Last Name: ______________________First Name: __________________Net ID: ___________

Discussion Section: _____________Discussion TA: ___________

Academic Integrity:

The following actions or activities during a University Examination are grounds for disciplinary action up to and including expulsion:

- Giving assistance to or receiving assistance from another student.
- Using unauthorized materials, including unauthorized electronic devices (phones, tablets, smart watches, laptops, etc.).

Turn off and put away all internet-capable electronic devices.

Calculators may not be shared.

Exam Duration:

You have 1.5 hours (90 minutes) to complete this examination.

Examination Type:

This is a Midterm Exam.

This is a closed-book examination. Put away all notes, textbooks, and study aids.

Instructions:

Use only a #2 (HB) pencil to fill in the answer sheet.

1. Name:
    a. Print your **Last Name** in the designated spaces on the answer sheet.
    b. Fill in the corresponding circles below each character in your **Last Name.**
    c. Print your **First Initial** in the designated spaces on the answer sheet.
    d. Fill in the corresponding circle below you **First Initial**.
2. NetID:
    a. Print your **NetID** in the designated spaces on the answer sheet.
    b. Fill in the corresponding circles below each character in your **NetID.**
3. Your UIN:
    a. Print you **UIN** in the **STUDENT NUMBER** spaces on your answer sheet.
    b. Fill in the corresponding circle below each number in your **UIN**.
4. **Sign your full name** on THE **STUDENT SIGNATURE** line on the answer sheet.

5. Print your **Discussion Section** on the **SECTION line** of your answer sheet. **Do not fill in the SECTION box.**
6. Your Exam **Test Form A.** Mark the circle for **A** in the **TEST FORM** box on your answer sheet.
7. Your exam should have **16 exam pages and 2** formula sheets**.** Count the number of pages in your exam booklet now.

Grading Policy:

This exam is worth **129** points. There are

three types of questions:

- **MC5:** *multiple-choice-five-answer questions, each worth 6 points.*
  Credit will be granted as follows:
  (a) If you mark only *one* answer and it is the correct answer, you earn **6** points.
  (b) If you mark *two* answers, one of which is the correct answer, you earn **3** points.
  (c) If you mark *three* answers, one of which is the correct answer, you earn **2** points.
  (d) If you mark no answers, or more than *three*, you earn **0** points.
- **MC3:** *multiple-choice-three-answer questions, each worth 3 points.*
  No partial credit. Mark *only one answer*
  (a) If your marked answer is the correct answer, you earn **3** points.
  (b) If your marked answer is wrong answer or if you mark no answers, you earn **0** points.
- **TF:** *true-false questions, each worth 2 points.*
  No partial credit. Mark *only one answer*.
  (a) If your marked answer is the correct answer, you earn **2** points.
  (b) If your marked answer is wrong or you marked neither answer, you earn **0** points.

The next two questions pertain to the situation described below.

Suppose there are two waves; one with functional form $y_1(x,t) = A_1 \cos(kx + \omega t)$ and another with functional form $y_2(x,t) = A_2 \cos(kx + \omega t + \phi)$. The variables are: A_1 =4, A_2 =3, and ϕ will be varied in the following questions.

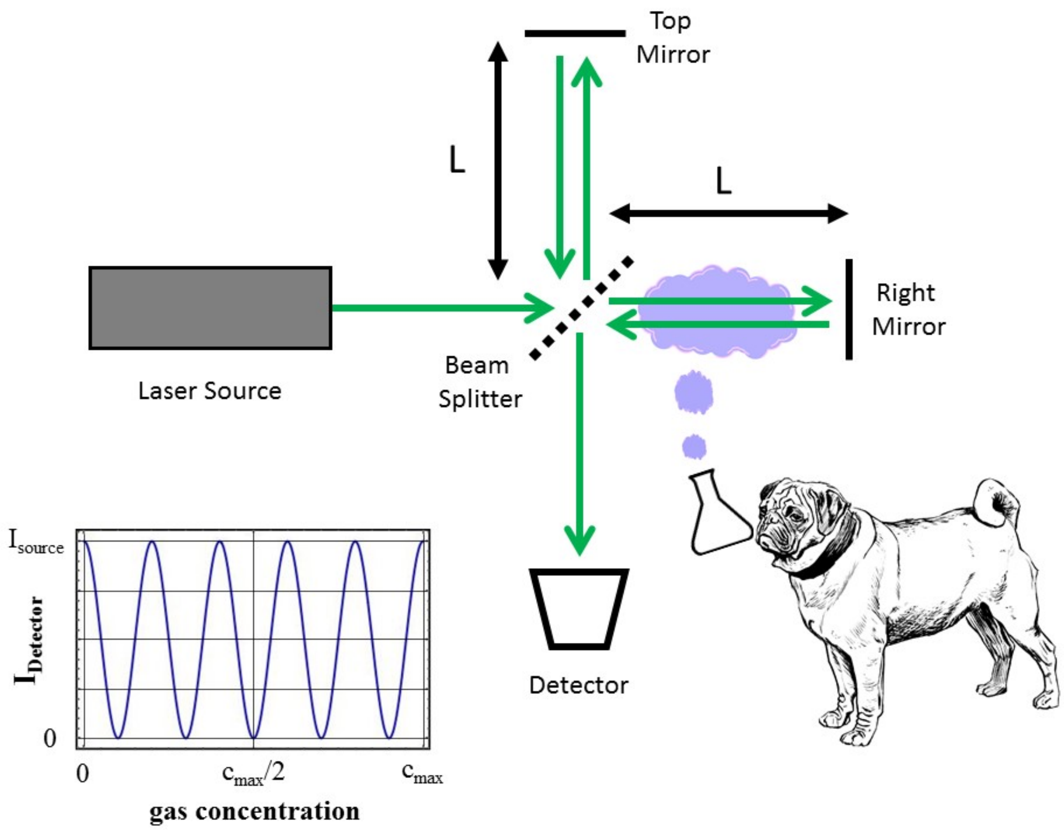
1) What is the amplitude of the combined wave $y_1 + y_2$ if ϕ = 0?

a. 25
b. 5
c. 7
d. 1
e. 49

2) What is the total **intensity** of the combined wave $y_1 + y_2$ if ϕ =$\pi/2$?

a. 1
b. 7
c. 25
d. 5
e. 49

The next two questions pertain to the situation described below.



Consider the Michelson interferometer drawn above. Light from a laser source, with wavelength λ = 532 nm and intensity I_{source}, enters the interferometer from the left. The source light is divided by a 50:50 beam splitter, traverses the two arms (top arm and right arm), and finally is recombined and directed to the intensity detector at bottom. The two arms have identical lengths from beam splitter to mirror, L = 1 m. Initially, with both arms completely empty (i.e., vacuum), all of the input light exits out of the lower port, and an intensity I_{source} is detected.

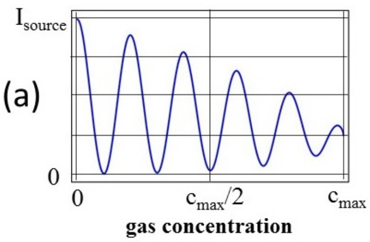
Professor Pug decides to slowly (and uniformly) fill the right arm of the interferometer with an unknown gas. As the concentration of this gas is increased from 0 to c_{max}, Prof. Pug observes that the detected intensity ($I_{Detector}$) goes from a maximum value to a minimum and back to a maximum 5 times (as shown in the inset plot).

3) What is the difference in the value of the index refraction of this gas at maximum concentration, n_{max}, from that of vacuum? In other words, what is the value of n_{max} - 1 ?

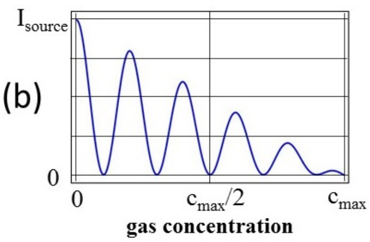
a. 1.33×10^{-6}
b. 6.65×10^{-7}
c. 5.32×10^{-7}
d. 2.66×10^{-6}
e. 9.4×10^{5}

4) Imagine that the unknown gas, in addition to changing the index of refraction in the right arm, also leads to a proportional loss of intensity in that arm with increasing concentration (with 100% intensity loss in the right arm at c_{max} and 50% intensity loss at $c_{max}/2$). In this scenario, which of the following plots (labeled **a-c**) would best describe the dependence of the detected lower port intensity on the gas concentration?

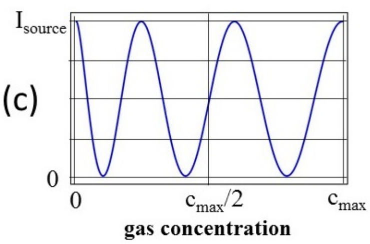
(a)



(b)

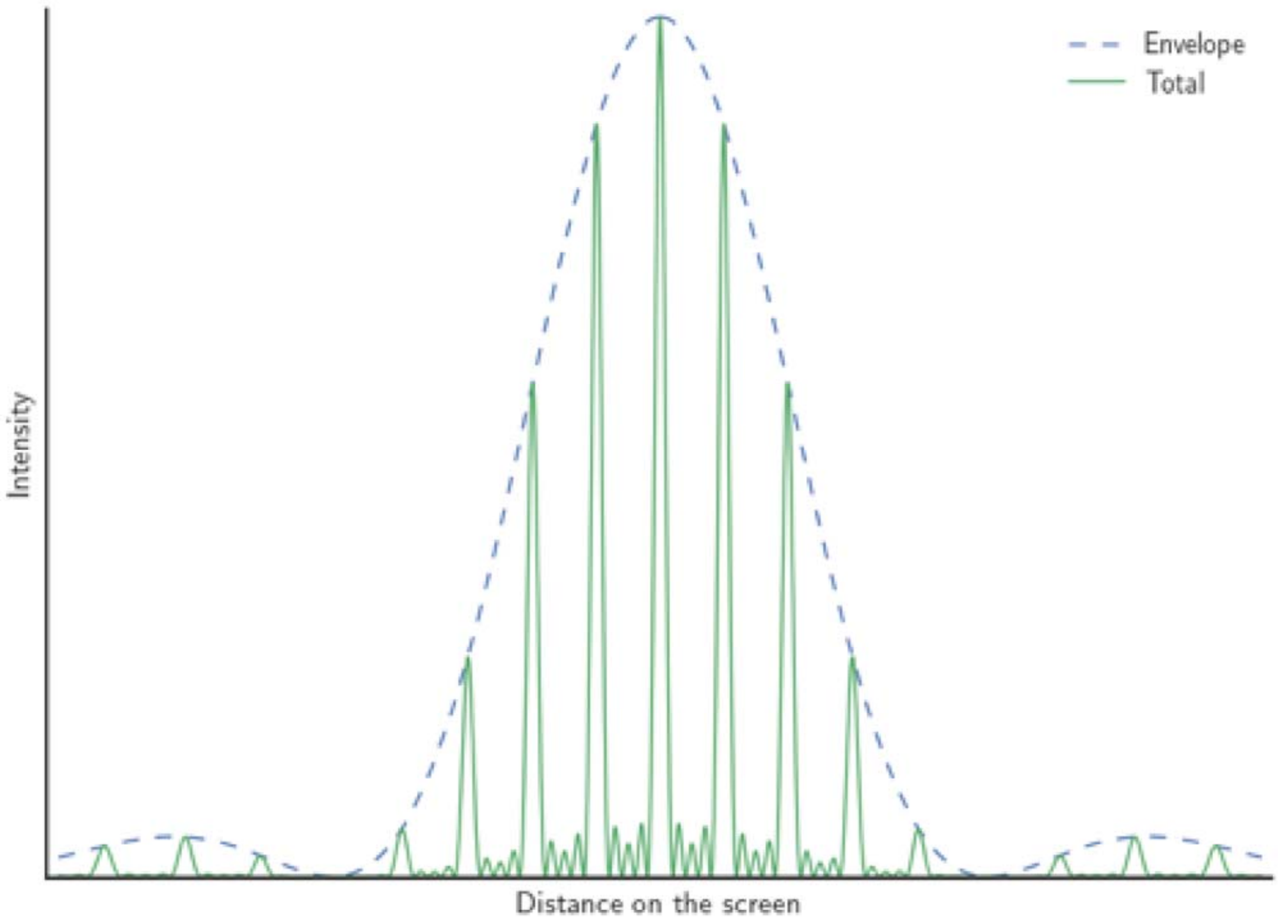


(c)



a. Plot (a)
b. Plot (b)
c. Plot (c)

The next three questions pertain to the situation described below.



A screen is set up 20 m away from a diffraction grating, with 5 slits 20 μm wide and with a distance 100 μm between the centers of the slits. It is irradiated with light of wavelength 415 nm. It produces a pattern similar to that pictured.

5) What is the distance between the maximum and the first zero of the diffraction envelope?

a. 5 m
b. 0.207 m
c. 0.415 m
d. 0.0208 m
e. 8.3×10^{-8} m

6) How wide are the interference peaks? That is, what's the distance from the central peak to the first zero in the total signal?

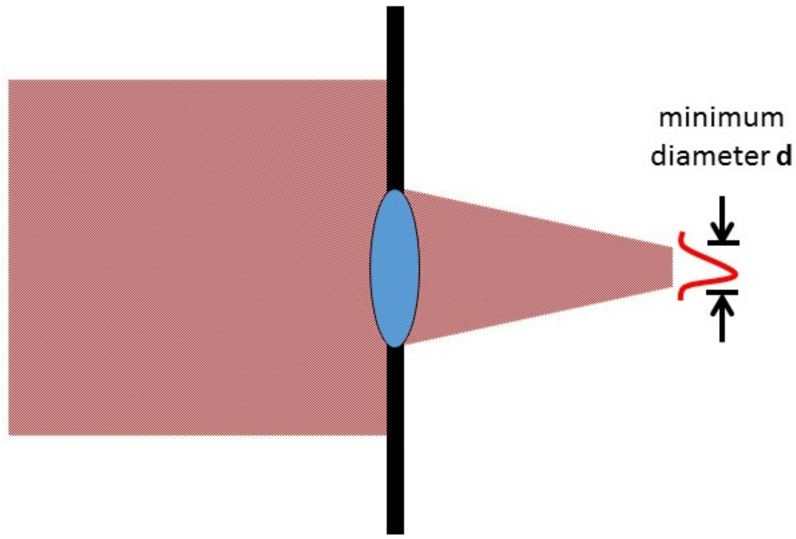
a. 8.3×10^{-4} m
b. 0.2 m
c. 0.0221 m
d. 0.0166 m
e. 8.3×10^{-6} m

7) What is the minimal change in wavelength that we could detect when looking at the first order maximum of the pattern?

a. 4.15×10^{-7} m
b. 8.3×10^{-8} m
c. 0.0207 m
d. 4.15×10^{-8} m
e. 1.04×10^{-7} m

The next two questions pertain to the situation described below.

Consider the figure below. A laser beam with diameter D_{beam} = 20 cm and wavelength of λ = 650 nm is directed towards, and "overfills", a smaller spherical lens (circular aperture diameter D_{lens} = 4 cm) having a focal length of 10 cm.



8) What is the smallest size (diameter) of the light beam as it is focused by the spherical lens?

a. 0.793 μm
b. 3.97 μm
c. 1.32 μm
d. 1.63 μm
e. 8.13 μm

9) Which of the following actions would decrease the minimum size of the focused laser beam? NOTE: answer choices **a-e** are found below.

(action #1) - decrease the laser wavelength

(action #2) - make the laser beam smaller, so that its diameter matches that of the lens

(action #3) - increase the laser wavelength

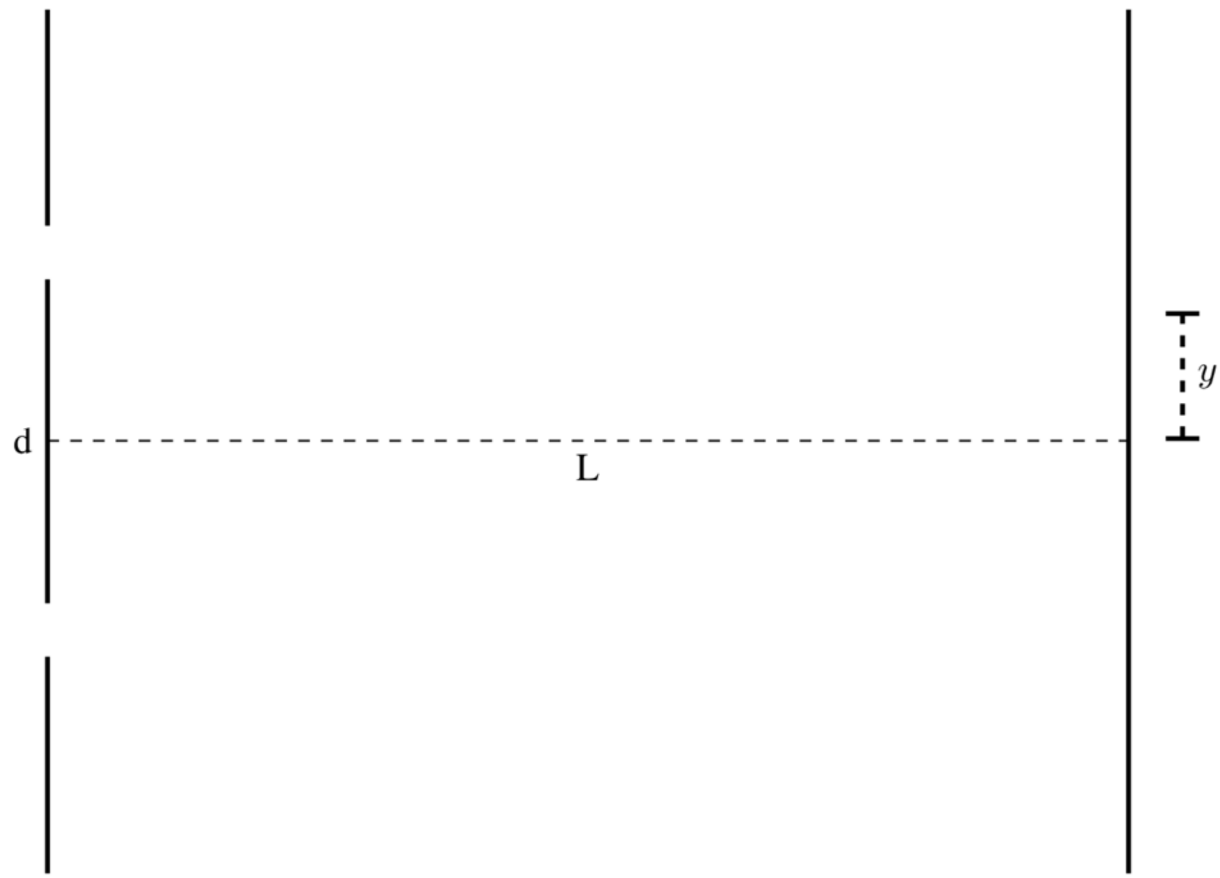
a. action #1 and action #2
b. action #3 only
c. action #2 only
d. action #1 only
e. action #2 and action #3

The mass of the muon is 207 times greater than the mass of the electron. Muons have the same charge as electrons. An electron and a muon are each in the respective ground states of two identical 1-D potential wells. Compare the deBroglie wavelengths of the particles, and the wavelength of a photon needed to excite the particle to the first excited state:

10)

a. $\lambda_{\text{electron deBroglie}} = \lambda_{\text{muon deBroglie}}$, $\lambda_{\text{photon to excite electron}} < \lambda_{\text{photon to excite muon}}$
b. $\lambda_{\text{electron deBroglie}} < \lambda_{\text{muon deBroglie}}$, $\lambda_{\text{photon to excite electron}} > \lambda_{\text{photon to excite muon}}$
c. $\lambda_{\text{electron deBroglie}} < \lambda_{\text{muon deBroglie}}$, $\lambda_{\text{photon to excite electron}} = \lambda_{\text{photon to excite muon}}$
d. $\lambda_{\text{electron deBroglie}} = \lambda_{\text{muon deBroglie}}$, $\lambda_{\text{photon to excite electron}} = \lambda_{\text{photon to excite muon}}$
e. $\lambda_{\text{electron deBroglie}} > \lambda_{\text{muon deBroglie}}$, $\lambda_{\text{photon to excite electron}} = \lambda_{\text{photon to excite muon}}$

The next three questions pertain to the situation described below.



A wave is incident on two small slits as shown in the diagram. You can assume that the source is very far away and so the wave is planar. The diagram is not to scale. There is a detector located a position y measured from the horizontal line crossing the midpoint of the center of the two slits, as shown in the diagram. The slits are spaced 3 m apart and the screen is 50 m away from the slits. The wave is propagating with wavelength 2 m and velocity 340 m/s.

11) What is the frequency of the wave?

a. 170 Hz
b. 680 Hz
c. 338 Hz
d. 342 Hz
e. 0.00588 Hz

12) What is the minimal value of y (greater than zero) for which the intensity is at a maximum?

a. 17.7 m
b. 0.667 m
c. 0.34 m
d. 0.73 m
e. 44.7 m

13) What is the minimal value of y (greater than zero) for which the intensity is at a minimum?

a. 44.7 m
b. 0.73 m
c. 0.34 m
d. 17.7 m
e. 0.667 m

The next two questions pertain to the situation described below.

Monochromatic light with wavelength λ is incident upon a gold plate (work function $\phi = 5.1$ eV). It is observed that electrons are ejected from the surface of the gold plate, traveling at speeds approaching 0.004 c (where c is the speed of light).

14) What is the value of the wavelength λ?

a. 243 nm
b. 1200 nm
c. 405 nm
d. 135 nm
e. 303 nm

15) What is the de Broglie wavelength of electrons ejected at speeds of 0.004 c ?

a. 0.61 nm
b. 303 nm
c. 540 nm

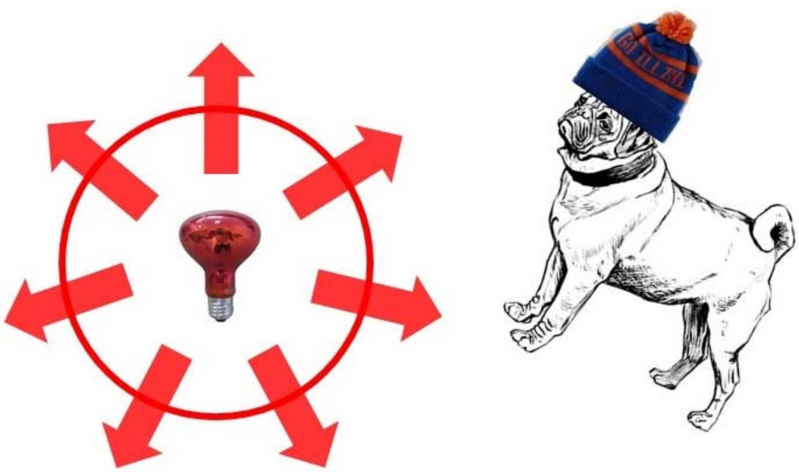
A photon of a wavelength 600 nm passes through a slit of width 10 μm . Initially it is going horizontally but after passing through a slit the photon propagation direction acquires some angular distribution. Use Heisenberg uncertainty to estimate the angular spread of the photon propagation direction.

16) The angular spread (in radians) of the momentum of the photon after passing through the slit is:

a. 0.001
b. 1
c. 0.1
d. 0.01
e. 0.0001

Recall that the surface area of a sphere with radius R is $4 \pi R^2$.

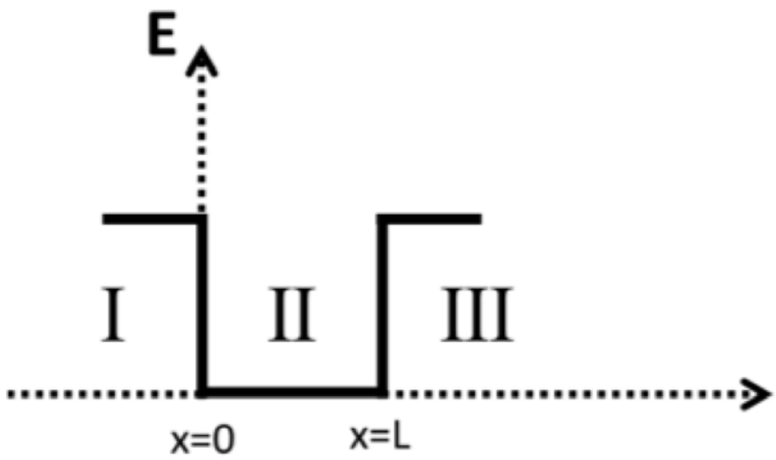
To warm up on a cold winter day in Urbana, Prof. Pug holds his paws (having a combined front surface area of 0.02 m^2) in front of a small 40 Watt infrared lamp (with 100% efficiency), at a distance of 2 meters away. The lamp illuminates uniformly in all directions, as shown in the figure below, and its emission can be approximated as having a common wavelength of 2 μm.



17) How many photons from the lamp hit the front of Prof. Pug's paws every second?

a. 0.026
b. 6.41×10^{17}
c. 4.03×10^{20}
d. 1.01×10^{20}
e. 1.6×10^{17}

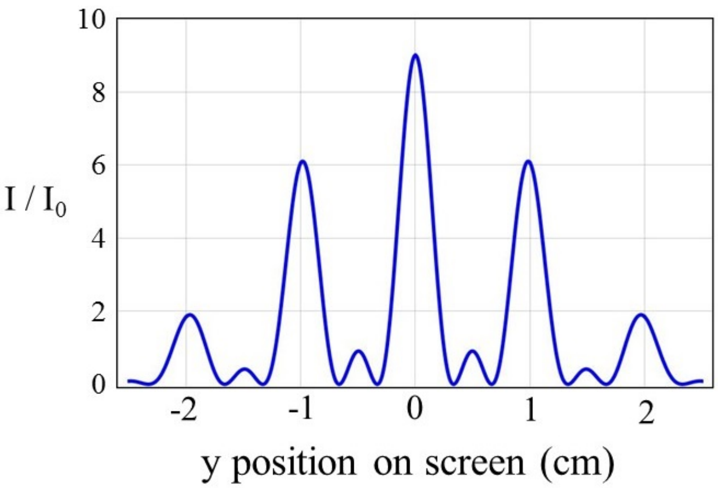
A finite potential well extends from x=0 to x=L as shown in the figure. The well is divided into 3 regions. Region I is to the left of the well, region II is in the well and region III is to the right of the well. We consider a bound state of this well.



18) Compare the probabilities, corresponding to different energy states, to find the particle outside the well, namely at x=11L. The probability for the ground state is denoted P1 and the probability for the first excited state is denoted P2.

a. P1=0 and P2>0
b. P1=0 and P2=0
c. P1 < 0 and P2 > 0
d. P2 > P1
e. P1 > P2

The next two questions pertain to the situation described below.

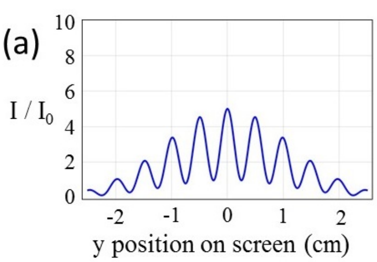


A coherent beam of metastable helium atoms passes through a pattern of three slits with common spacing d. Far away from these slits (distance L >> d, L >> λ, the wavelength of the atoms) the interference pattern shown in the figure above is detected on a phosphor screen. When considered alone, each of these equal-sized slits contributes an "intensity" of atoms I_0 on the detection screen.

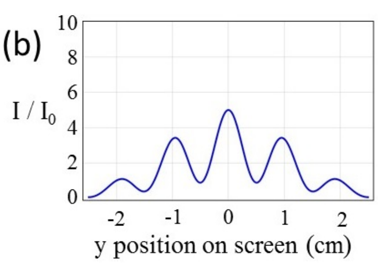
We then place a detector near the central slit. This detector can discern when the atoms pass through this central opening, but cannot distinguish which of the outer slits the atoms pass through for those events.

19) Which of the following plots best reflects the interference pattern that will be measured in the presence of this detector?

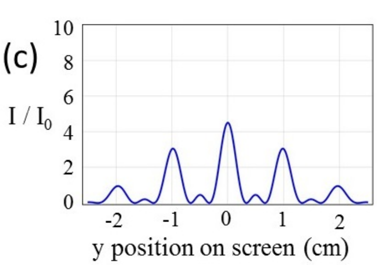
(a)



(b)



(c)



a. Plot (a)
b. Plot (b)
c. Plot (c)

20) This detector works by "scattering" photons from a laser focused to the central slit, and the laser directly opposes the incident velocity of the atoms. For this detector to unambiguously detect an atom passing through the central slit, each atom needs to "scatter" 10 photons of wavelength 1083 nm. Consider that for each "scattering" event, the atom absorbs the momentum of one photon.

Which condition best describes the change in atomic momentum for detection events?

a. $\Delta p \geq 1.22 \times 10^{-26}$ kg m/s
b. $\Delta p \geq 6.12 \times 10^{-27}$ kg m/s
c. $\Delta p \geq 6.12 \times 10^{-28}$ kg m/s

The next three questions pertain to the situation described below.

A one dimensional quantum particle having the same mass as the electron is placed in an infinite potential well 0.9 nm wide. Initially it is in its ground state (n=1), but a laser is used to excite it to its third excited state (n=4).

21) Which of the following wavelengths is the closest to the wavelength of the laser that does this?

a. 581 nm
b. 698 nm
c. 177 nm
d. 505 nm
e. 446 nm

22) Compare the magnitude of $|\Psi_n\,(\text{center})|^2$, relating to the probability of finding the particle in the area just near the center of the well, for the n=4 state versus the n=2 state.

a. $P_{middle}(n=4) > P_{middle}(n=2)$
b. $P_{middle}(n=4) < P_{middle}(n=2)$
c. $P_{middle}(n=4) = P_{middle}(n=2)$

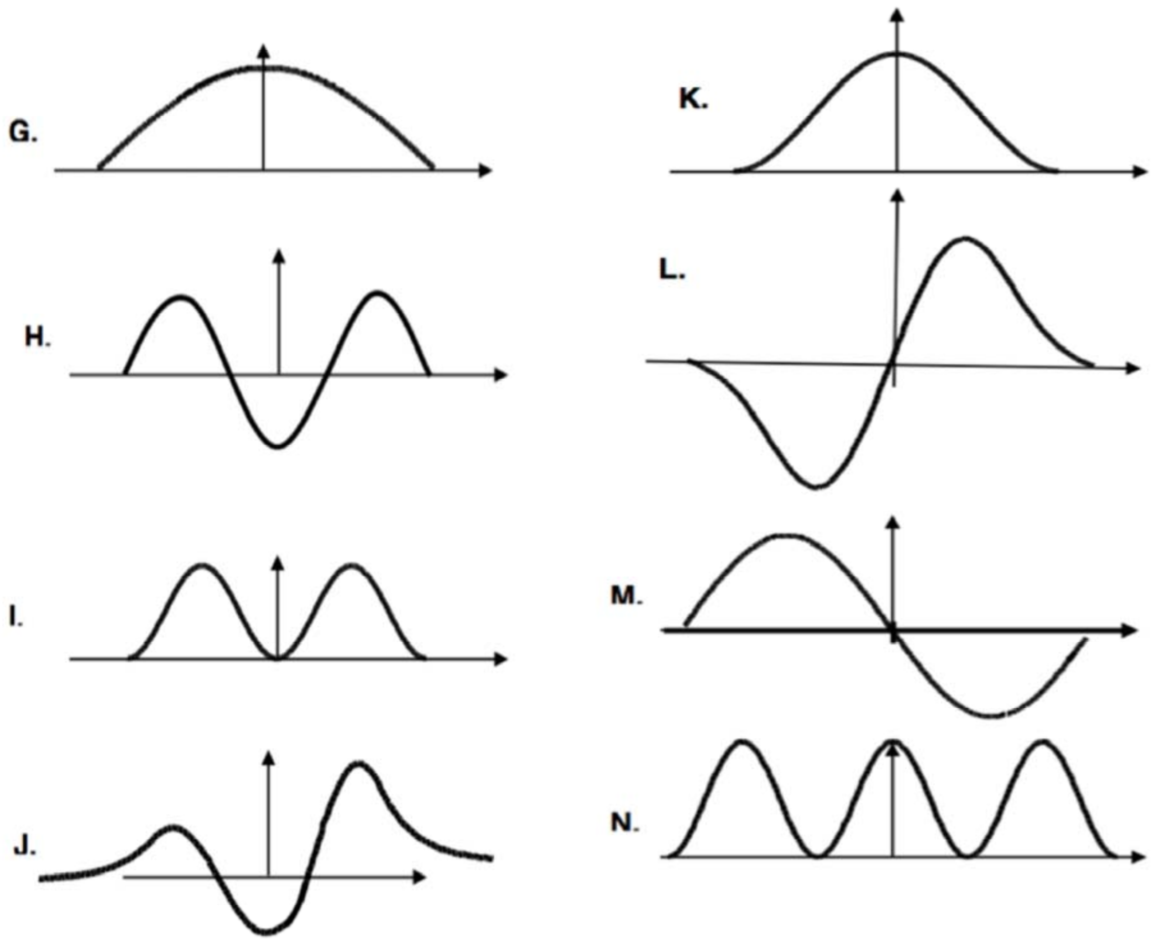
23) Instead of a particle having the same mass as an electron, we now put a particle with the mass of a muon (which is 207 times heavier than the electron) in the infinite potential well. Compare the energy difference between the n=2 state and the n=1 state, ΔE_{12}, for these two cases.

a. $\Delta E_{12}(\text{muon}) < \Delta E_{12}(\text{electron})$
b. $\Delta E_{12}(\text{muon}) = \Delta E_{12}(\text{electron})$
c. $\Delta E_{12}(\text{muon}) > \Delta E_{12}(\text{electron})$

The next two questions pertain to the situation described below.

Consider an electron in a small carbon nanotube with length 1.2 μm. We shine light onto the nanotube, and the electron is excited from its initial energy state to a higher one.

24) Which of the following could describe the electron's wavefunction after the photon absorption? Assume you can approximate the nanotube as a **finite** potential well.



a. I or N
b. All except I, N and K
c. H or M
d. L or J
e. G or K

25) Let's say we happen to actually find the electron in the first 10-nm part of the tube. What can you say about the subsequent likely maximum velocity of the electron (the component along the axis of the nanotube)? The answer, to an order of magnitude is,

a. ~1000 m/s
b. ~0
c. ~10,000 m/s

Physics 214 Common Formulae

SI Prefixes

Power	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^0		
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Physical Data and Conversion Constants

speed of light	$c = 2.998 \times 10^8$ m/s
Planck constant	$h = 6.626 \times 10^{-34}$ J·s $= 4.135 \times 10^{-15}$ eV·s
Planck constant / 2π	$\hbar = 1.054 \times 10^{-34}$ J·s $= 0.658 \times 10^{-15}$ eV·s
electron charge	$e = 1.602 \times 10^{-19}$ C
energy conversion	1 eV $= 1.602 \times 10^{-19}$ J
conversion constant	hc = 1240 eV·nm= 1.986×10^{-25} J-m
useful combination	$h^2/2m_e = 1.505$ eV nm^2
Bohr radius	$a_o = (4\pi\varepsilon_o)\hbar^2 / m_e e^2 = 0.05292$ nm
Rydberg energy	$hcR_\infty = m_e e^4 / 2(4\pi\varepsilon_o)^2\hbar^2 = 13.606$ eV
Coulomb constant	$\kappa = 1/(4\pi\varepsilon_o) = 8.99 \times 10^9$ N·m^2 / C^2
Avagadro constant	$N_A = 6.022 \times 10^{23}$ / mole
electron mass	$m_e = 9.109 \times 10^{-31}$ kg $= 0.511$ MeV/c^2
proton mass	$m_p = 1.673 \times 10^{-27}$ kg $= 938.3$ MeV/c^2
neutron mass	$m_n = 1.675 \times 10^{-27}$ kg $= 939.6$ MeV/c^2
hydrogen atom mass	$m_H = 1.674 \times 10^{-27}$ kg
Electron magnetic moment	$\mu_\varepsilon = 9.2848 \times 10^{-24}$ J/T $= 5.795 \times 10^{-5}$ eV/T
Proton magnetic moment	$\mu_p = 1.4106 \times 10^{-26}$ J/T $= 8.804 \times 10^{-8}$ eV/T

Trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$A_1\sin(\omega t+\phi_1) + A_2\sin(\omega t+\phi_2) = A_3\sin(\omega t+\phi_3)$$

$A^2 + B^2 + 2AB\cos\phi = C^2$ (ϕ here is the *external* angle)

Waves, Superposition

$$k \equiv \frac{2\pi}{\lambda} \quad \omega \equiv 2\pi f \quad T \equiv \frac{1}{f} \quad v = \lambda f = \frac{\omega}{k}$$

General relation for I and A: $I \propto A^2$, $A = A_1 + A_2 + \ldots$

Two sources: $I_{max} = |A_1 + A_2|^2$, $I_{min} = |A_1 - A_2|^2$

Two sources, same I_1: $I = 4I_1\cos^2(\phi/2)$ where $\phi = 2\pi\delta/\lambda$

Interference: Slits, holes, etc.

Far-field path-length difference: $\delta \equiv r_1 - r_2 \approx d\sin\theta$

Phase difference: $\dfrac{\phi}{2\pi} \equiv \dfrac{\delta}{\lambda} = \dfrac{d\sin\theta}{\lambda} \approx \dfrac{d\,\theta}{\lambda} \approx \dfrac{d}{\lambda}\dfrac{y}{L}$ if θ small

Principal maxima: $d\sin\theta_{max} = \pm m\lambda \quad m = 0, 1, 2, \ldots$

N slit: $I_N = I_1\left\{\dfrac{\sin(N\phi/2)}{\sin(\phi/2)}\right\}^2$ where $\phi = 2\pi d\sin\theta/\lambda$

Single slit: $\delta_a = a\sin\theta \quad a\sin\theta_{min} = \pm m\lambda$ with $m = 1, 2, 3\ldots$

$$\frac{\beta}{2\pi} \equiv \frac{\delta_a}{\lambda} = \frac{a\sin\theta}{\lambda} \approx \frac{a\,\theta}{\lambda} \approx \frac{a}{\lambda}\frac{y}{L}$$

Single slit: $I_1 = I_0\left\{\dfrac{\sin(\beta/2)}{\beta/2}\right\}^2$ with $\beta = 2\pi a\sin\theta/\lambda$

slit: $\theta_0 \approx \lambda/a$ *or* hole: $\theta_0 \approx 1.22\lambda/D \approx \alpha_c$

Approx. grating resolution: $\dfrac{\Delta\lambda}{\lambda} \geq \dfrac{1}{Nm}$

Quantum laws, facts….

UNIVERSAL: $p = \hbar k = h/\lambda \qquad E = hf = \hbar\omega$

Light: $E = hf = \hbar\omega = hc/\lambda = pc$

Slow particle: $KE = mv^2/2 = p^2/2m = h^2/2m\lambda^2$

Photoelectric effect: $KE_{max} = eV_{stop} = hf - \Phi$

UNIVERSAL: $\Delta x\,\Delta p_x \geq \hbar \qquad \Delta E\,\Delta t \geq \hbar$

$$\psi^*(x)\psi(x) \equiv |\psi(x)|^2$$

$$P_{ab} = \int_a^b |\psi(x)|^2\,dx, \quad a \leq x \leq b$$

(Slow) particle in fixed potential U:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + U(x)\psi(x) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

Physics 214 Common Formulae

Quantum stationary states (energy eigenstates):

$$\Psi(x,t)=\psi(x)e^{-i\omega t} \qquad \text{where } E=\hbar\omega$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+U(x)\psi(x)=\hbar\omega\,\psi(x)=E\,\psi(x)$$

In 1-D box: $n\lambda=2L$ where $n=1,2...$

$$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \qquad \text{for} \quad 0\le x\le L$$

$$E_n=\frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2=\left(\frac{h^2}{8mL^2}\right)n^2=E_1n^2 \qquad \text{(*last part*)}$$

Box, 3-D:

$$\psi(x,y,z)=\sqrt{\frac{8}{abc}}\sin\left(\frac{n_1\pi}{a}x\right)\sin\left(\frac{n_2\pi}{b}y\right)\sin\left(\frac{n_3\pi}{c}z\right)$$

$$E(n_1,n_2,n_3)=\frac{h^2}{8m}\left(\frac{n_1^2}{a^2}+\frac{n_2^2}{b^2}+\frac{n_3^2}{c^2}\right)$$

Simple Harmonic Oscillator (SHO):

$$E_n=\left(n+\tfrac{1}{2}\right)\hbar\omega \quad \text{where} \quad n=0,1,2...$$

$$\omega=\sqrt{k/m}$$

Free slow particle with definite p:

$$\Psi(x,t)=Ae^{i(kx-\omega t)} \ \textit{with}\ \hbar\omega=\hbar^2k^2/2m$$

H-like atom

potential $U(r)=-\dfrac{\kappa Ze^2}{r}$

$$E_n=\frac{-1}{4\pi\varepsilon_o}\frac{(Ze)^2}{2a_o}\frac{1}{n^2}=-\frac{1}{(4\pi\varepsilon_o)^2}\frac{me^4Z^2}{2\hbar^2n^2}$$

$$=-13.606\,\text{eV}\,\frac{Z^2}{n^2}$$

Ground state: $\psi_{1s}(r,\theta,\phi)=\dfrac{1}{\sqrt{\pi a_o^3}}e^{-r/a_o}$

Radial density for s-state: $P(r)\,dr=4\pi r^2\,|\psi(r)|^2\,dr$

Form of *n, l, m* eigenstate:

$$\psi_{n\ell m}(r,\theta,\phi)=R_{n\ell}(r)\,Y_{\ell m}(\theta,\phi)$$

$$Y_{00}=\frac{1}{\sqrt{4\pi}}, \quad Y_{10}=\sqrt{\frac{3}{4\pi}}\cos\theta,$$

$$Y_{1\pm1}=\mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\varphi}$$

Tunneling

$$T\approx Ge^{-2KL} \qquad \text{where} \qquad G=16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)$$

$$K=\sqrt{\frac{2m}{\hbar^2}\left(U_0-E\right)}=2\pi\sqrt{\frac{2m}{h^2}\left(U_0-E\right)}$$

Angular momentum and magnetism

Orbital: $L_z=m\hbar$ where $m=0,\pm1,\pm2,...\pm\ell$

$L^2=\ell(\ell+1)\hbar^2$ where $\ell=0,1,2,...$

Spin: $S_z=m_s\hbar$ where $m_s=\pm 1/2$

Magnetic energy: $U=-\vec{\mu}\cdot\vec{B}$

Force: $F_z=\mu_z\dfrac{dB_z}{dz}$ where $\mu_z\approx-\dfrac{e}{m_e}S_z$

Atomic orbital filling