Quantitative Analysis on the Tonal Quality of Various Pianos

Michael Chakinis, Swan Htun, Barrett Neath, Brianna Undzis
PHYS 398 DLP - University of Illinois at Urbana-Champaign
26 April 2019
Presentation Outline

● Theory
  ○ Auditory perception
  ○ Tuning methods
  ○ Inharmonicity

● Project Goals

● Methods
  ○ PCB construction
  ○ Recordings
  ○ Analysis

● 4. Results
  ○ Frequency shifts
  ○ Octave correspondence
  ○ Overtone amplitude
  ○ Self-dissonance

● 5. Conclusion

● 6. Discussion
Theory - What makes a chord sound good?

- Inner ear anatomy
  - Cochlear duct is a series of fluid-filled chambers responsible for auditory perception
  - Organ of Corti transforms pressure waves (sound) to electrical nerve signals using cilia
    - Different frequencies excite different regions of cilia → critical bands
Critical Band
Low Frequency versus High Frequency Bandwidth

- 3 dB
- Lower Frequency
- Center Frequency
- Upper Frequency
- Bandwidth

100 Hz

1 hertz

~15%
Theory - Equal temperament

● 12-tone equal temperament adopted in Western classical music for convenience with modern piano design and minimized dissonance
  ○ Other tuning methods can minimize dissonance in certain intervals but would result in increased dissonance in most other intervals
  ○ Equal temperament spreads this dissonance across entire piano

\[ f_n = f_a \left( \sqrt[12]{2} \right)^{(n-a)} \]

● Frequencies of successive notes separated by constant multiplicative factor of \( \sqrt[12]{2} \approx 1.059463 \)
A “pure” tone is characterized by a sine wave oscillating at a single frequency
  ○ Determining consonance and dissonance between two pure tones is as simple as comparing two frequencies

Pianos produce “complex” tones comprised of many frequencies (harmonics)
  ○ Determining consonance and dissonance becomes more complicated

\[ f_n = n f_0 \]
<table>
<thead>
<tr>
<th>Musical Note</th>
<th>Frequency (Hz)</th>
<th>$f_{\text{Note}}/f_{A_3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_3$</td>
<td>220.0</td>
<td>1</td>
</tr>
<tr>
<td>A sharp/B flat</td>
<td>233.1</td>
<td>$2^{1/12}$</td>
</tr>
<tr>
<td>$B_3$</td>
<td>246.9</td>
<td>$2^{2/12}$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>261.6</td>
<td>$2^{3/12}$</td>
</tr>
<tr>
<td>C sharp/D flat</td>
<td>277.2</td>
<td>$2^{4/12}$</td>
</tr>
<tr>
<td>$D_4$</td>
<td>293.7</td>
<td>$2^{5/12}$</td>
</tr>
<tr>
<td>D sharp/E flat</td>
<td>311.1</td>
<td>$2^{6/12}$</td>
</tr>
<tr>
<td>$E_4$</td>
<td>329.6</td>
<td>$2^{7/12}$</td>
</tr>
<tr>
<td>$F_4$</td>
<td>349.2</td>
<td>$2^{8/12}$</td>
</tr>
<tr>
<td>F sharp/G flat</td>
<td>370.0</td>
<td>$2^{9/12}$</td>
</tr>
<tr>
<td>$G_4$</td>
<td>392.0</td>
<td>$2^{10/12}$</td>
</tr>
<tr>
<td>A sharp/B flat</td>
<td>415.3</td>
<td>$2^{11/12}$</td>
</tr>
<tr>
<td>$A_4$</td>
<td>440.0</td>
<td>$2^{12/12}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Note</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_4$</td>
<td>261.6 Hz</td>
<td>523.2 Hz</td>
<td>784.8 Hz</td>
<td>1046 Hz</td>
</tr>
<tr>
<td>$C_5$</td>
<td>523.2 Hz</td>
<td>1046 Hz</td>
<td>1570 Hz</td>
<td>2093 Hz</td>
</tr>
<tr>
<td>$F_4$</td>
<td>349.2 Hz</td>
<td>698.4 Hz</td>
<td>1048 Hz</td>
<td>1397 Hz</td>
</tr>
<tr>
<td>F sharp</td>
<td>370.0 Hz</td>
<td>740.0 Hz</td>
<td>1110 Hz</td>
<td>1480 Hz</td>
</tr>
</tbody>
</table>
Theory - Inharmonicity

- The frequencies of harmonics begin to drift from integer multiples of the fundamental
  - Rigidity of piano does not propagate sound waves efficiently (acoustical impedance)
- Amount of inharmonicity is dependent on instrument/string characteristics (tension, stiffness, length)
- More elasticity = less inharmonicity
Project Goals

1. Quantitatively determine the differences between a tuned and an untuned piano

2. Determine the effect of frequency shift, octave correspondence, overtone amplitude, and self-dissonance on the tonal quality of a piano
Methods

- **Hardware**
  - PCB
    - Arduino microcontroller
  - Sensors
    - Electret microphone
    - LCD
    - Keypad
    - Current sensor
    - Mono amplifier
    - RTC
    - BME 680
    - SD breakout
Methods Continued

- Types of recordings
  - Tuned and untuned
    - Steinway
      - Grand
    - Yamaha
      - Upright and grand
    - Mason & Hamlin
      - Grand
  - Recently tuned and not recently tuned
  - Krannert Center for Performing Arts
Methods Continued

- Recording procedure
  - Originally every key and middle C (C4)
    - Pedals: sustain, damper, staccato
    - Similar information from subsequent octaves
  - Changed to octaves C2, C4, and C5 and middle C
    - Orange, green, indigo
    - Black and white
    - Only analyzed white keys
    - Allowed time between notes
Methods Continued

● Offline analysis
  ○ Python
  ○ Arduino to SD as binary
  ○ Binary to wave
    ■ Gollin’s code
  ○ Graph wave file
    ■ Amplitude vs. time
  ○ Duration of each note
  ○ Cut file for each note
    ■ Numpy FFT
      • Forward Discrete Fourier Transform
        ○ Acoustic power coefficient
      • Computes frequencies corresponding to coefficients
Methods Continued

Theoretical Fundamental Frequency: 82.41 Hz
Measured Fundamental Frequency: 81.4966 Hz
Methods Continued

Spectrogram
- C2 Scale, tuned Steinway
- Data transformed from time domain to frequency domain
  - Fourier transform
- Vertical line
  - Notes
- Color - intensity
Results

- General FFT
- Frequency shifts
- Octave correspondence
- Overtone amplitude
- Self-dissonance
Fast Fourier Transform

- As mentioned before, a FFT brings the audio file from the time domain into the frequency domain
- Using a FFT will produce frequency peaks where the fundamental pitch resides
- The tonal quality of a piano can be analyzed by using the difference between the measured and theoretical fundamental frequency
Fast Fourier Transform on C-Major Scale

[Graph showing Fast Fourier Transform of Tuned Yamaha Concert Grand C-Major Scale C4 Octave with peaks at C4, D4, E4, F4, G4, A4, B4, and C5.]
Frequency Shifts

- They are the largest contributor to impurities in tonal quality.
- When the frequency of a note deviates noticeably from its equal tempered frequency, it is perceived as sharp or flat
  - A frequency above the fundamental is sharp
  - A frequency below the fundamental is flat
Frequency Shifts Cont.

Fast Fourier Transform of D2 of a Tuned and Untuned Grand Steinway

Fast Fourier Transform of D4 of a Tuned and Untuned Grand Steinway
Frequency Shifts Cont.

Fast Fourier Transform of D6 of a Tuned and Untuned Grand Steinway

<table>
<thead>
<tr>
<th>Note</th>
<th>Grand Steinway (Tuned) 1st Fundamental (Hz)</th>
<th>Grand Steinway (Untuned) 1st Fundamental (Hz)</th>
<th>Abs. Diff (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C2</td>
<td>64.3134</td>
<td>64.7931</td>
<td>0.4797</td>
</tr>
<tr>
<td>D2</td>
<td>72.6644</td>
<td>72.4679</td>
<td>0.1965</td>
</tr>
<tr>
<td>E2</td>
<td>81.4966</td>
<td>81.9685</td>
<td>0.4719</td>
</tr>
<tr>
<td>F2</td>
<td>86.0679</td>
<td>86.5395</td>
<td>0.4716</td>
</tr>
<tr>
<td>G2</td>
<td>96.5554</td>
<td>97.1446</td>
<td>0.5892</td>
</tr>
<tr>
<td>A2</td>
<td>108.647</td>
<td>110.227</td>
<td>1.58</td>
</tr>
<tr>
<td>B2</td>
<td>122.037</td>
<td>123.212</td>
<td>1.175</td>
</tr>
<tr>
<td>C5</td>
<td>519.385</td>
<td>518.113</td>
<td>1.272</td>
</tr>
<tr>
<td>C6</td>
<td>1042.58</td>
<td>1048.58</td>
<td>6</td>
</tr>
<tr>
<td>D6</td>
<td>1168.99</td>
<td>1176.3</td>
<td>7.31</td>
</tr>
<tr>
<td>E6</td>
<td>1313.78</td>
<td>1322.94</td>
<td>9.16</td>
</tr>
<tr>
<td>F6</td>
<td>1393.56</td>
<td>1400.1</td>
<td>6.54</td>
</tr>
<tr>
<td>G6</td>
<td>1564.4</td>
<td>1576.34</td>
<td>11.94</td>
</tr>
<tr>
<td>A6</td>
<td>1758.03</td>
<td>1770.13</td>
<td>12.1</td>
</tr>
<tr>
<td>B6</td>
<td>1973.52</td>
<td>1990.78</td>
<td>17.26</td>
</tr>
</tbody>
</table>
Octave Correspondence

- Primary method used to tune pianos
  - Align the second harmonic of C4 with first fundamental of C5
Octave Correspondence Cont.

Fast Fourier Transform of C4 & C5 of a Tuned Grand Steinway Piano

Fast Fourier Transform of C4 & C5 of an Untuned Grand Steinway Piano
Overtone Amplitude

- The perceived frequency and tone of a note is due to the prevalence of its harmonic.
- When the acoustic power of a note’s upper harmonics begin to exceed that of its fundamental, the frequency of the fundamental begins to get overpowered.
Self-Dissonance

- When a piano is out of tune, a listener can often hear beats when it’s played
  - Two or more tones of similar frequencies interfering with each other
- An untuned piano can display doublet shaped peaks, whereas a tuned piano has a single peak
- Doublet shape is caused by dissonance.
  - Cannot form in lower octaves (one string per note)
  - Middle and upper octaves have multiple strings per note
Self-Dissonance

Fast Fourier Transform of Untuned Yamaha Concert Grand C6

Fast Fourier Transform of Tuned Yamaha Concert Grand C6
Discussion

- Sources of error
- Adjustments for future experiments
- Design proposal
Sources of Error

- Not all results are standardized across all four devices
- FFT peak values were determined manually
- More tuned than untuned pianos were recorded
Future Improvements

- Automating the code to generate the FFT peak value
- A higher quality microphone could be used
- Recording barometric pressure, temperature, and humidity may be useful
- Focus on a single piano for an extended period of time
Design Proposal

- This analysis can be used for a variety of piano technician needs
  - Piano appraisal
  - Training piano tuners
  - Verifying tonal quality before concerts

- The methods used in this paper can be used to create a software for personal use
  - Takes a scale as an input
  - Eliminates white noise
  - Analyzes FFT
  - Generates and compares Railsback curve
Conclusion

- Perceived tonal quality doesn’t entirely depend on frequency shifts
- Tuned pianos exhibit small frequency differences, strong octave correspondence, smooth overtone amplitude patterns, and low self dissonance
- Untuned pianos exhibit large frequency differences, poor octave correspondence, erratic overtone amplitude patterns, and noticeable self-dissonance
References