Physics 326 Final Exam #2 Spring 2015
Friday May 14, 8:00 am – 11:00 am

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so **anything you write on these question pages will NOT be graded**

You have **3 hours** to work the problems.

At the beginning of the exam:
1) Write your **name** and **netid** on your answer booklet(s).
2) Turn your **cell phone off**.
3) Put away all calculators, phones, computers, notes, and books.

During the exam:
1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...
2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you understand what you're doing and are **not just guessing**. Also a good **annotated sketch** is often the best explanation of all!
3) **All question parts on this exam are independent**: you can get full points on any part even if your answers to all the other parts are incorrect. Therefore you should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. IMPORTANT: If you think you've made a mistake but can't find it, **explain what you think is wrong** ➔ you may well get partial credit for noticing your error!
5) It is fine to leave answers as **radicals or irreducible fractions** (e.g. $10\sqrt{3}$ or 5/7).

Remember: There are many **Math Tables** provided ➔ do **Use Them!**

When you're done with the exam:

Turn in only your **answer booklet**. (Keep the exam questions).

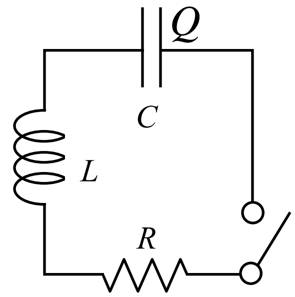
Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.

You must derive any expression that is not on the formula sheet.
You may of course use from memory any formulae from a previous class, e.g. 325, 225, 211.

Problem 1 : RLC Oscillator



A capacitor C, inductor L, and resistor R are connected in series. Let Q be the charge on the right-hand plate of the capacitor, as indicated in the figure. Kirchoff's equation for the charge $Q(t)$ on the capacitor is:

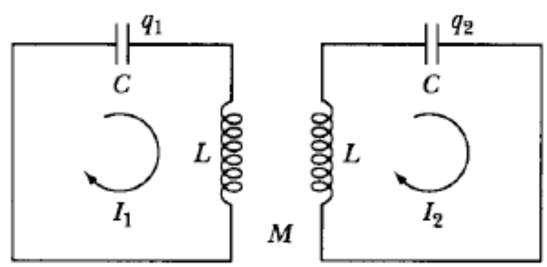
$$\ddot{Q}L + \dot{Q}R + \frac{Q}{C} = 0$$

The values of the circuit elements are $R = 5$, $L = 1/2$, and $C = 1/8$ (all in SI units). Initially, the switch is open and the capacitor is charged to Q_0. At time $t = 0$, the switch is closed, and an initial current $\dot{Q}\big|_{t=0} = -2Q_0$ flows. Calculate the charge $Q(t)$ for all times $t > 0$.

Problem 2 : Coupled AC Circuits

The two AC circuits shown at right are coupled together via the mutual inductance M between their nearby inductors. The Kirckhoff equations for this system are

$$Q_1 + LC\ddot{Q}_1 + MC\ddot{Q}_2 = 0 \text{ and}$$
$$Q_2 + LC\ddot{Q}_2 + MC\ddot{Q}_1 = 0$$



Find the general solutions for the charges on the capacitors, $Q_1(t)$ and $Q_2(t)$. Hint: normal coordinates.

Problem 3 : Hohmann Transfer to Earth

A spaceship is in a circular orbit of radius 0.2 A.U. = 1/5 A.U. around the Sun and wishes to transfer its trajectory to match the Earth's orbit. To conserve fuel, the ship's crew use a 2-impulse Hohmann transfer. <u>Hohmann reminder</u>: the crew fire their engines twice, first to leave their original orbit, and later to settle into their final orbit. Both engine firings are brief impulses that are parallel to the ship's current velocity.

(a) Calculate the <u>semi-minor axis</u>, b, of the transfer orbit.

(b) Calculate the thrust factor, $\lambda \equiv v_{\text{after}} / v_{\text{before}}$, for the <u>first</u> firing of the ship's engines.

Problem 4 : Theorists and Experimentalists

At a particular physics institute, the theorists work on the top floor of a building of height 30 m, while the experimentalists work on the ground floor. They meet on the ground floor at 9 am, then the theorists zip up to the top floor in the elevator and start work at their desks. (Ignore the tiny amount of time spent in the elevator.) At 5 pm, the theorists and experimentalists meet again on the ground floor. Over the course of the day, the theorists have aged by $T_t = 8$ hours $\approx 3 \times 10^4$ seconds, while the experimentalists have aged by a slightly different amount of time T_e.

(a) Calculate the approximate age difference $|T_t - T_e|$ between the theorists and experimentalists, in seconds.

(b) Who has aged more, the theorists or the experimentalists? Be sure to justify your answer.

Problem 5 : Laser Pulse near a Black Hole

Two stationary space stations are parked at fixed locations near a black hole of mass M. Station A is located at $r_A = 3M$, while station B is located directly above station A at $r_B = 4M$. (By "directly above", I mean that the two stations and the center of the black hole lie on the same radial line.) At time $t = 0$, Station A fires a laser pulse toward station B. Calculate the time $t = T$ at which the laser pulse arrives at station B.
Integration hint: You may find the substitution $x \equiv r - 2M$ helpful. **IMPORTANT**: The t and r variables in this question are all Schwarzschild ("Bookkeeper") coordinates; no local measurements of any sort are involved.

Problem 6 : Football in Space

An American football has a total mass $M = 1$ that is evenly distributed across its surface. The football is placed so that its center-of-mass is at the origin and its axis of symmetry runs along the z axis. In this (x, y, z) coordinate system, the football's inertia tensor for rotation around the origin=CM is

$$\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

(a) An astronaut throws the football so that its spin is almost entirely along the football's axis of symmetry … but not quite. The initial rotation vector $\vec{\omega}$ has these properties:

- Its magnitude is $\omega = 10\,\pi$ $\rightarrow$ its linear frequency is thus $f = \omega / 2\pi = 5$ rotations-per-second.
- Its direction makes an angle of 0.01 radians with respect to the football's axis of symmetry. (This angle is so small that you can Taylor-approximate it without concern.)

Approximately how many rotations-per-second does the symmetry axis make, as viewed by the astronaut ?

(b) The football is now placed at rest with its center-of-mass at the origin and its symmetry axis along the z axis. At time $t = 0$, the astronaut imparts a brief kick to the bottom end of the football, which is located at the point $(x, y, z) = (0, 0, -2)$. The delivered impulse is $\vec{k} = k\,\hat{x}$ where k is a constant with units of momentum. Calculate the trajectory $x(t), y(t), z(t)$ of the bottom end of the football for all times $t > 0$.