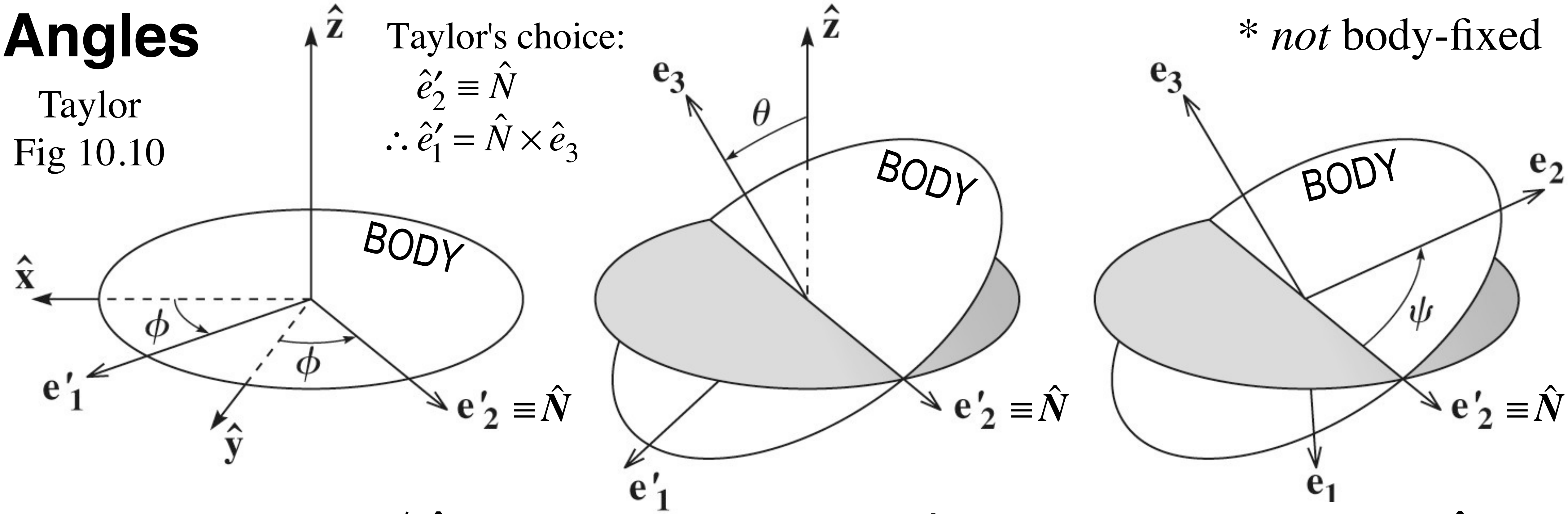
Euler Angles

$\hat{N} \equiv$ **Line of Nodes** $\parallel \hat{z} \times \hat{e}_3 \parallel$ intersec^n of $(\hat{x},\hat{y})$ & $(\hat{e}_1,\hat{e}_2)$

* *not* body-fixed

Taylor
Fig 10.10

Taylor's choice:
$\hat{e}_2' \equiv \hat{N}$
$\therefore \hat{e}_1' = \hat{N} \times \hat{e}_3$



(a) **Twist** $\phi\,\hat{z}$ (b) **Tilt** $\theta\,\hat{e}_2'$ (c) **Spin** $\psi\,\hat{e}_3$

$$\boxed{\vec{\omega} = \dot{\phi}\,\hat{z} + \dot{\theta}\,\hat{e}_2' + \dot{\psi}\,\hat{e}_3}$$

$\hat{z} = \cos\theta\ \hat{e}_3 - \sin\theta\ \hat{e}_1'$ from figure, so :

$$\vec{\omega} = \left(-\dot{\phi}\sin\theta\right)\hat{e}_1' + \dot{\theta}\,\hat{e}_2' + \left(\dot{\psi} + \dot{\phi}\cos\theta\right)\hat{e}_3$$

$$= \qquad \vec{\omega}_{12} \qquad + \qquad \vec{\omega}_3$$

For axi-symmetric body ($I_1 = I_2$) :

$$\vec{L} = I_1\vec{\omega}_{12} + I_3\vec{\omega}_3$$

$$T = \tfrac{1}{2} I_1 \omega_{12}^2 + \tfrac{1}{2} I_3 \omega_3^2 \ \rightarrow \ \textbf{Lagrangian}\ \mathcal{L}(\phi,\theta,\psi)$$