1 Basic Special Relativity Effects from Lorentz Transformation

1. Time dilation of moving clock:
   $\Delta x' = 0 \rightarrow \Delta t = \gamma \Delta t'$

2. Length measurement of moving object:
   $\Delta t = 0 \rightarrow \Delta x = \Delta x' / \gamma$

3. Simultaneous events in S′:
   $\Delta t' = 0 \rightarrow \Delta t = \gamma \beta \Delta x' / c$

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2 Derivation of LT

1. LT must be **linear**, as straight lines (constant v) map onto straight lines to preserve relativity

2. **Inverse** S↔S′ equivalent to $t \leftrightarrow -t$
$\therefore$ (1) $x' = ax + bt$ and (2) $x = ax' - bt'$

3. **Relative speed** of S,S′ is v
$\therefore$ $x' = 0$ maps onto $x = vt$
$\therefore$ (1) $x' = 0 = a(vt) + bt \rightarrow b = -av$

4. **Light ray** $x = ct$ maps onto $x' = ct'$:
(1) $x' = ct' = a(x - vt) = a(c - v)t$ and
(2) $x = ct = a(x' + vt') = a(c + v)t'$
$\therefore$ $t'/t =$ <algebra> $\rightarrow a = \gamma$

3 Argument for Speed Limit *c*

- Hypothesis: X travels FTL from A to B
$\therefore$ Emission at A *causes* detec^{n} at B

- FTL $\rightarrow I_{A\text{-}B}$ is spacelike (negative)
$\therefore$ Can change frames so that $t_B < t_A$
$\therefore$ A *cannot* have caused B

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4 Derive velocity addit^{n}

Boost space-time interval
$\Delta x'^{\mu} = (cT', u_x'T', u_y'T', u_z'T')$
betw two points on trajectory of particle moving with speed u'
$\rightarrow$ get Δx^{μ} and so (u_x, u_y, u_z)

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2 Derivation of LT

1. LT must be **linear**, as straight lines (constant v) map onto straight lines to preserve relativity

2. **Inverse** S↔S′ equivalent to $t \leftrightarrow -t$
$\therefore$ (1) $x'=ax+bt$ and (2) $x=ax'-bt'$

3. **Relative speed** of S,S′ is v
$\therefore$ $x'=0$ maps onto $x=vt$
$\therefore$ (1) $x'=0=a(vt)+bt \rightarrow b=-av$

4. **Light ray** $x=ct$ maps onto $x'=ct'$:
(1) $x'=ct'=a(x-vt)=a(c-v)t$ and
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5 Derive Doppler shift

Calculate intersection of two wave crests with path of moving observer S′; boost to S′ frame to get $\Delta t'=1/f'$

6 Derive relativ. mech.

Motivation:
- Incorporate $v \leq c$ speed limit
- Incorporate photon, with $E=pc$

Photon-in-a-box thought expt: preserve principle of inertia by assigning photon mass $m=E/c^2$

Hypotheses for normal particles:
- inertial mass in $p=mv$ grows w v
- total energy $E=mc^2$ as for photon
- keep $F=dp/dt$ and W= $\int F \bullet dl$

$\rightarrow$ derive new energy-momentum relation $E^2 = (pc)^2 + (m_0c^2)^2$ where m_0 is **rest mass** of particle

$\rightarrow$ find $m_0=0$ for photon and inertial mass $m=\gamma m_0$ for massive particles