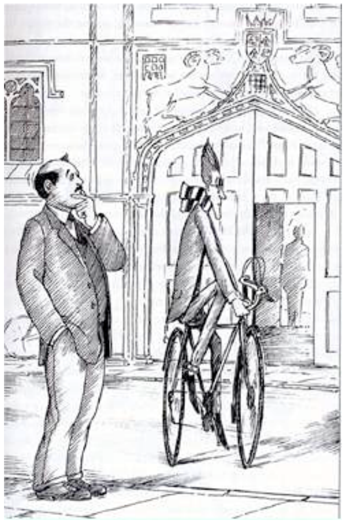
The Speed of Light *c* in vacuum is the same for all inertial observers, independent of the motion of the source.

$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

- **Time Dilation**: moving clocks tick slower by factor γ
- **Length Contraction**: moving objects are shorter by factor γ along direction of motion
- **Loss of Simultaneity**: events that occur at the same time but different positions in one frame are not simultaneous in another frame



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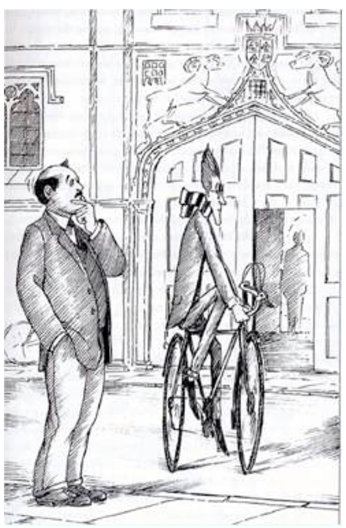
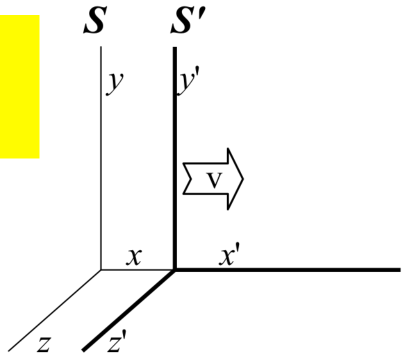


Lorentz Boosts



1

$$\Delta t' = \gamma(\Delta t - \beta \Delta x/c)$$
$$\Delta x' = \gamma(\Delta x - \beta \Delta t\, c)$$
$$\Delta y' = \Delta y$$
$$\Delta z' = \Delta z$$

- **Convention**: S′ moves at speed $v = \beta c$ in $+x$ direction relative to S.
- **Inverse**: swap S ↔ S′ by changing the sign of β.

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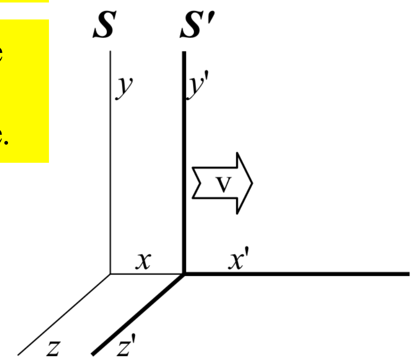

2

1

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Invariant Interval

$$I = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

is invariant under boosts

Timelike $I_{A\text{-}B} > 0$
- object can travel from A-B
- boost can change Δx_{AB} sign

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The **proper time** interval $\Delta\tau_{AB} \equiv \sqrt{I_{AB}}/c$ is the "**watch-time**" that elapses on the wristwatch of an inertial observer who travels from A to B.

Causality: Causal relationships exist where "event A causes event B", implying that A must occur *before* B

3

Nothing – even information – can travel faster than c.

Minkowski Diagrams

- **worldline**: path of an object in (ct,x) diagram
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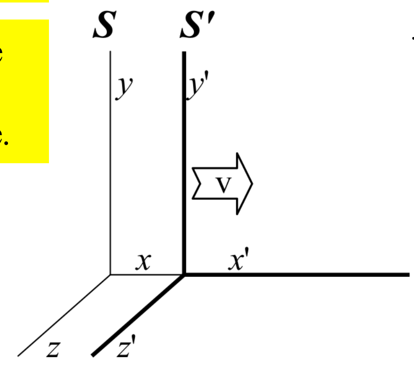

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4-Vectors

$$x^\mu \equiv (ct, x, y, z) \qquad \Delta x'^\mu = \Lambda\ \Delta x^\mu$$

$$\Lambda \equiv \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- **Doppler shift** of light ray moving parallel to S′

$$\frac{f'}{f} = \sqrt{\frac{1-\beta}{1+\beta}}$$

4

- **Velocity addition** (parallel case)

$$u = \frac{u' + v}{1 + u'v/c^2}$$

Causality: Causal relationships exist where "event A causes event B", implying that A must occur *before* B

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Week 4

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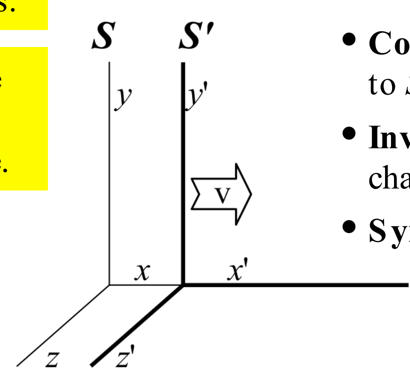
Lorentz Boosts and 4-Vectors



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$$u_x = \frac{u'_x + v}{1 + u'_x v / c^2}$$

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5

- Boost **frequency** of light ray (parallel case)

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Dynamics

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad W = \int \vec{F} \bullet d\vec{l} = \Delta E$$

$$\vec{p} = m\vec{v} \qquad E = mc^2$$

6

$$E = \sqrt{(pc)^2 + (m_0 c^2)^2}$$

$$\vec{p} = \gamma m_0 \vec{v} \qquad E = \gamma m_0 c^2 \qquad m = \gamma m_0$$

$$KE \equiv E - m_0 c^2 \qquad \beta = \frac{pc}{E}$$

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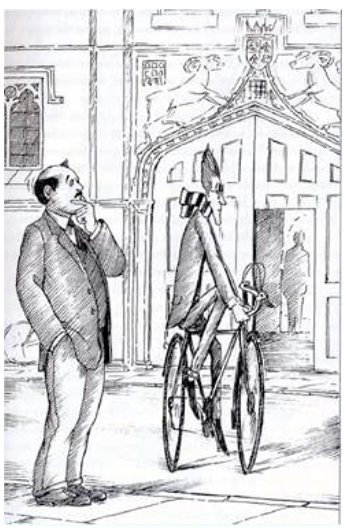

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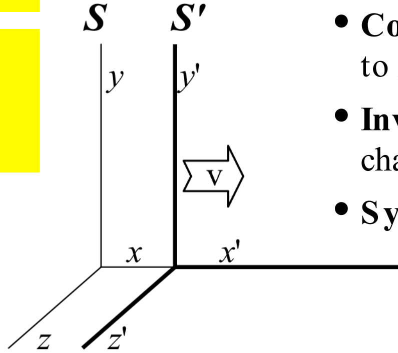

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$$\vec{p} = \gamma m_0 \vec{v} \qquad E = \gamma m_0 c^2 \qquad \beta = \frac{pc}{E}$$

$$KE \equiv E - m_0 c^2$$

E,p conserved

- **(Rest) mass not conserved**, can be converted ↔ energy
- **photon**: m_0=0, γ equ's useless

Causality: Causal relationships exist

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