Microwave cavities

Physics 401, Fall 2013
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Agenda

• Waves in waveguides
• Standing waves and resonance
• Setup
• Experiment with microwave cavity
Reminder: Propagation of Plane Waves

Maxwell's Equations

\[ \nabla \vec{D} = 0 \]
\[ \nabla \vec{B} = 0 \]
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]
\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \]

uniform plane wave traveling in z-direction \( \Rightarrow \vec{H} \perp \vec{E} \)

\[ \vec{E}_x = E_0 e^{i(\omega t - k z)} \]

wave equation

\[ \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2} \]

general form of solution

\[ E_z(z,t) = f\left(t - \frac{z}{v}\right) + g\left(t + \frac{z}{v}\right) \]

propagation speed

\[ v = \frac{1}{\sqrt{\varepsilon \mu}} \]
\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \]

E vs H

\[ H_y = \sqrt{\frac{\varepsilon}{\mu}} E_x \]
\[ E_x = Z H_y \]
Wave Propagation in Wave Guides

\[ E_y = E_0 \sin k_x x \cdot e^{i(\omega t - kz)} \]
Standing Waves in Cavities

\[ E_y = E_y(z) \]

\[ E_y = E_y(x \text{ or } z) \]
Resonances for transverse Electric Waves

\[ \omega_{mnp}^2 = v_0^2 \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 + \left( \frac{p\pi}{c} \right)^2 \right] \]

-\textit{phase velocity}

**TE}_{101} \text{ mode: } m=1, n=0, p=1

\[ \omega_{101}^2 = v_0^2 \pi^2 \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{1}{c} \right)^2 \right] \]
Equivalent Circuit

coaxial wave guide

inner conductor
outer conductor

coupling loop

cavity

line

$Z_0$

Impedance of wave guide

11/4/2013
Coupling between Wave Guide and Cavity

**Impedance of wave guide**

\[ Z_0 \]

\[ Q_L = \frac{\omega L}{R + Z_0} \]

\[ Q_L = \frac{\omega L}{R \left(1 + \frac{Z_0}{R}\right)} = \frac{Q_0}{(1 + \beta)} \quad , \]

\( \beta \): coupling coefficient

Maximum power transfer:

\[ Z_0 = R \rightarrow \beta = 1 \]

\[ \Rightarrow Q_L = \frac{1}{2} Q_0 \quad , \]

quality factor without external load
Microwaves in Cavities. Overview of the Experiment.
Microwaves in Cavities. The Setup of the Experiment.
Experiment. Wavelength measurement.

Use detector to find distance between minimums in the slotted line (wave guide)
Use detector to find distance between minimums in the slotted line (wave guide). Distance between consequent minima correspond $\lambda/2$. 

Experiment. Wavelength measurement.
Experiment. Cavity resonance.

Use plunger to change the dimension of the cavity in z-direction and search for maxima in power stored using the cavity detector. Identify TE_{101} and TE_{102}.
Experiment. Cavity resonance.

\[ \omega_{102}^2 = \nu_0^2 \pi^2 \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{2}{c} \right)^2 \right] \]

\[ f_{102} = \frac{\nu_0}{2} \sqrt{\left( \frac{1}{a} \right)^2 + \left( \frac{2}{c} \right)^2} \]

**Graphs:**
- **Graph 1:** TE_{102}
  - \( c_0 = 13.88 \)
  - \( c_1 = 13.86 \)
  - \( c_2 = 13.92 \)
  - \( I \) vs \( l \) (cm)
- **Graph 2:** TE_{102}
  - \( Q = \frac{f_0}{\Delta f} \sim 450 \)
  - \( \Delta f \) vs \( f \) (GHz)
Experiment. Cavity resonance.

By moving the plunger we changing the resonance frequency of the cavity.

Frequency of the oscillator
Experiment. Cavity resonance. Oscillator tuning.

Wavetek FG

\[ V_m \]

\[ 10V \]

\[ 0 \]

Sync out

\[ V_{\text{Tune IN}} \]

\[ \mu \text{Wave OUTPUT} \]

\[ V_{\text{Tune OUT}} \]

Scope is in X-Y mode

ch1

ch2

detector

Cavity
1. Oscilloscope should run in X-Y mode
2. To plot the I(f) dependence you have to download both Ch1 and Ch2 data
3. Use triangular waveform as a voltage applied to modulation input of the oscillator
4. Use a proper time scale setting on the scope which could estimated from scanning frequency
5. Apply the calibration equation to calculate the frequency of the oscillator from the modulation voltage

\[ f = 0.03706 V_{mod} + 2.9349 \]
Experiment. Cavity resonance. Oscillator tuning.

Voltage tunable oscillator ZX95-3250a-S+ from Mini-Circuits
FM Calibration for microwave oscillator

**ZX95-3250a-S+**

**Equation**
\[ y = \text{Intercept} + \frac{B_1 \times x + B_2 \times x^2}{2} \]

**Weight**
No Weighting

**Residual Sum of Squares**
8.01482E-5

**Adj. R-Square**
0.99977

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<thead>
<tr>
<th>frequency</th>
<th>Value</th>
<th>Standard Error</th>
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<tr>
<td>Intercept</td>
<td>2.91874</td>
<td>8.0905E-4</td>
</tr>
<tr>
<td>B1</td>
<td>0.03588</td>
<td>3.43148E-4</td>
</tr>
<tr>
<td>B2</td>
<td>-4.41E-5</td>
<td>3.20212E-5</td>
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**f (GHz)**

**V_Tune IN (V)**
Experiment. Cavity resonance.

- TE101
- $Q = 340$
- $Q = 133$
By changing the coupling between oscillator and cavity, we can control the quality factor of the cavity resonance. However, at the same time, we change the power delivered to the cavity.
While in resonance: turn orientation of the input loop from the vertical direction in 10° steps to 360°. Read cavity detector.
Experiment. Coupling: Detecting of the Magnetic field.

Experimental result. Fitted to $A|\cos(\alpha + \phi)|^n + A_0$
Presence of dielectric reduces length of cavity at a given resonance frequency $\omega_0$.
This effect grows with the electric field strength $E_y$.

(0) Without dielectric the cavity length at resonance is $c_0$.
(1) Place dielectric into cavity and move in 0.5cm steps, $l_i$.
(2) At each place tune plunger to resonance and record $c_i$.
(3) Plot $\Delta c_i = |c_0 - c_i|$ versus $l_i$ : this measures now $E_y$ vs $l_i$. 
Electric Field Distribution.

**Electric field distribution**

**TE$_{102}$ mode**

- Cavity retune (cm)
- Dielectric position z (cm)

*Courtesy of P. Debevec*
Electric Field Distribution.

A small writeup correction:

4. Part IV: (a) Plot $\Delta c_\ell$ versus $\ell$. (b) On the same graph, plot $\sin\left(\frac{\pi \ell}{c}\right)$ versus $\ell$. (c) This works for TE101 mode but more general it should be: $\sin\left(\frac{p\pi \ell}{c}\right)$ where $p$ is mode index (TE$_{mnp}$); $p=1$ for TE$_{101}$ and $p=2$ for TE$_{102}$ modes.
Calculation of the Quality factor of the Unloaded Cavity

Quality factor ($\text{TE}_{101}$ mode) of unloaded cavity can be calculated as:

$$Q_0 = \frac{abc(a^2 + c^2)}{\delta \left[ 2b(a^3 + c^3) + ac(a^2 + c^2) \right]}$$

$\delta$ is the skin depth at frequency $\omega_0$

$$\delta = \sqrt{\frac{2\rho}{\mu\omega}}$$

$\rho$ – resistivity of the cavity material

$\mu = \mu_r\mu_0 \approx \mu_0 = 4\pi \times 10^{-7}$

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Calculation of the Quality factor of the Unloaded Cavity

For red brass $\rho = 6 \times 10^{-8} \Omega m$
$\mu \approx 4\pi \times 10^{-7}$
$\delta = 2.25 \times 10^{-6} m$

$a = 7.22 cm$, $b = 3.42 cm$, $c = 6.91 cm$ (TE$_{101}$)

$$\delta = \sqrt{\frac{2\rho}{\mu\omega}}$$

$$Q_0 = \frac{abc\left(a^2 + c^2\right)}{\delta\left[2b\left(a^3 + c^3\right) + ac\left(a^2 + c^2\right)\right]}$$

$Q_0 \approx 7700$