Fall 2015 – Physics 225 Final Exam
Friday, December 11 (7:00 pm – 10:00 pm)

This is a closed book exam. You may not use any notes, calculators, or other electronic devices that are not required for health reasons. Everything you want graded must be written in your answer booklet. You have **3 hours** to work the problems.

At the beginning of the exam:

- Write your **name** and **netid** on your **answer booklet(s)** and **exam booklet.**
- Put away all calculators and cell phones, and make sure your cell phones are **off**.

During the exam:

- **Show your work** and/or reasoning. **Answers with no work or explanation get 0 points.**
- **Don't write long essays** explaining your reasoning. We only need to see enough work to show that you understand what you're doing and not just guessing.
- **Attempt all the questions!** If you get stuck, move on to the next question and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple. All question parts on this exam are independent, i.e. you can get full points on any part even if your answers to all the other parts are incorrect.
- It is **fine** to leave answers as **radicals or irreducible fractions** (e.g. $10\sqrt{3+\pi}$ or 5/7).
- Specify the **units** for all **numerical answers**.
- Partial credit will be given for incorrect answers provided that the work is legible and understandable and some of it is correct. If you think you've made a mistake but can't find it, **explain what you think is wrong**: you may well get partial credit for *noticing* your error.

> Throughout this exam, feel free to **drop factors of *c*** during your calculations if you feel comfortable doing so. However, you must **put the *c*'s back correctly in your final answers** (in units and/or expressions) or points will be deducted.

When you're done with the exam:

- **TURN IN EVERYTHING**: your **answer booklet(s)**, **exam questions**, and **formula sheets.**

Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.

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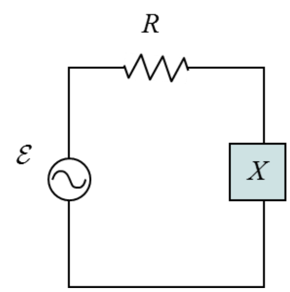
Please remember: No Work = No Points!

You must provide ***some*** **explanation** – a few words or a labeled sketch – for even the simplest questions so I know you're not just guessing!

Problem 1 [12 points] – The black box

Consider the circuit shown below, containing a resistor, R = 2 Ω, in series with a black box, X, whose contents is unknown. You hook up a driving voltage, $\varepsilon(t) = 20\cos(\omega t)\ V$, and investigate the effect of different driving frequencies, ω. Your investigations reveal:

1. $\tilde{Z} \to R$ as $\omega \to 0$,
2. $\tilde{Z} \to R$ as $\omega \to \infty$,
3. $\tilde{Z} = 5 + i5\ \Omega$ for $\omega = 120\ rad/s$.



where $\tilde{Z}$ is defined to be the complex impedance for the *entire* circuit (resistor plus box).

(a) What is the simplest circuit which could be in the box and result in the above observations? Draw a picture showing the circuit, and explain how you know your answer is correct. You don't need to determine the exact value of resistances and impedances of your individual circuit elements, just how they are hooked up.

(b) For $\omega = 120\ rad/s$, what the phase of the current in the circuit relative to the driving voltage? What is the maximum instantaneous current, $I_{\max}$?

Problem 2 [12 points] – Deep space explorer

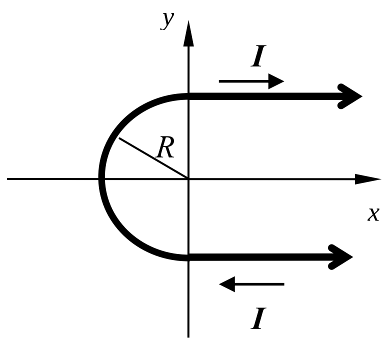
The Department of Unlikely Developments at NASA unveils its plans for a new deep space explorer, which they suggest will leave Earth with an amazing speed, $v = \frac{c}{2}$, and remain at constant velocity thereafter. The explorer is designed to send reports back to Earth via radio signal once an hour, according to its own internal clocks.

(a) If the explorer sends the first message one hour after launch (in its own reference frame), when would NASA engineers receive the n^{th} report from the explorer, in the reference frame of Earth? Define the launch of the explorer as (c*t*, *x*) = (0,0) in both reference frames.

(b) NASA wants to receive the messages at a frequency $f = 100\ MHz$. At what frequency must they program the explorer to transmit?

Problem 3 [14 points] – A current "U-turn"

Consider the two semi-infinite wires, situated in the *xy*-plane, and connected by a semi-circle of radius, *R*. A current, I, flows in from $+\infty$ (in the $-\hat{x}$-direction) along one wire positioned at $y=-R$, then flows clockwise around the semi-circle, and out to $+\infty$ (in the $\hat{x}$-direction) along the second wire positioned at $y=R$. (See the plot below.)



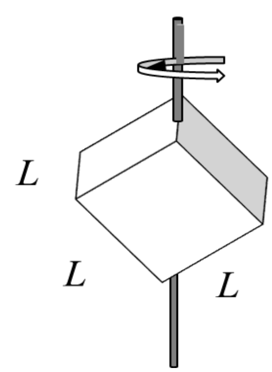
Use the Biot-Savart law,

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int I d\vec{l}_q \times \frac{\vec{r}-\vec{r}_q}{|\vec{r}-\vec{r}_q|^3},$$

to calculate the magnetic field at the origin (center of the semi-circle) induced by this current. When performing this calculation, remember the handy list of integral identities at the back of the exam.

Problem 4 [12 points] – A spinning block

Consider a cube with sides of length, *L*, and a constant mass density. ρ_0. The block is mounted on a pivot so that it is free to spin about an axis which passes through the center of the block, defined as the origin, and is parallel to the diagonal of one of the cube's *faces* (corner-to-corner of one square). Calculate the moment of inertia, $I_{\hat{a}} = \int |\vec{r}\times\hat{a}|^2\, dm$, for the cube about this rotation axis, $\hat{a}$.

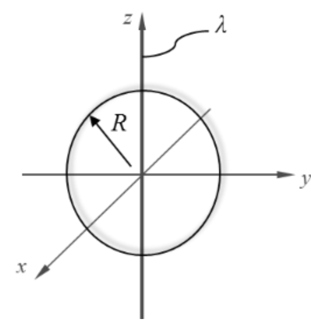


Problem 5 [12 points] – An infinite wire and some shells

You have previously learned that an infinite wire positioned on the *z*-axis and with charge per unit length, λ, gives rise to an electric field described by the equation:

$$\vec{E}(\vec{r}) = \frac{\lambda}{2\pi s\varepsilon_0}\hat{s},$$

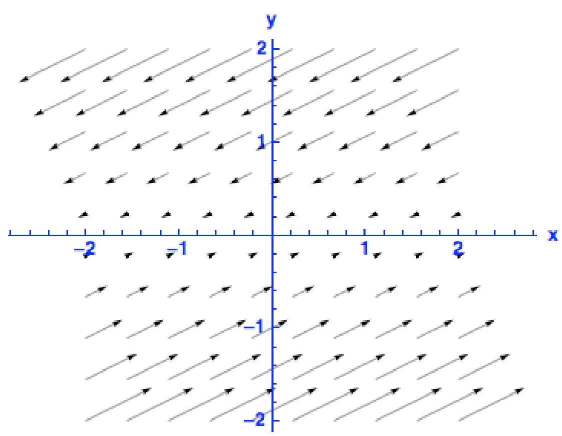
where s and $\hat{s}$ have their usual meaning in the cylindrical coordinate system.



Consider such a line of charge running through the centre of a spherical shell of radius, *R*, centered at the origin . What is the electric flux, $\Phi = \iint \vec{E}\cdot d\vec{A}$, passing from through the spherical surface?

Problem 6 [10 points] – A mystery field

The graph below shows an electric field $\vec{E}(x, y)$ in the *xy*-plane. The field has no *z*-dependence or *z*-component.



(a) What is the direction of the **curl** of $\vec{E}(x, y)$ at each of the points (*x*,*y*) = (1,1) and (1,–1)? If the curl is zero, say so; otherwise give its direction using Cartesian unit vectors.

(b) What is the sign of the **divergence** of $\vec{E}(x, y)$ at each of the points (*x*,*y*) = (1,1) and (1,-1)? You only need to specify whether the divergence is positive, negative or zero.

Important: Be brief, but you must supply a few words explaining the reasons for your answers.

Problem 7 [16 points] – A nucleus decays

A nucleus A is produced in a laboratory with energy 6 GeV and with momentum 3 GeV/c in the $+x$ direction. After traveling a short distance, the nucleus decays. After the decay, there are three particles: a nucleus B moving in the $+x$ direction, and two photons moving in the $+y$ and $-y$ directions respectively. Each photon has an energy of 0.5 GeV.

(a) What is the rest mass of nucleus B?

(b) Calculate the 4-momentum of the "upper" photon in the rest frame of nucleus A.
(By "upper" I mean the photon that goes in the $+y$ direction in the lab.)

(c) Check your answer to (b) by calculating the invariant quantity associated with the photon's 4-momentum. What answer *do* you get, and what answer *should* you get for this invariant?

Problem 8 [12 points] – Gregor the Incredible

Gregor the Incredible is a circus performer, who rides a motorcycle inside the "Sphere of Death" as a part of his act. His favorite part of the routine is a stunt called the "Dizzy Spiraller", which involves riding along a path well parameterized by the following equations:

$$t: 0 \to T,$$

$$r(t) = R,$$

$$\theta(t) = \frac{4\pi}{T^2}\left(t - \frac{T}{2}\right)^2,$$

and $$\phi(t) = \frac{16\pi t}{T}.$$

Here t represents time, and R and T are constants with appropriate units. (r, θ, Φ) are components in the spherical coordinate system, where the origin is defined as the center of the Sphere.

(a) Describe in words what Gregor is doing for this stunt, and make a quick sketch in your answer booklet of the path travelled. Be as specific as possible in your description.

(b) Write down an equation for the total length travelled by Gregor while performing this stunt. Proceed with the calculation as far as you can without performing an integral and then stop. You need not perform the integral, but your formula should contain only known quantities and integration constants. Your answer is to contain no vectors of any kind.