^{-\kappa\sqrt{mV}L},$$