Physics 225 – Homework #6 **due in 225 homework box[1] by Thu, 2 pm**

All solutions must clearly show the steps or reasoning you used to arrive at your result. You will lose points for poorly written solutions or incorrect reasoning / explanations; answers given without explanation will not be graded. Include your **NAME** *and* **DISCUSSION SECTION.**

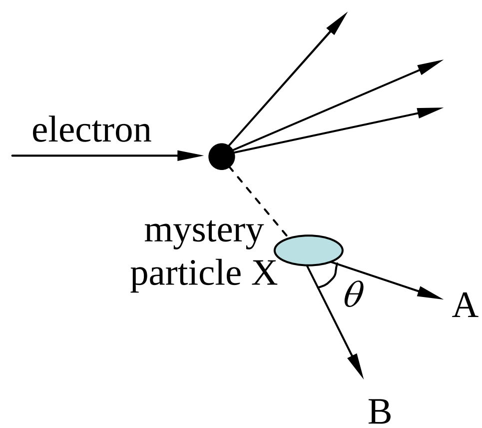
Problem 1 (8 points): $\pi^0 \rightarrow \gamma\gamma$ decay in 1D

(a) A neutral pion of rest mass m_π decays into two photons. In the laboratory frame, the pion is moving with momentum ¾ $m_\pi c$. Photon #1 is emitted in the same direction as the original pion while photon #2 emerges in the opposite direction. Find the energy of each photon in the laboratory frame, expressing your answer in terms of m_π and physical constants.

(b) What is the speed of the pion in the laboratory frame?

Problem 2 (12 points): Creation of a Mystery Particle

In the laboratory frame of a particle physics experiment, a high energy electron is made to collide with a stationary nucleus. The collision causes the creation of many particles. The outgoing particle tracks are shown in the figure. One particular particle causes excitement among the experimenters: a mystery particle (they call it "X") is seen to travel away from the collision for a while, then decay to two identical known particles A and B. The dashed line in the figure shows the trajectory of particle X, from the time it is formed to the time it decays.



The experimenters find that particles A and B have the same momentum $p_A = p_B = 50$ MeV/c in the lab frame, and that the opening angle θ between their directions is 30°, also in the lab frame. The known rest masses of the two decay particles are also the same: $m_A = m_B = 140$ MeV/c^2.

(a) Calculate the energy E_A of particle A in the laboratory frame in units of MeV.

(b) What is the energy of particle A in its own rest frame? Express your answer in MeV.

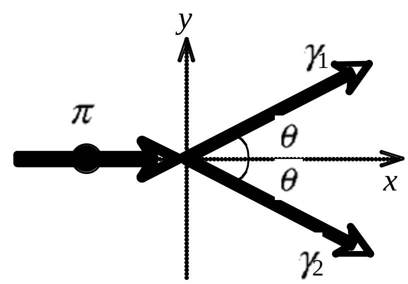
(c) Calculate the speed v_A of particle A in the laboratory frame. Express your answer as a fraction of the speed of light c (e.g. 0.8 c).

(d) Calculate the rest mass m_X of mystery particle X in units of MeV/c^2.

[1] All physics homework boxes are in the 2nd floor overpass between Loomis and MRL (to the north of Loomis).

Problem 3 (10 points): $\pi^0 \to \gamma\gamma$ decay in 2D

(a) A neutral pion of rest mass m_π decays, yet again, into two photons. In the laboratory frame, the pion is moving in the $+x$ direction and has energy E_π. As showin in the figure, the two photons emerge in the xy-plane in a symmetric configuration where each photon's trajectory makes the <u>same angle</u> θ with respect to the $+x$ axis. Calculate this angle θ.
Hint: you should end up with an expression for <u>cos θ</u>, which is a perfectly fine way to leave your answer. (The cosine uniquely determines an angle that can only vary from 0° to 180°.)



(b) Your solution should reveal that the angle θ varies with the pion's energy E_π. To gain some intuition about how this common decay in subatomic physics behaves, first consider the **high energy** regime where $E_\pi >> m_\pi c^2$. Obtain an approximate expression for θ to <u>lowest non-vanishing order</u> in the small quantity $(m_\pi c^2 / E_\pi) << 1$. Your expression should be for the <u>actual angle θ</u> this time, <u>not</u> cos θ. (We're trying to gain some intuition here, and it's much easier to do visualize an angle than its cosine!) Note: you will need a Taylor approximation that you *haven't used before*. Do you remember a couple of lectures ago when we presented **The Taylor Collection**: the most common leading-order Taylor approximations we use in physics? You'll need that collection, so consult your lecture notes.[2]

(c) What about the **low energy** regime? What angle θ does your solution give you in the limit where the pion energy E_π is as small as it can possibly be? (No Taylor needed here, just a pure limit.)

(d) Combine your high-energy approximation from (b) and your low-energy limit from (c) to make a rough **plot** of the decay angle θ versus E_π. (Just interpolate smoothly from the low-energy limit to the high-energy behavior $\to$ nothing weird happens in between for this function.)
Be sure to label your axes, and to indicate clearly any important values on your curve, e.g. where the curve starts, where it crosses an axis, any asymptotic value it approaches, etc.

Congratulations! You have just attacked a very common problem in particle physics in *exactly* the way any experienced physicist would $\to$ calculate the solution, then figure out what it is *telling you* by examining its approximate behavior in different limiting cases.

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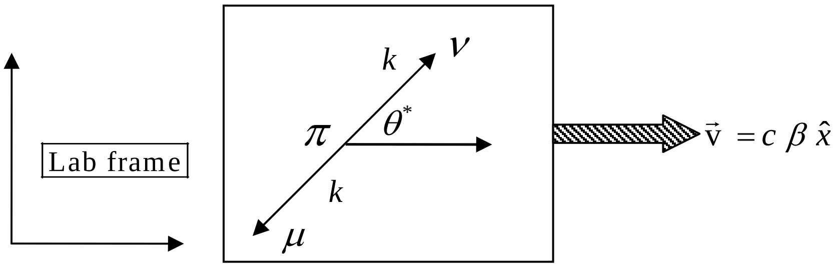
[2] If due some tragic circumstance you cannot locate The Taylor Collection in your lecture notes (coffee spill, hungry pet, unfortunate handwriting, aliens, etc) you can find it on the <u>midterm exam formula sheet</u>, which is posted on our website's homework page. The Collection is right above the table of trig values at the bottom of the page.

Problem 4 (10 points): High energy neutrino production at Fermilab

A pion with a lab energy of E_π^{lab} of about 100 GeV travels along the positive x-axis and decays into a muon and a neutrino. Let k be momentum of the muon and neutrino in the pion rest frame. The situation is depicted in the below figure. Assume throughout that the neutrino is massless, $m_\pi c^2 = 0.140\ \text{GeV}$, and $m_\mu c^2 = 0.105\ \text{GeV}$.



(a) Calculate ck where k is the pion rest-frame momentum depicted in the figure in terms of $m_\pi c^2$ and $m_\mu c^2$.

(b) We will learn next week that energy and momentum boosts in a way very similar to time and position. In particular $\begin{pmatrix} E_\nu^{\text{lab}}/c \\ p_{x\nu}^{\text{lab}} \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} k \\ k\cos\theta^* \end{pmatrix}$.

Use this formula to find the lab energy of the neutrino E_ν^{lab} in terms of $\gamma, \beta, ck,$ and $\cos\theta^*$.

(c) Show that $E_\nu^{\text{lab}} = E_\pi^{\text{lab}}\ F\ \cos^2\dfrac{\theta^*}{2}$ and find F in terms of $m_\pi c^2$ and $m_\mu c^2$. Then evaluate F numerically. Please assume that $E_\pi^{\text{lab}} / m_\pi c^2$ is sufficiently large so that $\gamma\beta \approx \gamma$.