Useful equations

Physical constants

speed of light	c	2.998×10^{8} m/s
Planck constant	h	6.626×10^{-34} J s
		4.135×10^{-15} eV s
	$\hbar$	1.054×10^{-34} J s
		0.658×10^{-15} eV s
electron volt	eV	1.602×10^{-19} J
electron charge	e	1.602×10^{-19} C
Bohr radius	a_0	0.05292 nm
electron mass	m_e	9.109×10^{-31} kg
		0.511 MeV/c^2
proton mass	m_p	1.673×10^{-27} kg
		938.3 MeV/c^2
neutron mass	m_n	1.675×10^{-27}kg
		939.6 MeV/c^2
hydrogen mass	m_H	1.674×10^{-27} kg

Trigonometric identities

$$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos^2\alpha+\sin^2\alpha = 1$$

$$A^2+B^2+2AB\cos\phi = C^2$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}, \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

Complex numbers

$$i = \sqrt{-1}$$

$$e^{i\theta} = \cos(\theta)+i\sin(\theta)$$

$$z = x+iy$$

$$z^* = x-iy$$

$$|z|^2 = zz^* = x^2+y^2$$

Waves

Symbol	Name	SI units
k	Wave number	m^{-1}
λ	Wavelength	m
ω	Angular frequency	rad/s
ϕ	Phase	radians
T	Period	s
f	Frequency	s^{-1}
I	Intensity	$\mathrm{W/m^2}$
A	Amplitude	$\sqrt{\mathrm{W/m^2}}$

$$f = \omega/2\pi = 1/T$$

$$k = 2\pi/\lambda$$

$$I_{avg} = \frac{|A|^2}{2}$$

Intensity of superposition of two waves of equal magnitude

$$I_{\text{total}} = 2A^2\cos^2\left(\frac{kr_1+\phi_1-kr_2-\phi_2}{2}\right)$$

Diffraction

Double-slit:	$m\lambda = d\sin\theta_{max}$
Single-slit:	$a\sin\theta_0 = \lambda$
Circular aperture:	$D\sin\theta_0 = 1.22\lambda$

Photons

$$p = \hbar k = h/\lambda$$

$$E = hf = \hbar\omega = \frac{1240\text{ eV nm}}{\lambda}$$

$E = pc$ $\quad f\lambda = c$

F = dp/dt (Note that p and F are vector quantities)

Photoelectric effect: KE = hf-[work function]

Wave functions

$$\rho(x,t) = \Psi(x,t)\Psi^*(x,t)$$
$$P(a < x < b) = \int_a^b \rho(x,t)dx$$
$$\int_{-\infty}^{\infty} \rho(x,t)dx = 1$$

Measurement rule

$$\Psi = a\Psi_1 + b\Psi_2 + \dots$$
$$P(1) = \frac{|a|^2}{|a|^2 + |b|^2 + \dots}$$

Quantum matter

Wave function of momentum p

$$\Psi(x) = Ae^{ikx}$$
$$p = \hbar k$$

Heisenberg Uncertainty Principle

$$\Delta x \Delta p \geq \hbar/2$$

Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + U(x)\Psi(x) = E\Psi(x)$$

Infinite square well eigenstates

$$\Psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L}) & \text{if } 0 < x < L \\ 0 & \text{otherwise,} \end{cases}$$
$$E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

Harmonic oscillator eigenstates

Ground state

$$\Psi_0(x) = Ae^{-\alpha x^2}$$
$$\alpha = \frac{1}{2\hbar}\sqrt{mk}$$
$$E = \frac{\hbar^2 \alpha}{m} = \frac{\hbar}{2m}\sqrt{mk}$$

Spectrum

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega, n = 0, 1, 2, \dots$$

Multiple non-interacting electrons

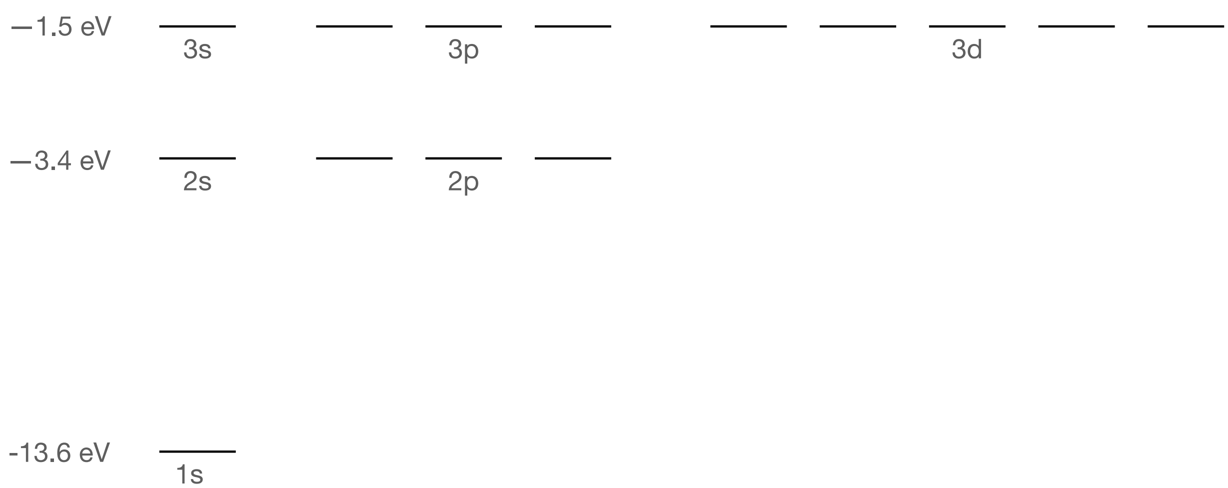
- Ground state: Two electrons per energy level, starting from the bottom.
- Excited states: Other fillings, keeping to the 2 electrons/level rule.
- Energy is the sum of the occupied levels.

Band structure

- Metal: no gap
- Insulator: big gap
- Semiconductor: small gap

Hydrogen energy levels

$$E_n = \frac{-13.6Z^2}{n^2}\ \text{eV}$$



Two-state wave functions

Polarization direction	State
Vertical	Ψ_v
Horizontal	Ψ_h
Diagonal (45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h + \Psi_v)$
Diagonal (-45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h - \Psi_v)$
Circular (right-handed)	$\frac{1}{\sqrt{2}}(\Psi_h + i\Psi_v)$
Circular (left-handed)	$\frac{1}{\sqrt{2}}(\Psi_h - i\Psi_v)$

Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i\downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i\downarrow)$

Measurement rule for a particle with wave function Ψ and a state with definite direction S: $P(S) = |S^* \cdot \Psi|^2$