

When to Use Energy

Proof of (T+U) conservation *used* $\vec{F} = m\vec{a}$... so

$\dot{T} + \dot{U} = 0$ always **gives you back a force-based EOM**.

It's often a more efficient way to obtain the EOM than using forces or torques, but more strategy tips are needed:

When do we **NEED** to use $d(T+U) = 0$?

When force details are unknown.

- **Elastic Collisions**

All we know about the forces is that
"elastic" \equiv **T is conserved** in the collision.

- **Known ΔE added / subtracted from system**
e.g. particle explodes, releasing known ΔE
 \rightarrow must use energy analysis to include this info.

When can we **NOT** use $d(T+U) = 0$?

When it's not true OR can't be calculated.

- $U(\vec{r}, t)$ with **explicit t -dependence**
- Forces that **can't be described** by $U(\vec{r})$ but **do work**
e.g. kinetic friction (depends on normal force),
drag force (depends on velocity),
force fields with $\vec{\nabla} \times \vec{F} \neq 0$
- **Energy leaves / enters system in non-calculable way**
e.g. inelastic collisions where $\Delta U^{\text{INT}} \neq 0$
- System is **losing mass**, e.g. rocket motion

Collisions

When details of interaction force not known, some other information must be provided.

Elastic Collision : total T is conserved

- Collision takes place in infinitesimal time interval $\Delta t \approx 0$
 \therefore **impulse** collision force : $F_{\text{impulse}} \cdot \Delta t = \Delta P$
 $\infty \cdot 0 = \text{finite}$
 $\therefore \Delta \vec{r} = 0$ (no time for p'cles to move) $\rightarrow \Delta U^{\text{EXT}}(\vec{r}) = 0$
- **Particle structure unchanged** by collision $\rightarrow \Delta U^{\text{INT}} = 0$
i.e. no energy can escape into deforming / heating / fragmenting the particles
- "Elastic" also implies that energy **cannot escape** to **surrounding medium**, e.g. via sound waves

Inelastic Collision : total $T+U^{\text{EXT}}$ *not* conserved

- **Total Mass is conserved** (non-relativistic systems)
- Classic example : **Fusion / Fission** of $N \leftrightarrow 1$ particles
When particles fuse, KE of incoming particles converted into heat of fused particle $\therefore \Delta U^{\text{INT}} \neq 0$
 \rightarrow Often useful : **go to CM Frame**
- **Abrupt ΔT_{total}** , i.e. that occurs over $\Delta t \approx 0$, is a signature of an inelastic process
 $U^{\text{EXT}}(\vec{r}) = 0$ can't change over $\Delta t \approx 0 \therefore \Delta \vec{r} \approx 0$
 $\therefore \Delta(T+U^{\text{EXT}}) = \Delta T \neq 0 \rightarrow \text{inelastic}$