Physics 225: Final Exam Formula Sheet

$$t' = \gamma(t - \beta x/c) \qquad t = \gamma(t' + \beta x'/c)$$
$$x' = \gamma(x - \beta c t) \qquad x = \gamma(x' + \beta c t')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{p} = m\vec{v} \qquad W = \int \vec{F} \bullet d\vec{l} = \Delta E$$

$$E = mc^2 = \sqrt{(pc)^2 + (m_0 c^2)^2} \qquad KE = E - m_0 c^2$$

$$m = \gamma m_0 \qquad \vec{p} = \gamma m_0 \vec{v} \qquad \beta = \frac{pc}{E} \qquad \gamma = \frac{E}{m_0 c^2}$$

$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

$$I = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = (c\Delta\tau)^2$$

$$\Lambda^{\mu}_{\nu} \equiv \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x^{\mu} \equiv (ct, x, y, z) \qquad d\tau = \frac{dt}{\gamma_u} \qquad \gamma_u = \frac{1}{\sqrt{1-(u/c)^2}}$$

$$\eta^{\mu} \equiv \frac{dx^{\mu}}{d\tau} = \gamma_u \left(c, u_x, u_y, u_z\right)$$

$$p^{\mu} \equiv m_0 \eta^{\mu} = \left(\frac{E}{c}, p_x, p_y, p_z\right) \qquad \frac{f'}{f} = \sqrt{\frac{1-\beta}{1+\beta}} \qquad E_{\gamma} = hf$$

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u_y' = \frac{u_y/\gamma}{1 - u_x v/c^2}$$

$$u_z' = \frac{u_z/\gamma}{1 - u_x v/c^2}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int dq \, \frac{\vec{r} - \vec{r}_q}{\left|\vec{r} - \vec{r}_q\right|^3} \qquad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{|\vec{r} - \vec{r}_q|} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

where

$$dq = \rho \, dV_q \text{ or } \sigma \, dA_q \text{ or } \lambda \, dl_q \qquad \vec{J}(\vec{r}) \equiv \frac{d\vec{I}}{dA_{\perp}} \longrightarrow I = \int_{\text{Surface}} \vec{J} \cdot d\vec{A}$$

$$\vec{E} = -\vec{\nabla} V \qquad V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{l} \qquad \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int I \, d\vec{l}_q \times \frac{\vec{r} - \vec{r}_q}{\left|\vec{r} - \vec{r}_q\right|^3}$$

From Physics 212: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc by S}}}{\varepsilon_0} \qquad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$ charged sphere (exterior) or point charge

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc by C}} \qquad E = \frac{\lambda}{2\pi\varepsilon_0 s}$$ charged ∞ line

$$\oint_{\partial S} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint_S \vec{B} \cdot d\vec{A} \qquad E = \frac{\sigma}{2\varepsilon_0}$$ charged ∞ sheet

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$$\tilde{I}(t) = I_{\max} e^{i\omega t}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\tilde{V}_R(t) = \tilde{I}(t) R$$

$$\tilde{V}_L(t) = \tilde{I}(t) \tilde{X}_L$$

$$\tilde{V}_C(t) = \tilde{I}(t) \tilde{X}_C$$

$$\tilde{\mathcal{E}}(t) = \tilde{I}(t) \, \tilde{Z}$$

$$\tilde{X}_L = iX_L = i\omega L$$

$$\tilde{X}_C = -iX_C = \frac{1}{i\omega C}$$

- Cofactor: $(M_C)_{ij} \equiv (-1)^{i+j} \times$ det of (M with row-i, col-j removed)
- Determinant of 3×3 matrix: $|M| = \sum_{j=1}^{3} M_{ij}(M_C)_{ij}$ for any i
- Transpose: $(M^T)_{ij} \equiv M_{ji}$
- Inverse: $M^{-1} = (M_C)^T / |M|$

$$f(x_0) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

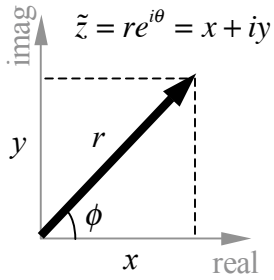
For $x \approx 0$:

- $(1+x)^n \approx 1 + nx$
- $\sin x \approx x$
- $\sin^{-1} x \approx x$
- $\cos x \approx 1 - \frac{x^2}{2}$
- $\cos^{-1} x \approx \frac{\pi}{2} - x$
- $\tan x \approx x$
- $\tan^{-1} x \approx x$
- $e^x \approx 1 + x$
- $\ln(1+x) \approx x$

For $x \to \infty$: if a finite & $n > 0$,
$(x^n + a) \to x^n$ & $(x^{-n} + a) \to a$

θ	sin	cos	tan
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	∞

$$e^{i\theta} = \cos\theta + i\sin\theta$$



$$\tilde{z}^* \equiv x - iy$$

$$|\tilde{z}| \equiv \sqrt{\tilde{z}^* * \tilde{z}} = r$$

Integral Table

$$\int_0^{2\pi} \sin^2\phi \, d\phi = \int_0^{2\pi} \cos^2\phi \, d\phi = \pi$$

$$\int \cos^n\theta \sin\theta \, d\theta = -\frac{\cos^{n+1}\theta}{n+1}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln\left(x + \sqrt{a^2 + x^2}\right)$$

$$\int \frac{x\,dx}{(a^2 + x^2)^{3/2}} = \frac{1}{2}\ln\left(a^2 + x^2\right)$$

$$\int \frac{x\,dx}{a^2 + x^2} = \sqrt{a^2 + x^2}$$

$$\int \frac{x\,dx}{\sqrt{a^2 + x^2}} = -\frac{1}{\sqrt{a^2 + x^2}}$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

$$\int \sqrt{x^2 \pm a^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

$$\int \frac{(x - a\cos\theta)\sin\theta \, d\theta}{(x^2 + a^2 - 2ax\cos\theta)^{3/2}} = \frac{1}{x^2}\frac{a - x\cos\theta}{\sqrt{x^2 + a^2 - 2ax\cos\theta}}$$