PHYS 225 (Relativity & Math Applications), Spring 2024

Course Information

Course website: `http://courses.physics.illinois.edu/phys225/sp2024`

Meeting time and location: Tuesdays 4:00-4:50pm, Loomis 151

Credit Hours: 2 (undergraduate)

Instructor:
Prof. Yonatan (Yoni) Kahn
Email: `yfkahn@illinois.edu`
Loomis Lab 415

Discussion TAs:
Rachel Nguyen, `rnn4@illinois.edu`
Diana Forbes, `dianamf2@illinois.edu`
Saavanth Velury, `svelury2@illinois.edu`

Office Hours: Wednesdays 4pm-6pm and Thursdays 3:30pm-5:30pm, Loomis 276

Prerequisites: Credit or concurrent registration in PHYS 212.

Learning Objectives

Relativity: By the end of the course, students will be able to

- compute the effects of time dilation, length contraction, and relativistic Doppler shifts and explain their practical relevance
- derive the Lorentz transformation equations from the postulates of relativity
- manipulate 4-vectors with Lorentz transformations
- solve basic problems in relativistic kinematics using invariance and energy-momentum conservation
- explain why Maxwell's equations are consistent with relativity

Math Methods: By the end of the course, students will be able to:

- multiply matrices and understand the relationship to group properties such as commutativity
- use Taylor series to compute approximations and infinitesimal transformation matrices
- solve first-order ordinary differential equations by separation of variables
- compute fluently with vector differential operators and convert between the differential and integral forms of Maxwell's equations
- manipulate complex numbers
- solve the wave equation using Fourier transformation

Course Description

PHYS 225, "Relativity & Math Methods," is a quantitative survey of Einstein's theory of special relativity, which since its publication in 1905 has fundamentally changed the way we think about space, time, and the universe. Interleaved throughout the course are important mathematical methods in physics, which will be applied to several examples from relativity and electrodynamics to illustrate their importance in solving physical problems. A particular emphasis is placed on the role of symmetries and an introduction to the mathematics of group theory.

Course Format

The course consists of one 50-minute lecture per week, which introduces material that is reinforced with a 110-minute weekly discussion section where students solve problems together. The material is further reinforced with weekly homework assignments expected to take approximately 2–3 hours each. Progress is assessed with a midterm exam and a final exam.

Learning Resources

The following textbook is *required*:

D. Morin, **Special Relativity for the Enthusiastic Beginner**
It is available as a Kindle e-book on Amazon, or in the Illini Union bookstore.

The following textbook is *optional*:

R. Shankar, **Basic Training in Mathematics: A Fitness Program for Science Students**
It is also available in the Illini Union bookstore.

Course Requirements and Assessments

Course grading will proceed in compliance with University policy as given in Article 3, Part 1 of the Student Code.

Grading will be based on a combination of attendance and participation (via "1-minute papers" to be completed at the beginning and end of each class – you must turn in *both* to get credit), weekly homework assignments, weekly discussion sections, a midterm exam, and a final exam. Your final grade for PHYS 225 will be based upon your total score on all the components of the course. The total possible score is 1000 points. Attendance and participation will count for 200 points, homework for 200 points, discussions for 100 points, midterm exam for 225 points, and final exam for 275 points. One pair of 1-minute papers, one discussion, and your lowest homework score will be dropped without penalty. Up to 80% credit will be awarded for finishing homework up to one week late.

You also have the opportunity to earn up to 20 bonus points during the semester. Attending office hours and asking a question to the instructor or one of the TAs will count for 1 bonus point (for a total of up to 15). A short written assignment analyzing Einstein's original 1905 paper will count for 5 bonus points. Bonus points are added to your non-exam scores up to, but not exceeding, a 500-point total (remember, all of the course activities other than the exams count up to 500 points). **Only bonus points that will bring your non-exam total up to 500 points may be used.**

Letter grade cutoff values can be found on the course website.

Statement on use of AI tools

Submission of AI-generated solutions to any assignment – including those produced by ChatGPT, Bard, Claude, or other large language model chatbots – is not acceptable. Artificial intelligence is a fascinating new tool, but its ability to reason mathematically and physically is not yet reliable. **Use AI at your own peril: it may give you the wrong answers, even if you are simply asking for conceptual help.** You will not have such assistance on the exams, and if your reasoning is flawed as a result of relying on incorrect information from AI tools, **you will lose substantial credit.**

Absence Policy

The full attendance policy can be found on the course website, and is summarized here.

- Students are expected to attend the discussion section in which they are registered. Section swapping is not allowed. Section changes will be allowed only until the midterm exam on 3/5/24.

- If you show up to discussion late by 10 minutes or more, you will lose all participation points associated with that discussion session.

- If you show up to an exam late, you will have only the amount of time left on the timer to complete the exam.
- Two types of absence can be recorded in the gradebook: excused absences (EX) and unexcused absences (ABS). The *only* course components eligible to be issued a grade of EX are discussions and exams. Unexcused absences equate to a grade of zero (0) for the missed course component.
- Excused absences will be granted and documented in accordance with University policy as described in Article 1, Part 5 Class Attendance, of the Student Code.
- You are allowed a maximum of three (3) excused absences in discussion. Starting with the 4th absence, the absence will be marked ABS.
- **Failing to show up for a final exam will result in an AB grade resulting in failure (F) for the course.**

Disability Access Statement

To obtain disability-related academic adjustments and/or auxiliary aids, students with disabilities must contact the course instructor and the as soon as possible. To ensure that disability-related concerns are properly addressed from the beginning, students with disabilities who require assistance to participate in this class should contact Disability Resources and Educational Services (DRES) and see the instructor as soon as possible. If you need accommodations for any sort of disability, please speak to the instructor after class, or make an appointment during office hours. DRES provides students with academic accommodations, access, and support services. To contact DRES you may visit 1207 S. Oak St., Champaign, call 333-4603 (V/TDD), or e-mail disability@illinois.edu. `http://www.disability.illinois.edu/`.

Schedule

Students are expected to complete the assigned reading *before* the lecture on Tuesday. Optional readings are indicated in parentheses. Homework is generally assigned on Thursdays and due the following Thursday, *except* for the week of the midterm when the homework will be shorter and due the day before the midterm (Monday), and the final week of the course when the homework is due on Wednesday, the day before Reading Day.

- **Week 1** (1/16) – course intro, constancy of speed of light, relativity of simultaneity.
  - Reading: Morin 1.1–1.2, Appendix F
  - Objectives: define Galilean coordinate transformations, give an example of loss of simultaneity, apply relativistic velocity addition formula.

- **Week 2** (1/23) – three fundamental effects of relativity, twin paradox.
  - Reading: Morin 1.3–1.4
  - Objectives: rigorously define concepts of length and time, construct physical scenarios where the "rear clock ahead" effect, time dilation, and length contraction appear, explain the resolution of the twin paradox.
  - HW #1 due 1/25
- **Week 3** (1/30) – vectors, matrices, and index notation.
  - Reading: (Shankar 8.1)
  - Objectives: multiply a vector by a matrix and write this using Einstein summation convention, explain why linear transformations and vectors are used in physics, perform active and passive rotations in 2 dimensions.
  - HW #2 due 2/1
- **Week 4** (2/6) – derivation of the Lorentz transformations, 4-vectors.
  - Reading: Morin Appendix D, Morin 2.1.3
  - Objectives: state the two postulates of relativity and use them to derive Lorentz transformations, recover the three fundamental effects from Lorentz transformations, write the Lorentz transformations in terms of 4-vectors.
  - HW #3 due 2/8
- **Week 5** (2/13) – matrix multiplication, derivation of relativistic velocity addition, Taylor series.
  - Reading: Morin Appendix G, Morin 2.2 (Morin 2.7)
  - Objectives: multiply two square matrices of various sizes, derive velocity addition formula from Lorentz matrix multiplication, use Taylor series to compute numerical and analytic approximations for small quantities.
  - HW #4 due 2/15
- **Week 6** (2/20) – symmetries and group theory.
  - Reading: (Shankar 8.2, 8.4)
  - Objectives: state the three properties which define a group, define the properties of an orthogonal matrix, perform Taylor expansions of group matrices to obtain infinitesimal elements, show that unidirectional boosts and rotations about a fixed axis are abelian groups.
  - HW #5 due 2/22

- **Week 7** (2/27) – invariant interval, proper time, Minkowski diagrams, spacetime metric, rapidity.
  - Reading: Morin 2.3–2.4
  - Objectives: show that Lorentz transformations preserve the invariant interval, explain relationship between Lorentz group and orthogonal group, parameterize a boost in terms of rapidity, draw a Minkowski diagram for spacetime events, classify events as timelike/spacelike/lightlike.
  - HW #6 due 2/29
- **Week 8** (3/5) – **Midterm Exam**
  - HW #7 due 3/4 **(note Monday due date!)**
- **Week 9** – **SPRING BREAK**
- **Week 10** (3/21) – relativistic energy and momentum, relativistic force in 1D, separation of variables for first-order ODEs.
  - Reading: Morin 3.1, 3.5.1 (Shankar 10.3)
  - Objectives: define relativistic energy and momentum, derive non-relativistic kinetic energy using Taylor series, solve first-order ODEs using separation of variables.
- **Week 11** (3/19) – relativistic force in 2D, vector fields and curvilinear coordinate systems, vector differential operators.
  - Reading: Morin 3.5.2 (Shankar 7.1–7.3, 7.5–7.8)
  - Objectives: derive formulas for coordinate vectors and differential operators in cylindrical and spherical coordinate systems, show Laplacian operator is rotationally invariant.
  - HW #8 due 3/28
- **Week 12** (4/2) – lots of 4-vectors, relativistic inner product, Doppler effect.
  - Reading: Morin 4.1–4.3, 2.5
  - Objectives: define a 4-vector in terms of its Lorentz transformations; derive 4-velocity, energy-momentum 4-vector, acceleration and force 4-vectors from displacement 4-vectors, derive the relativistic Doppler effect, give physical scenarios exhibiting the relativistic Doppler effect.
  - HW #9 due 4/4
- **Week 13** (4/9) – relativistic kinematics, natural units.

  - Reading: Morin 3.3–3.4, 4.4
  - Objectives: convert fluently between natural and SI units, solve basic relativistic kinematics problems using the relativistic dot product and conservation laws.
  - HW #10 due 4/11

- **Week 14** (4/16) – line and surface integrals, Stokes' theorem, differential forms of Maxwell's equations.
  - Reading: (Shankar 7.4, 7.9, 7.11)
  - Objectives: compute simple line and surface integrals with a choice of coordinates, derive the differential Maxwell's equations from the integral forms using Stokes' theorem, show how relativistic length contraction and velocity addition imply that electric fields turn into magnetic fields in a moving frame.
  - HW #11 due 4/18

- **Week 15** (4/23) – complex numbers, introduction to Fourier transformations, complex wave notation.
  - Reading: (Shankar Ch. 5)
  - Objectives: add and multiply complex numbers, apply Euler's identity, parameterize a wave using complex numbers, use Fourier transformations to solve simple partial differential equations.
  - HW #12 due 4/25

- **Week 16** (4/30) – wave equation from Maxwell's equations, solutions as lightlike 4-vectors using Fourier transformation.
  - Reading: (Shankar 7.11, Einstein's 1905 paper "On the Electrodynamics of Moving Bodies")
  - Objectives: derive the wave equation for photons from Maxwell's equations, solve using Fourier transformation, show that the wave operator is invariant under Lorentz transformations.
  - HW #13 due 5/1 **(note Wednesday due date!)**

- **Finals Week** – **Final Exam** Tuesday 5/7 1:30-4:30pm, location TBA
  - Bonus assignment due on 5/7 at 1:30pm to Gradescope