Physics 225

Relativity and Math Applications

Fall 2012

Unit 1

Special Relativity: Do you *c* what I *c*?

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[1] The innovative guided-discovery format of these units was developed by Prof. George Gollin for the Phys 212 honors sections he introduced in 2003. The cover-page graphic is by Ryan Bliss (http://digitalblasphemy.com).

Unit 1: Special relativity-- time dilation and Lorentz contraction

The speed of light is finite, and constant.

The consequences of that brief, innocent-looking statement are the weirdest things you'll see in physics before you meet quantum mechanics. Today you're going to derive those consequences yourself, from that one statement.

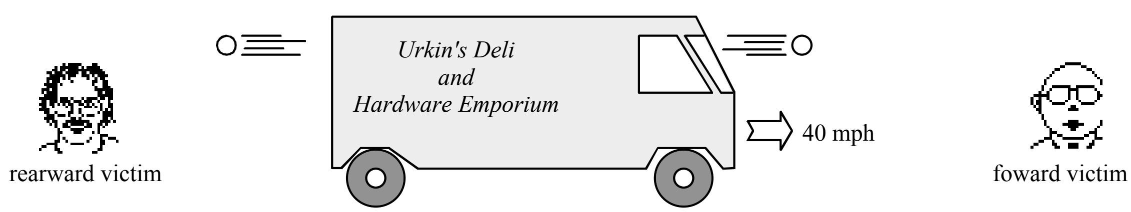
All the strange features of special relativity come from the fact that the speed of light in vacuum is exactly 2.9972458×10^8 meters per second. It doesn't matter whether the source of light moves with respect to the measuring apparatus: any device measuring *c* (the speed of light) will obtain, within experimental error, this value. Nothing else works this way: sound travels at fixed speed with respect to the air, for example. The value 2.9972458×10^8 meters per second really is exact: it serves as a definition, along with some time standard, of the length of one meter. The speed of light is very close to 1 foot per nanosecond, so we'll use that as a convenient approximation over the next few weeks. (This fact is also the one and only useful feature of the "foot", an otherwise abominable unit ☺).

Here's what you'll discover today:

- Events that are simultaneous in one frame of reference are not necessarily simultaneous in another frame.
- The rate at which time passes in a moving frame of reference is slowed down.
- Moving objects become shorter along their direction of motion.

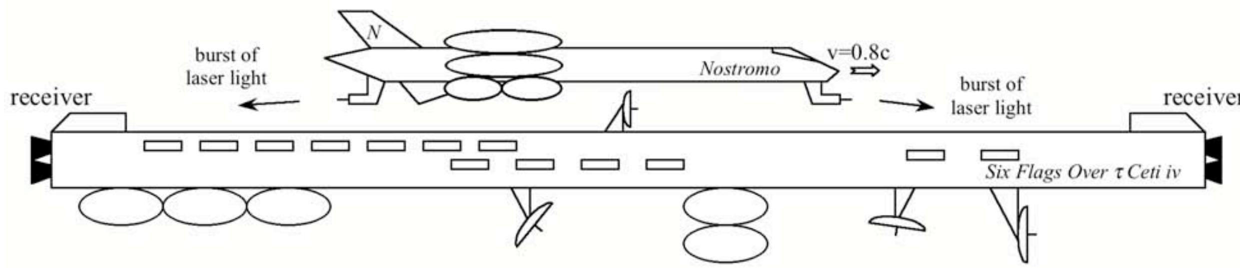
Section 1.1: Constant *c* and simultaneity

Mr. Urkin, sociopathic co-proprietor of Urkin's Deli and Hardware Emporium, is capable of throwing a snowball at 50 miles per hour. While riding as a passenger in a delivery truck traveling 40 miles per hour, he hurls snowballs at stationary pedestrians ahead of, and behind, the moving truck.



(a) Calculate the speeds of the snowballs from the perspectives of the two pedestrians shown in the diagram.

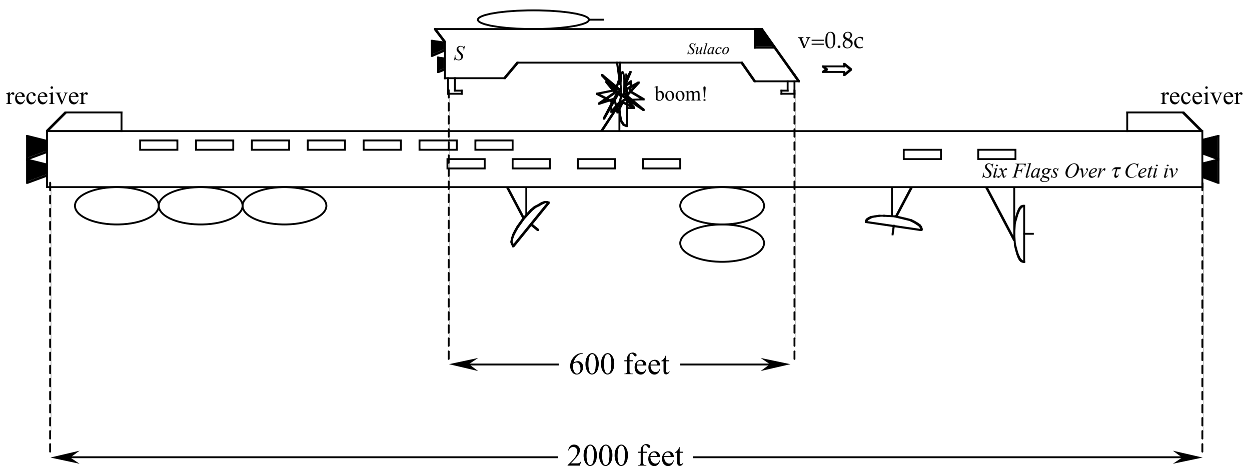
(b) The starship *Nostromo,* drifting in deep space, passes a large interstellar recreational facility which is also adrift, as shown in the figure. Observers on the recreational facility see the *Nostromo* as moving to the right, with speed 0.8 *c*. While the ship is between the ends of the rec facility, it fires its communications lasers to send signal pulses in the forward and aft directions to receivers built into the ends of the rec facility.



According to technicians on the recreation facility, how fast are the two light pulses traveling as they pass through the receiver hardware built into each end of the rec facility?

Note that (slowly moving) snowballs do not behave the same way as bursts of light!

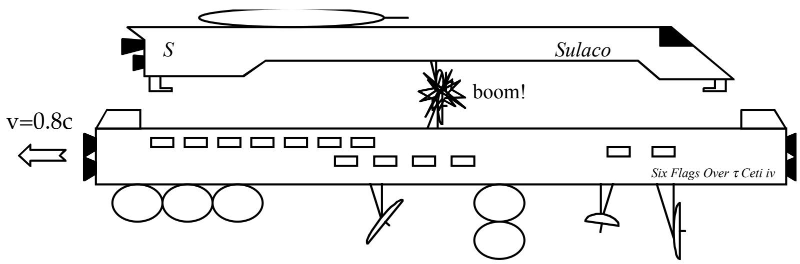
The *Sulaco,* another fast ship, glides past the same drifting interstellar recreational facility. As before, observers on the recreational facility see the ship moving to the right with speed 0.8 *c*. Unfortunately, *Sulaco's* X-band antenna strikes the rec facility's TV parabola, interrupting the final episode of *Pride and Prejudice. Sulaco's* wrecked antenna is midway along the ship's length; the rec facility's ruined parabola is also midway along the facility's length. Observers on the recreation facility see the *Sulaco's* length as 600 feet. The rec facility's length, according to the manufacturer, is 2000 feet.



(c) According to technicians on the recreation facility, when does the flash of light from the collision reach the rec facility's right- and left-receivers (assume the flash occurs at $t = 0$)?

(d) According to technicians on the recreation facility, when does light from the collision reach *Sulaco's* aft periscope (located at the rear of the ship)? When does light reach the forward periscope?
Hint: you should get two *different* values this time.

From the perspective of the *Sulaco,* the collision happens because the incompetently-piloted recreational facility smashes into *Sulaco's* antenna as it lumbers past at 0.8 *c*, moving to the left, as shown in the figure.



(e) According to cosmonauts on the *Sulaco*, does the flash of light from the collision reach the *Sulaco's* forward and aft periscopes simultaneously?

(f) According to cosmonauts on the *Sulaco*, does the flash of light from the collision reach the rec facility's right- and left-receivers simultaneously?

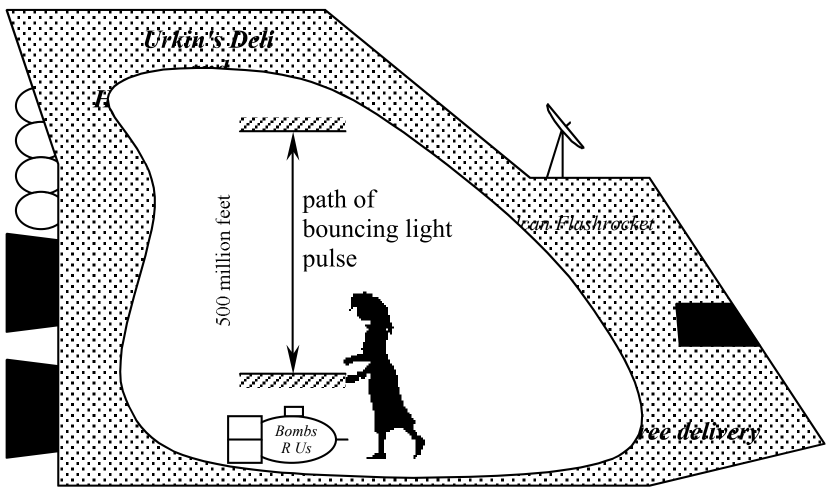
(g) Now imagine we use the receipt of the light flash as a way to *synchronize the clocks* on the rec facility: we set a pair of stopped clocks, one at each end of the rec facility, to $t = +1000$ nsec. When the light flash arrives at the left end, we start the left clock ticking. When the light flash arrives at the right end, we start the right clock ticking. How do the clocks look from the two frames of reference (recreation facility's and *Sulaco's*)? We're just looking for a qualitative answer here, not a calculation.

Discovery #1: **Loss of Simultaneity**

You've just discovered the first, bizarre consequence of special relativity: loss of simultaneity. Specifically, events which occur at the same time in one frame, but at different locations, *do not* generally occur at the same time *in a moving frame*!

Section 1.2: A light clock and time dilation

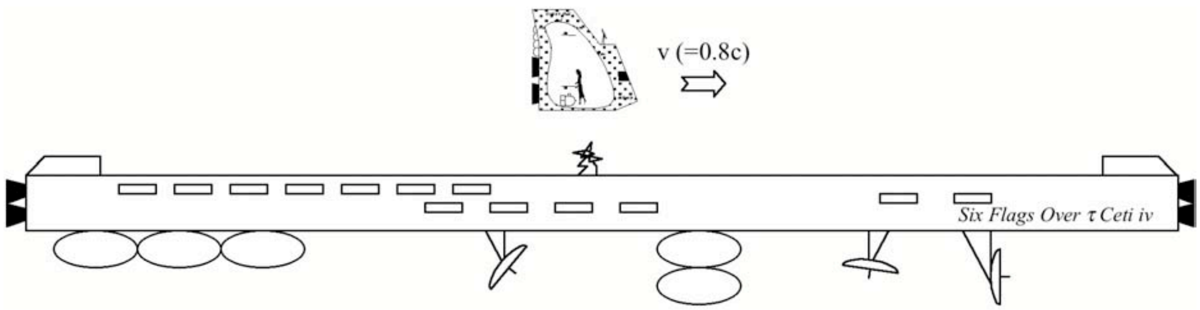
Mrs. Urkin, the irritable, but surprisingly fit co-proprietress of Urkin's D&HE, drifts in deep space aboard the gigantic starship *Vulcan Flashrocket*. Since she realizes that the speed of light is approximately one foot per nanosecond, Mrs. Urkin constructs a clock using two mirrors, separated by a distance of 500 million feet, and a bouncing light pulse. The pulse is of brief duration-- the laser that produced it fired for only a nanosecond-- and it bounces between the mirrors without attenuation.



As shown in the figure, she holds the lower mirror in one hand, moving it briefly into place to reflect the descending light beam. After each successful reflection she removes the mirror. Because Mrs. Urkin knows her resting pulse rate, she has constructed the device so that she will need to move the mirror into position immediately after each heart beat. It is fortunate that she does not suffer from cardiac arrhythmia since the bomb stored below the light clock will detonate if its optical sensor is struck by the light beam.

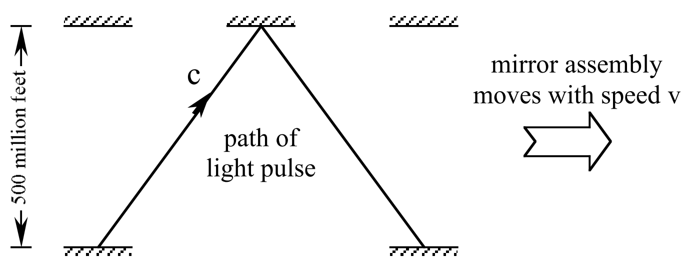
(a) What is Mrs. Urkin's pulse rate?

The *Flashrocket* is passed by that hazardous interstellar recreational facility while Mrs. Urkin is occupied with her light clock. Fortunately, no collision takes place. From the perspective (the "frame of reference") of technicians on the rec facility, the *Flashrocket* is seen to glide past the facility at speed v, as shown in the figure below.



(b) As seen from the rec facility's perspective, does Mrs. Urkin's bomb detonate?

As seen from the recreational facility, Mrs. Urkin's light pulse travels along a diagonal path, as shown below. Keep in mind that the *Flashrocket* is seen to move with speed *v*, and that the light pulse is seen to travel one foot per nanosecond!



(c) How long does it take for the light pulse to make one bottom-to-top-to-bottom trip, according to the observers on the rec facility? Express your answer in terms of *v*, *c*, and the vertical distance *D* = 500 million feet between the mirrors.

(d) If we define Δt to be the time required for one bottom-to-top-to-bottom light pulse cycle in Mrs. Urkin's frame of reference, and $\Delta t'$ to be the time required for one cycle according to observers on the recreational facility, what is the ratio $\Delta t' / \Delta t$? Express your result in terms of v and c.

(e) If $v = 0.8\ c$, what is Mrs. Urkin's pulse rate according to observers on the recreational facility?

Discovery #2: Time Dilation

- To Mrs. Urkin, everything seems fine. To observers in the other frame, Mrs. Urkin is *doing everything slowly* → time is "dilated" (slowed down) in a moving frame.
- This *must* be true if something (light) exists which has the same velocity in all frames regardless of any source-observer relative motion → you derived the effect using only that one assumption!
- All systems used to measure time are affected the same way (heart beat, light clocks, mechanical clocks,...)

It's time for some new notation. The dimensionless variable

$$\boxed{\beta \equiv v/c}$$

is used constantly in relativity to describe speeds close to the speed of light. Naturally, β is always less than 1. The factor $1/\sqrt{1-(v^2/c^2)}$ also appears everywhere in relativity so it, too, gets its own symbol:

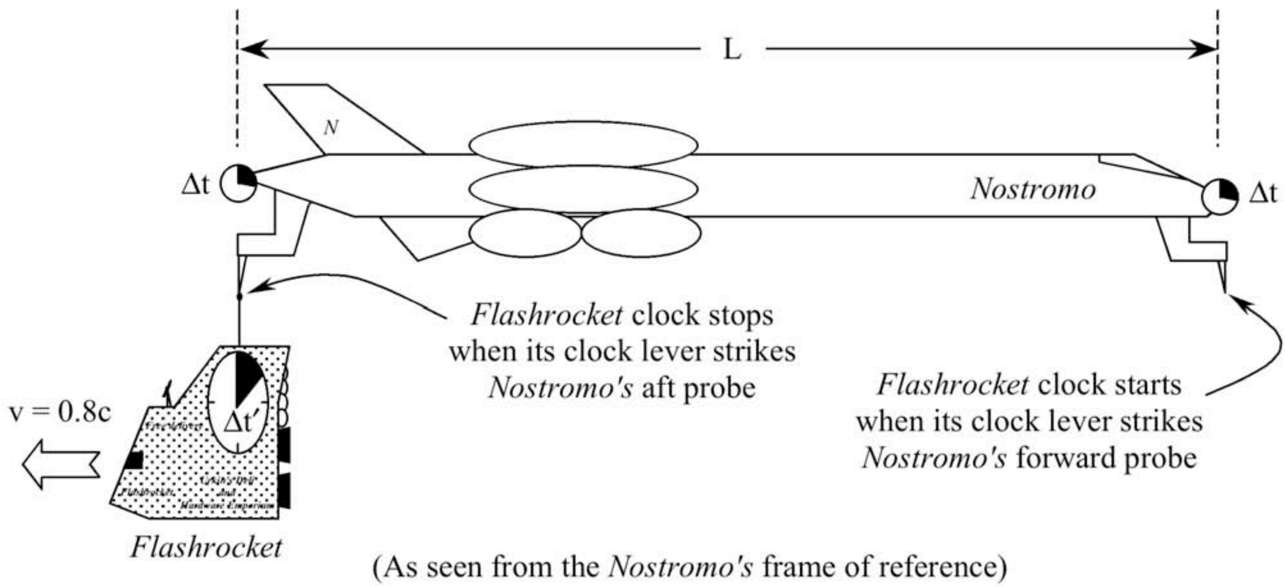
$$\boxed{\gamma \equiv 1/\sqrt{1-\beta^2}}$$

This "**gamma factor**" is always *greater* than 1. It is precisely the factor by which **time is** "**dilated**" in a moving frame.

Section 1.3: Measuring the length of a moving object using a single clock

Floating comfortably aboard the *Vulcan Flashrocket,* Mrs. Urkin measures the length of the *Nostromo* by timing how long it takes for the *Nostromo* to coast past her position, moving to the right at speed v.

Before we study Mrs. Urkin's measurement, let's head to the *Nostromo* and see what its crew are doing. They see the *Flashrocket* gliding past with speed v, and decide to use this opportunity to measure the length of their own ship (let's call it L). They measure Mrs. Urkin's velocity to be $v = 0.8$ c, then they time how long it takes the *Flashrocket* to fly the full length of the *Nostromo*. Let's call the time interval they measure Δt.

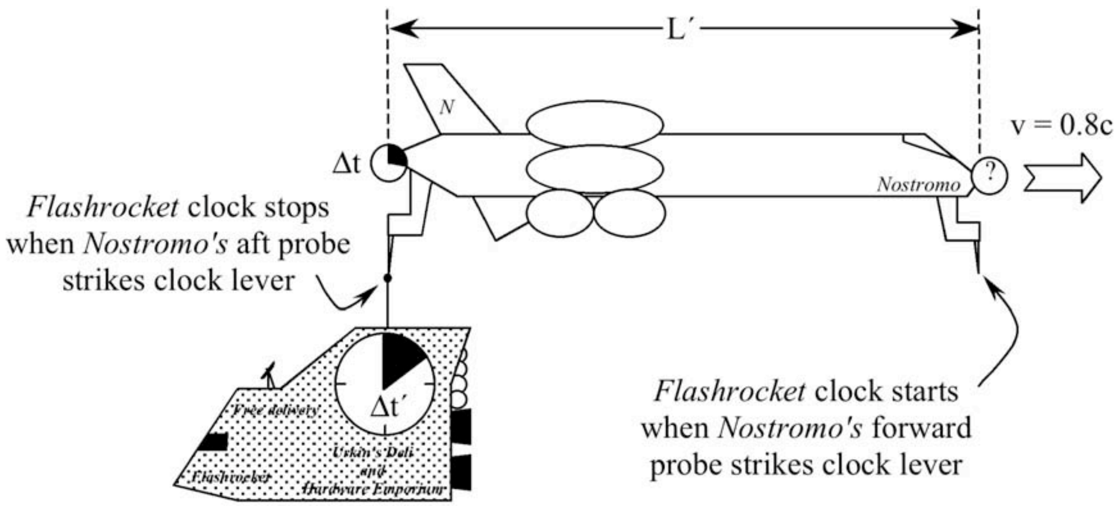


(As seen from the *Nostromo's* frame of reference)

(a) What is L in terms of Δt? (L is the *Nostromo's* rest length. This is a ridiculously easy problem!)

(b) As the *Nostromo* crew watches the *Flashrocket* gliding by, they naturally see that Mrs. Urkin's clock is ticking slowly. While their own clocks register a time interval of Δt, the hands on Mrs. Urkin's clock change by only $\Delta t'$. What is the ratio $\Delta t' / \Delta t$? Express your answer in terms of the gamma factor $\gamma \equiv 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(v/c)^2}$, and evaluate this factor numerically.
(Be careful … do use your result from the previous page, but think physically first – the "primes" *may* have changed hands …)

Mrs. Urkin's also decides to use this opportunity to measure the length of the Nostromo. She uses the same technique: she knows how fast the Nostromo is moving, so she will simply time how long it takes her to travel from one end to the other. In her rest frame, the measurement looks as shown in the figure. Her clock is started the probe mounted on the *Nostromo's* forward end and stopped by the probe on its tail. The interval she records is $\Delta t'$, which you determined in the previous question.



(As seen from the *Flashrocket's* frame of reference)

(c) Define L' to be the *Nostromo* length measured by Mrs. Urkin; the time interval which passed on the *Flashrocket* clock is $\Delta t'$. In terms of v and $\Delta t'$, what is the value she obtains for L'? (Clearly, we have another difficult problem here!)

(d) What is the ratio L' / L ?

Discovery #3: Length Contraction (aka "Lorentz Contraction")

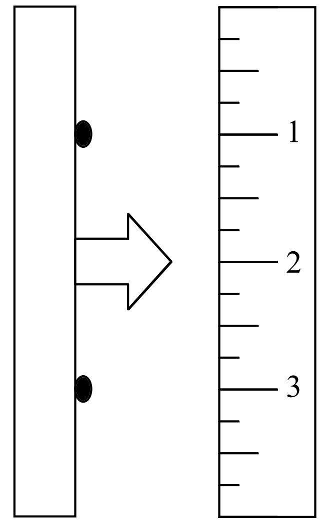
All methods of performing a length measurement (tape measure, timing how long it takes a moving object to coast past a fixed point, ...) will yield consistent results in *one* frame of reference. Since "timing a moving object" is one of those methods, relativistic *time dilation* necessarily implies *length contraction* → To wit:

- To a static observer, objects *become shorter* when they move faster!
- This "**length contraction**" factor is precisely $1/\gamma$.

Section 1.4: Is there any "transverse" contraction?

You have just discovered that lengths *parallel* to their direction of motion are contracted (shortened) in the reference frame of a stationary observer. What about lengths *perpendicular* to their direction of motion?

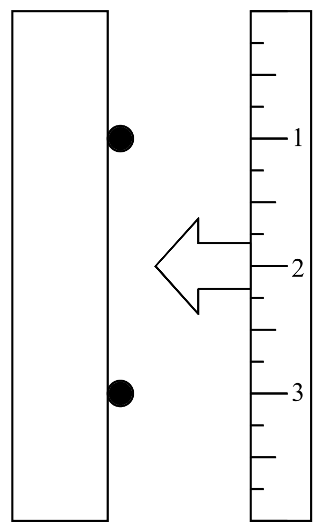
Consider a long, vertical stick that carries a pair of paint balls and is moving to the right. As shown in the figure, the paint balls are vertically separated while their motion is horizontal. The stick slams into a stationary, vertical ruler, and the paint balls make marks at their points of impact.



paint balls on moving stick

(a) How far apart are the paint marks left on the ruler?

Imagine that a second observer views the <u>very same</u> ruler-stick collision, but from a frame in which the stick is at rest and the ruler is in motion.



paint balls on stationary stick

(b) After the collision, where will the paint marks on the ruler be according to this second observer? (Remember, it's the *exact same ruler* as in the previous question …)

(c) What conclusion can you draw concerning lengths that are <u>perpendicular</u> to their direction of motion relative to an observer?

Your stick-figure drawing:

(d) Let's put all this together. You are standing on a sidewalk and a bicyclist is racing toward you at about half the speed of light. What does the bicyclist look like compared with when he is standing still? Smaller, bigger, narrower, longer, taller, shorter … ? Make a stick-figure drawing of the zooming cyclist, then flip the page to see a famous physicist's sketch of this unlikely but interesting situation. ☺

Summary

- The variables β and γ are the ABC's of Special Relativity: $\beta \equiv v / c$ and $\gamma \equiv 1 / \sqrt{1-\beta^2}$
  β is always *less* than 1, while the "gamma factor" γ is always *greater* than 1.
- **Time** is messed up: moving clocks take longer per tick by a factor of γ. And all time references are affected identically: light clocks, heartbeats, etc … This effect is called **time dilation**.
- **Length** is messed up: moving objects are shorter by a factor of $1/\gamma$, but *only* along their direction of motion. This effect is called **Lorentz contraction**.
- **Simultaneity** is messed up: clocks at the front of a moving object read earlier times that clocks at the back. In other words, events which occur at the same time, but at different positions, in one frame do *not* occur simultaneously in a moving frame. This effect is called **loss of simultaneity**.
- These conclusions are valid if anything in nature (light, neutrinos, gravitons,...) is found to move with a constant velocity that is *independent* of source-observer motion.
- Some things don't change when one switches frames of reference (e.g. bomb did/didn't explode).

Optional reading, just for fun:

Mr. Tompkins in Wonderland, George Gamow. In 1940, George Gamow (one of the architects of the Quantum Revolution) wrote this wonderful little book, where he described the dreams of a certain Mr. Tompkins who falls asleep during a lecture on Relativity. Mr. Tompkins dreams that he is in a world where the speed of light is 20 miles per hour and the bizarre consequences of relativity are obvious indeed. "Mr. Tompkins" is a brilliant attempt to explain relativity in an intuitive way, and you'd be hard pressed to find a physicist who has not encountered it. It's been reprinted at least 16 times. *Mr. Tompkins in Paperback* also includes George Gamow's second offering of this type, "Mr. Tompkins Explores the Atom", and both volumes have been recently updated by Russell Stannard in *The NEW World of Mr. Tompkins*.

Einstein's Dreams, Alan Lightman. Warner Books (paperback), 179 pages, 1994. Comments from Amazon.com: "The book takes flight when Einstein takes to his bed and we share his dreams, 30 little fables about places where time behaves quite differently. In one world, time is circular; in another a man is occasionally plucked from the present and deposited in the past: "He is agonized. For if he makes the slightest alteration in anything, he may destroy the future ... he is forced to witness events without being part of them ... an inert gas, a ghost ... an exile of time."

Einstein's Bridge, John Cramer. Avon Books (paperback), 1998. A high energy physics experiment at the SSC lab in Texas opens a channel into another universe. There are good things and bad things lurking out there, and both find their way through the portal. I suppose this really is about a quantum version of General Relativity, rather than Special Relativity, but it was really fun to read.