Our Kepler formula-set is now updated to include: (1) the possibility of <u>repulsive</u> $1/r^2$ forces with negative force-constants γ (2) relations needed for scattering problems, namely formulae for the scattering angle θ and impact parameter b for unbounded Kepler orbits as well as general cross-section formulae.

• **Coordinates & Reduced Mass** : $\vec{r}_1 = \vec{R} + \frac{m_2}{M}\vec{r}\,, \qquad \vec{r}_2 = \vec{R} - \frac{m_1}{M}\vec{r}\,, \qquad \mu = \frac{m_1 m_2}{M}$

• **Centrifugal** force & PE : $\vec{F}_{cf} = \frac{L^2}{\mu r^3}\hat{r}\,, \qquad U_{cf} = \frac{L^2}{2\mu r^2}\,,$ effective radial $U^* = U + U_{\text{cf}}$

• **Angular EOM** : $\dot{\phi} = \frac{L}{\mu r^2}$ • **Radial EOMs** : $\mu\ddot{r} = F(r) + F_{\text{cf}}(r)\,, \qquad E = T + U(r) = \frac{1}{2}\mu\dot{r}^2 + U_{\text{cf}}(r) + U(r)$

• **Path Equation** : $u(\phi) \equiv 1/r(\phi) \quad \rightarrow \quad u'' + u = -\frac{\mu F(1/u)}{L^2 u^2}$ and $u' = -\frac{\mu\dot{r}}{L}$

• **Conics** : With (r,ϕ) centered on a <u>focal point</u> and E ≡ Ellipse, H ≡ Hyperbola

$$\frac{1}{r} = \frac{a}{b^2}(\pm 1 + e\cos\phi) \text{ with } \begin{cases} +\text{: E or H-near-branch} \\ -\text{: H-far-branch} \end{cases}, \qquad e = \frac{c}{a} = \frac{\sqrt{a^2 \mp b^2}}{a} \text{ with } \begin{cases} -\text{: E} \\ +\text{: H} \end{cases}$$

• **Kepler Orbits** $F = -\frac{\gamma}{r^2}$: $r(\phi) = \frac{r_0}{\text{sgn}[\gamma] + e\cos\phi}$ with $r_0 = \frac{L^2}{\mu|\gamma|} = \frac{b^2}{a} = a\left|1-e^2\right|\,, \quad E = \mp\frac{|\gamma|}{2a} = \frac{|\gamma|(e^2-1)}{2r_0}$

Bounded orbits: $\tau^2 = \frac{4\pi^2\mu}{\gamma}a^3\,, \qquad r_0 = \frac{2\,r_{\text{min}}r_{\text{max}}}{r_{\text{min}} + r_{\text{max}}}\,, \qquad e = \frac{r_{\text{max}} - r_{\text{min}}}{r_{\text{max}} + r_{\text{min}}}$

Unbounded orbits: scattering angle $\theta = \pi - 2\alpha$ with $\tan\alpha = \frac{b}{a}$, impact parameter $b =$ semi-minor axis b ☺

• **XSec** : $d\Omega \equiv \frac{dA}{r^2} = \begin{cases} \sin\theta\, d\theta\, d\phi \\ d\theta_x\, d\theta_y \end{cases}, \quad \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$ with $\theta =$ scattering angle • **Lumi**: $\mathcal{L} = n_A N_e\,, \quad \frac{dN_{\text{ev}}}{dt} = \mathcal{L}\sigma$