

1-Body Motion $\vec{f} = \dot{\vec{p}} = m\ddot{\vec{r}} \quad \vec{p} = m\vec{v} = m\dot{\vec{r}}$

$\vec{l} \equiv \vec{r} \times \vec{p} \quad \vec{\tau} \equiv \vec{r} \times \vec{f} = \dot{\vec{l}} \quad \vec{f}_{\text{grav}} = -(GMm/r^2)\hat{r}$

$\vec{f}_{\text{air}} = -(bv + cv^2)\hat{v} = \vec{f}_{\text{lin}} + \vec{f}_{\text{quad}} \quad \text{Reynolds } R \equiv Dv\rho/\eta$

sphere, diameter D: $b = \beta D \quad c = \gamma D^2 \quad R = 48 f_{\text{quad}} / f_{\text{lin}}$

Rocket Motion $m\dot{\vec{v}} = -\dot{m}\vec{v}^{\text{ex}} + F^{\text{EXT}}$

Miscellaneous $1 \text{ m/s} \approx 2.2 \text{ mph}$

$f_{\text{friction}} \begin{cases} = \mu_{\text{kin}} N \\ \leq \mu_{\text{static}} N \end{cases} \quad \vec{f}_{\text{EM}} = q(\vec{E} + \vec{v} \times \vec{B})$

Collective Motion * assuming Newton's 3rd Law $\rightarrow F^{\text{INT}}$ cancel

Notation for collective properties

- unsubscripted capital letter $\rightarrow$ "TOTAL", except for ...
- unsubscripted capital position, velocity, accel $\rightarrow$ "OF THE CM"
- ◇ subscript $\neq$ coordinate index $\rightarrow$ "OF"
- ◆ no superscript $\rightarrow$ "RELATIVE TO ORIGIN"
- ◆ superscript () $\rightarrow$ "RELATIVE TO (POINT)"
- ◆ superscript prime' $\rightarrow$ "RELATIVE TO THE CM"

CM: $M\vec{R} \equiv \sum_i m_i \vec{r}_i \quad \vec{P} = M\dot{\vec{R}} \quad \vec{L}_{\text{CM}} = \vec{R} \times \vec{P}$

Rotating Body: for any BODY-FIXED vector $\vec{B}$, $\dot{\vec{B}} = \vec{\omega} \times \vec{B}$

Moment of Inertia: for any BODY-FIXED point B,

$$I_{\hat{\omega}}^{(B)} \equiv \sum m_i |\vec{r}_i^{(B)} \times \hat{\omega}|^2 \quad L_{\omega}^{(B)} = I_{\hat{\omega}}^{(B)} \omega \quad T^{(B)} = \frac{1}{2} I_{\hat{\omega}}^{(B)} \omega^2$$

Uniform Gravity: If $\vec{f}^{\text{EXT}} = m\vec{g}$ only $\rightarrow \vec{F}^{\text{EXT}} = M\vec{g} \quad \vec{\tau}^{\text{EXT}} = \vec{R} \times M\vec{g} \quad \vec{\tau}'^{\text{EXT}} = 0 \quad U^{\text{EXT}} = MgH$

Variational Calc / Mech * Gen. coord q_i must be indep

$$S \equiv \int_{t_1, \vec{q}_1}^{t_1, \vec{q}_1} dt L(q_i, \dot{q}_i, t) \quad \delta S = 0 \rightarrow \frac{\partial L}{\partial q_i} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) \text{ for each } q_i$$

$$H \equiv \sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L \quad \frac{dH}{dt} = -\frac{\partial L}{\partial t}$$

Mechanics Principle of Least Action :

$$L = T - U \rightarrow \delta S = 0 @ \text{ true } \{q_i(t)\}$$

$$\text{Gen. force } Q_i \equiv \frac{\partial L}{\partial q_i}, \text{ momentum } p_i \equiv \frac{\partial L}{\partial \dot{q}_i}$$

H equals $T+U$ when $\exists$ no t -dep constraints

Accel Frames $\vec{f} \equiv \vec{F}/m \rightarrow \vec{f}_{\text{lin}}^* = -\vec{A}_0 \quad \vec{f}_{\text{cf}}^* = (\vec{\Omega} \times \vec{r}^*) \times \vec{\Omega} = \Omega^2 s^* \hat{s}^* \quad \vec{f}_{\text{Cor}}^* = 2\vec{v}^* \times \vec{\Omega} \quad \vec{f}_{\text{azim}}^* = \vec{r}^* \times \dot{\vec{\Omega}}$

For $\vec{B}$ constant
in S^* frame: $\left. \frac{d\vec{B}}{dt} \right|_{S^*} = \vec{\Omega} \times \vec{B}$

For general vector $\vec{A}$: $\left. \frac{d\vec{A}}{dt} \right|_{S^*} = \vec{\Omega} \times \vec{A} + \left. \frac{d\vec{A}}{dt} \right|_{S^*}$