Physics 326   Midterm Exam #1   Spring 2015
Monday Feb 23, 12:30 pm – 1:50 pm

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so
anything you write on these question pages will NOT be graded

You have **80 minutes** to work the problems.

At the beginning of the exam:
1) Write your **name** and **netid** on your answer booklet(s).
2) Turn your **cell phone off**.
3) Put away all calculators, phones, computers, notes, and books.

During the exam:
1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...

2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you understand what you're doing and are **not just guessing**. Also a good **annotated sketch** is often the best explanation of all!

3) **All question parts on this exam are independent**: you can get full points on any part even if your answers to all the other parts are incorrect. Therefore you should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.

4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. IMPORTANT: If you think you've made a mistake but can't find it, **explain what you think is wrong** ➔ you may well get partial credit for noticing your error!

5) It is fine to leave answers as **radicals or irreducible fractions** (e.g. $10\sqrt{3}$ or 5/7).

Remember: There are many **Math Tables** provided ➔ do **Use Them!**

When you're done with the exam:
Turn in only your **answer booklet**. (Keep the exam questions).

Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.

You must derive any expression that is not on the formula sheet.
You may of course use from memory any formulae from a previous class, e.g. 325, 225, 211.

Problem 1 : Boing

A spring of length $d = 100$ and spring constant $k = 2/3$ is suspended from a fixed support and a mass $m = 3$ is attached to its bottom end. Uniform gravity $g = 10$ points downward and there is no air resistance. The mass m is initially held motionless at a distance of $d_0 = 150$ below the fixed support; at $t = 0$, it is released from rest.

How close does the mass m ever get to the fixed support? i.e. Find the minimum distance between the mass and the support.

Problem 2 : Boing Bwing

A cart, a spring, and a pendulum are connected as shown in the figure. Qualitatively describe each normal mode of the system with sketches. (No calculations are necessary ... and for this question only, no explanations are necessary either.)
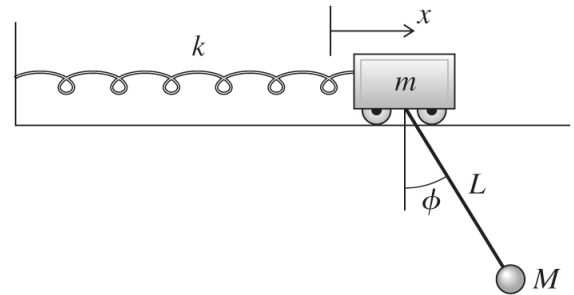
Problem 3 : Bzzing

A cart of mass $m = 1$ is connected to a wall by a spring of strength $k = 4$ and unstretched length d. Due to a little friction in the wheels, the cart experiences a drag force $\vec{F} = -2m\beta\vec{v}$ where β is very small. A motor exerts a driving force $F_x = F_0 \sin(\omega t)$ on the cart, where ω can be varied. The resulting system has a Q-factor of $Q = 10$.
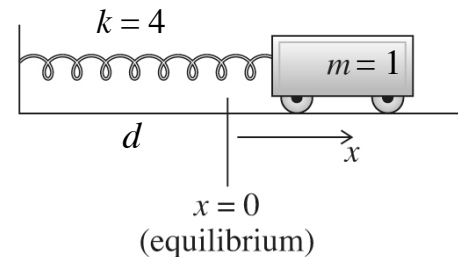
(a) At various driving frequencies ω, a student measures the cart's oscillation amplitude after it has been oscillating for a long time. She finds that:

  ● when $\omega = \omega_1$, the maximum amplitude is achieved, and is equal to $d/4$

  ● when $\omega = \omega_2$, the amplitude drops to $d/(4\sqrt{2})$

Calculate numerical values for ω_1 and ω_2 that give these results. (One pair of values that work is enough.)

(b) At what driving frequency ω are the cart's oscillations **45° out of phase** with the driving force?
Express your answer algebraically, in terms of $\omega_0 \equiv \sqrt{k/m}$ and β. (One ω value that works is enough.)

Problem 4 : Boing Boing Boing
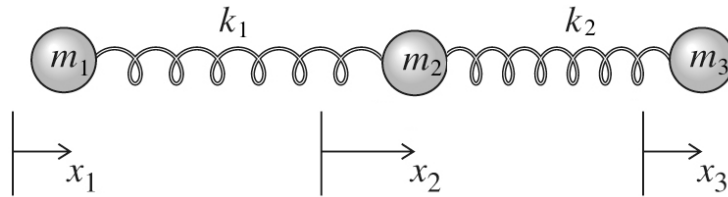
A 3-atom molecule is arranged in a line, and the atoms can only move to the left and to the right.
As shown, the atoms' masses are m_1, m_2, and m_3, and the effective spring constants between them are k_1 and k_2.
The "mass matrix" and "spring matrix" for small oscillations of this molecule around equilibrium are:

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \text{ and } \mathbf{K} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 2 \end{pmatrix} \text{ using the coordinates } \vec{x} \equiv \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

(a) Write down the equation of motion for the acceleration $\ddot{x}_2$ of mass m_2.

(b) Determine the normal mode $\vec{x}(t)$ corresponding to the eigenfrequency $\omega = 1$.

(c) Determine a set of initial conditions (conditions at some time $t = 0$) that would cause the system to move **purely** in its **lowest-frequency** normal mode.

(d) Find the numerical values of k_1, k_2, m_1, m_2, and m_3.

Problem 5 : Boing Boing

Two springs have the same unstretched length d but different spring constants, k_1 and k_2. The springs are tied together end-to-end with a massless piece of string; this <u>combination</u> behaves exactly like a <u>single spring</u> of spring constant k. **Derive** the formula for k in terms of k_1, k_2, and/or d.